Mimetic Gravity

Alexander Vikman

Institute of Physics
of the Czech Academy of Sciences

Fyzikální ústav
Akademie věd ČR, v. v. i.

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This talk is mostly based on

- arXiv: **1512.09118,**
  K. Hammer, A. Vikman
- arXiv: **1412.7136,** JCAP 1506 (2015) 06, 028
  L. Mirzagholi, A. Vikman
- arXiv: **1403.3961,** JCAP 1406 (2014) 017
  A. H. Chamseddine and V. Mukhanov, A. Vikman
- arXiv: **1003.5751,** JCAP 1005 (2010) 012
  I. Sawicki, E. Lim, A. Vikman

Mimetic Matter

`Chamseddine, Mukhanov (2013)`

- One can *encode* the conformal part of the *physical* metric in a scalar field:

$$g_{\mu\nu} = \tilde{g}_{\mu\nu}\left(\tilde{g}^{\alpha\beta}\,\partial_\alpha\phi\,\partial_\beta\phi\right)$$

physical metric of free fall

auxiliary metric, dynamical variable

$$S\left[\tilde{g}_{\mu\nu},\phi,\Phi_m\right] = \int \mathrm{d}^4x\left[\sqrt{-g}\left(-\frac{1}{2}R\left(g\right) + \mathscr{L}\left(g,\Phi_m\right)\right)\right]_{g_{\mu\nu}=g_{\mu\nu}(\tilde{g},\phi)}$$

“matter”

Mimetic Matter

Chamseddine, Mukhanov (2013)

- $$g_{\mu\nu} = \tilde{g}_{\mu\nu} \left(\tilde{g}^{\alpha\beta}\,\partial_\alpha\phi\,\partial_\beta\phi\right)$$

physical metric of free fall *auxiliary metric, dynamical variable*

- The theory becomes invariant with respect to Weyl transformations:

$$\tilde{g}_{\mu\nu} \rightarrow \Omega^2(x)\,\tilde{g}_{\mu\nu}$$

- The scalar field obeys the relativistic Hamilton-Jacobi equation:

$$g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi = 1$$

$$g^{\mu\nu} = \tilde{g}^{\mu\nu}\left(\tilde{g}^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi\right)^{-1}$$

the Hamilton-Jacobi equation

$$g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi = 1$$

corresponding four-velocity

$$u_\mu = \partial_\mu\phi$$

is geodesic

$$a_\mu = u^\lambda \nabla_\lambda u_\mu = \nabla^\lambda\phi\left(\nabla_\lambda\nabla_\mu\phi\right) = \frac{1}{2}\partial_\mu\left(\partial\phi\right)^2 = 0$$

Modification of the Einstein equation

EMT for all other matter

$$\frac{\delta S}{\delta \tilde{g}^{\mu\nu}} \quad G_{\mu\nu}(g) - T_{\mu\nu}(g) - (G(g) - T(g))\,\partial_\mu\phi\partial_\nu\phi = 0$$

3rd order equation for ϕ

cf Masahide Yamaguchi talk

c.f. Einstein equations with dust or DM:

$$G_{\mu\nu} = T_{\mu\nu} + \rho u_\mu u_\nu$$

$$\rho = G - T$$

Dark Matter

energy density: ρ

4-velocity: $u_\mu = \partial_\mu\phi$

$$\frac{\delta S}{\delta \phi}$$

$$\left[(G(g) - T(g))\,\phi^{;\mu}\right]_{;\mu} = 0$$

3rd order equation for $\tilde{g}_{\mu\nu}$

$$S\left[\tilde{g}_{\mu\nu},\phi,\Phi_m\right] = \int \mathrm{d}^4x \left[\sqrt{-g}\left(-\frac{1}{2}R\left(g\right)+\mathscr{L}\left(g,\Phi_m\right)\right)\right]_{g_{\mu\nu}=g_{\mu\nu}\left(\tilde{g},\phi\right)}$$

with $g_{\mu\nu}\left(\tilde{g},\phi\right) = \tilde{g}_{\mu\nu}\,\tilde{g}^{\alpha\beta}\,\partial_\alpha\phi\,\partial_\beta\phi$

Brans-Dicke $\omega = -3/2$

$$S_0\left[\tilde{g},\phi,\Phi_m\right] = -\int \mathrm{d}^4x\,\sqrt{-\tilde{g}}\left(X\,R\left(\tilde{g}\right)+\frac{3}{2}\cdot\frac{\tilde{g}^{\alpha\beta}X_{,\alpha}X_{,\beta}}{X}-\mathscr{L}_m\left(\tilde{g},\phi,\Phi_m\right)\right)\,,$$

where $X = \frac{1}{2}\tilde{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}$

is *not* in the Horndeski (1974) construction of the most general scalar-tensor theory with *second order* equations of motion

But it is still a system with one degree of freedom + standard two polarizations for the graviton!

Lagrange Multiplier and Weyl-Invariance

Hammer, Vikman (2015)

$$S_0\left[\tilde{g},\phi,X,\lambda\right] = -\int \mathrm{d}^4x\,\sqrt{-\tilde{g}}\left[X\,R\left(\tilde{g}\right) + \frac{3}{2}\cdot\frac{\tilde{g}^{\alpha\beta}X_{,\alpha}X_{\beta}}{X} + \lambda\left(X - \frac{1}{2}\tilde{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}\right)\right]$$

Weyl-invariance:

$$\tilde{g}_{\mu\nu} \rightarrow \Omega^2\left(x\right)\tilde{g}_{\mu\nu}\,,$$

$$\phi \rightarrow \phi\,,$$

$$X \rightarrow \Omega^{-2}\left(x\right)X\,,$$

$$\lambda \rightarrow \Omega^{-2}\left(x\right)\lambda\,,$$

gauge invariant variables

$$\tilde{g}_{\mu\nu} = \left(2X\right)^{-1} g_{\mu\nu}\,,$$

$$\phi = \phi\,,$$

$$X = X\,,$$

$$\lambda = 2X\,\rho$$

$$S_0[g,\phi,\rho] = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}R\left(g\right) + \frac{\rho}{2}\left(g^{\alpha\beta}\phi_{,\alpha}\phi_{\beta} - 1\right)\right]\,.$$

Mimetic Dark Matter via Lagrange Multiplier

Chamseddine, Mukhanov; Golovnev; Barvinsky (2013)
Lim, Sawicki, Vikman (2010);

- constraint via Lagrange multiplier $\rho\left((\partial\phi)^2 - 1\right)$

$$S\left[g,\phi,\rho,\mathrm{SM}\right] = \int \mathrm{d}^4x\sqrt{-g}\left(-\frac{1}{2}R + \frac{1}{2}\rho\left((\partial\phi)^2 - 1\right) + \mathscr{L}_{\mathrm{SM}}\right)$$

$$T_{\mu\nu} = \rho\, u_\mu u_\nu$$

Dark Matter

Lagrange multiplier is the energy density

$$u_\mu = \partial_\mu\phi$$

- dust / DM via Lagrange multiplier

$$\rho\left((\partial\phi)^2 - 1\right)$$

- Cosmological Constant / DE via Lagrange multiplier

$$\Lambda\left(\nabla_\mu V^\mu - 1\right)$$

`M. Henneaux and C. Teitelboim (1989)`

Disformal Transformation

Nathalie Deruelle and Josephine Rua (2014), Domènech et al. (2015)

One obtains the same dynamics
(***the same Einstein equations***),
if instead of varying the Einstein-Hilbert action
with respect to the metric $g_{\mu\nu}$

one plugs in a *disformal transformation* (Bekenstein 1993)

$$g_{\mu\nu} = F\left(\Psi, w\right)\ell_{\mu\nu} + H\left(\Psi, w\right)\partial_\mu\Psi\partial_\nu\Psi$$

with $w = \ell^{\mu\nu}\partial_\mu\Psi\partial_\nu\Psi$ and $w^2 F\frac{\partial}{\partial w}\left(H + \frac{F}{w}\right) \neq 0$

and varies with respect to $\ell_{\mu\nu},\ \Psi$

Mimetic gravity is an exception! And it does provide new dynamics!

Next term in the gradient expansion

Chamseddine, Mukhanov, Vikman (2014)

$$\gamma\left(\Box\phi\right)^{2}$$

There are ***no new degrees of freedom***, because higher time derivatives can be eliminated by differentiating the Hamilton-Jacobi equation.

$$S_{\gamma}\left[\tilde{g},\phi\right]=-\int\mathrm{d}^{4}x\,\sqrt{-\tilde{g}}\left[X\,R\left(\tilde{g}\right)+\frac{3}{2}\cdot\frac{\tilde{g}^{\alpha\beta}X_{,\alpha}X_{,\beta}}{X}-\frac{1}{2}\gamma\left(\Box_{\tilde{g}}\varphi+\frac{\tilde{g}^{\alpha\beta}\phi_{,\alpha}X_{\beta}}{X}\right)^{2}\right],$$

Hammer, Vikman (2015)

sound speed $c_s^2=\dfrac{\gamma}{2-3\gamma}$

Unitary gauge without any other matter and projectable Horava-Lifshitz gravity

$$\phi = t$$

$$ds^2 = N^2 dt^2 - \ell_{ik}\left(dx^i + N^i dt\right)\left(dx^k + N^k dt\right) \ ,$$

constraint / Hamilton-Jacobi equation $N = 1$

$$S_\gamma = S_{\mathrm{HL}} + S_\Delta$$

$$S_\Delta\left[N, N_i, \ell_{ik}\right] = \frac{\gamma}{2}\int dt d^3x N\sqrt{\ell}\left[\frac{1}{N^2}\left(K - \frac{\dot{N}}{N^2} + \frac{N^i\partial_i N}{N^2}\right)^2 - K^2\right]$$

where we denote extrinsic curvature $K_{ik} = \dfrac{1}{2N}\left(\partial_t \ell_{ik} - D_i N_k - D_k N_i\right)$

$$\frac{\delta S_\Delta}{\delta N} = \gamma\sqrt{\ell}\left[\partial_t\left(NK\right) - D_i\left(\left(NK\right)N^i\right) - \left(NK\right)^2\right] \ ,$$

Just redefines ρ

$$\frac{\delta S_\Delta}{\delta N^i} = \frac{\delta S_\Delta}{\delta \ell_{ik}} = 0 \, .$$

Conclusions and Further Directions

- New large class of higher-derivative Weyl-invariant scalar-tensor theories beyond Horndeski, *cf Yamaguchi's talk*
- Can be fluid-like DM on cosmological scales *cf Ivanov's talk*
- Caustics? *cf Babichev's talk*
- Practically the same as the low energy limit of the Horava-Lifshitz gravity, *see Ramazanov's talk*
- Galactic Halos?
- Gravitational collapse?
- Accretion of the mimetic DM?

Thanks a lot for attention!