Living with ghosts in Hořava–Lifshitz gravity

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Based on works made in collaboration with
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General Relativity

Extremely powerful at 0.1 mm $\lesssim l \lesssim$ kpc:

perihelion of Mercury, bending of light, gravitational waves etc.

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- Galaxy rotation curves $\Longrightarrow$ Dark Matter. BSM particles?
- Accelerated expansion of the Universe $\Longrightarrow$ Dark Energy.

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$G = M_{Pl}^{-2} \Longrightarrow$ Gravity is not (perturbatively) renormalizable.

Problem manifests at least at $l \lesssim M_{Pl}^{-1} \sim 10^{-33}$ cm

Hořava-Lifshitz gravity: basic idea — Hořava'09

$$\mathbf{x} \to b\mathbf{x} \qquad t \to b^z t \qquad \text{NB: } z = 1 \Longrightarrow \omega^2 = \mathbf{p}^2$$

$$z = 1 \to 3 \Longrightarrow \omega^2 \to \mathbf{p}^2 \ (IR) \qquad \omega^2 \to \frac{\mathbf{p}^6}{M_{UV}^4} \ (UV)$$

M_{UV} is some new fundamental scale. $\mathcal{L}_{GR} \to \mathcal{L}_{GR} + \frac{1}{M_{UV}^4} R\Delta R + ...$

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$$\frac{1}{\omega^2 - \mathbf{p}^2} \longrightarrow \frac{1}{\omega^2 - \frac{\mathbf{p}^6}{M_{UV}^4}}$$

$z = 3 \Longrightarrow$ power-counting renormalizable theory of gravity

NB In particle sector $M_{LV} \gtrsim 10^{10}$ GeV.

Problems with recovery of GR at large scales!

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Start from bad new: $g^{00} = 1 + 2\Phi$ is a degree of freedom in HL gravity.

Φ is a tachyon in generic inhomogeneous backgrounds $\Longrightarrow$ strong coupling.

Charmousis et al'09 Blas et al'09

In GR Φ is non-dynamical -can be switched to 0 by the gauge choice.

Projectable HL model

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Germani et al'09 Blas et al'09

$$g^{00} \to g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$$

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NB By far, projectable HL gravity is the only renormalizable model of gravity in 4 dimensions. Barvinsky et *al*'15

NB The other ways to cure the problem exist Blas et al'09

Low energy limit $|\mathbf{p}| \ll M$: GR + (Imperfect) 'Fluid'

$$S_{IR} = S_{GR} + \int d^4x\sqrt{-g} \cdot \frac{\Sigma}{2} \cdot (g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 1) + \int d^4x\sqrt{-g} \cdot \frac{\gamma}{2}(\Box\varphi)^2$$

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NB $\quad S_{IR} \supset -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\cdot\lambda\cdot K^2 \qquad \frac{1-\lambda}{8\pi G} = \gamma \qquad \gamma \ll M_{Pl}^2$

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$$\Longrightarrow T_{\mu\nu} = \Sigma\cdot\partial_\mu\varphi\partial_\nu\varphi + \frac{\gamma}{2}(\Box\varphi)^2 g_{\mu\nu} + \ldots$$ Imperfect Dark Matter

Discussed in the talk by Alex Vikman

Chamseddine et al'13 '14 Capela and SR'14 Mirzagholi and Vikman'15

Dark Matter from projectable HL originally by S. Mukohyama'09

Tensor/Scalar modes

Tensor modes are unaffected by the 'Fluid'.

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$$ds^2 = (1+2\Phi)dt^2 - (1-2\Psi)d\mathbf{x}^2 \qquad \Psi = \Phi$$

$$\ddot{\Psi} + c_s^2 \mathbf{p}^2 \Psi = 0 \ .$$

γ-term leads to non-zero sound speed Chamseddine et *al.*'14

$$\omega^2 = c_s^2 \mathbf{p}^2 \qquad \boxed{c_s^2 = 4\pi\gamma G}$$

Quantum structure of projectable HL gravity

Quadratic action for the potential Ψ,

$$S_\Psi = \frac{1}{8\pi G}\int d^4x \left(-\frac{1}{c_s^2}\dot{\psi}^2 - \Psi\Delta\Psi\right) \Longrightarrow$$

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- $c_s^2 > 0 \Longrightarrow$ ghosts (Ψ-particles carry negative energy)
- $c_s^2 < 0 \Longrightarrow$ tachyons $\Longrightarrow \Psi \propto e^{|\omega|t}$ (potential blows up exponentially)

Ghosts

Particles with negative energy states lead to the vacuum decay.

Say, Vacuum $\to$ 2 photons + 2 ghosts

In Lorentz-invariant theories: decay rate $\Gamma \sim \frac{1}{M_{Pl}^4} \int d^4\mathbf{P} \cdot \mathbf{P}^4 \to \infty$

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In Lorentz-violating theory $|\mathbf{P}| \lesssim \Lambda$

Cline et al'04

In HL UV operators do not cure ghosts $\Longrightarrow \Lambda = M_{UV}$-does not work

Strong coupling

Cutoff Λ=strong coupling scale.

Breakdown of linear theory at $\boxed{\Lambda_p \sim M_{Pl} c_s^{3/2}}$ $\quad \psi^3 \sim \psi^2$

This is a Lorentz-violating cutoff. Blas et al'09 Koyama and Arroja'09

Assumption: no ghosts above strong coupling scale.

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Bad: $\Lambda_p \lesssim M_{UV} \Longrightarrow$ loss of perturbativity $\Longrightarrow$ how to renormalize?

Good: $\boxed{\Lambda_p \gtrsim 10^{-3}\ \text{eV} \sim (1\ \text{mm})^{-1}}$ NB Not fulfilled in HL with tachyons

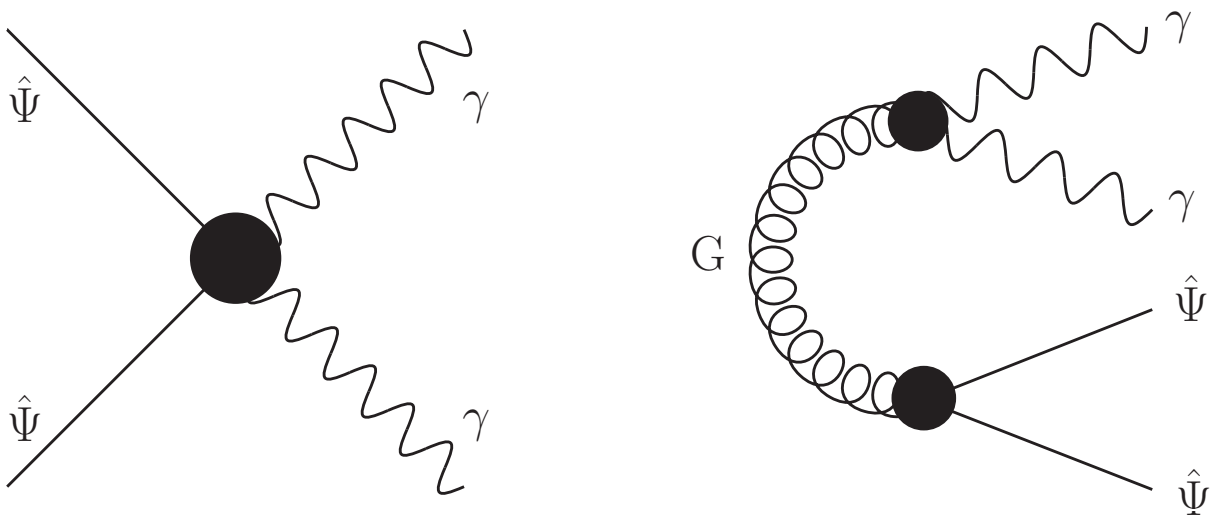
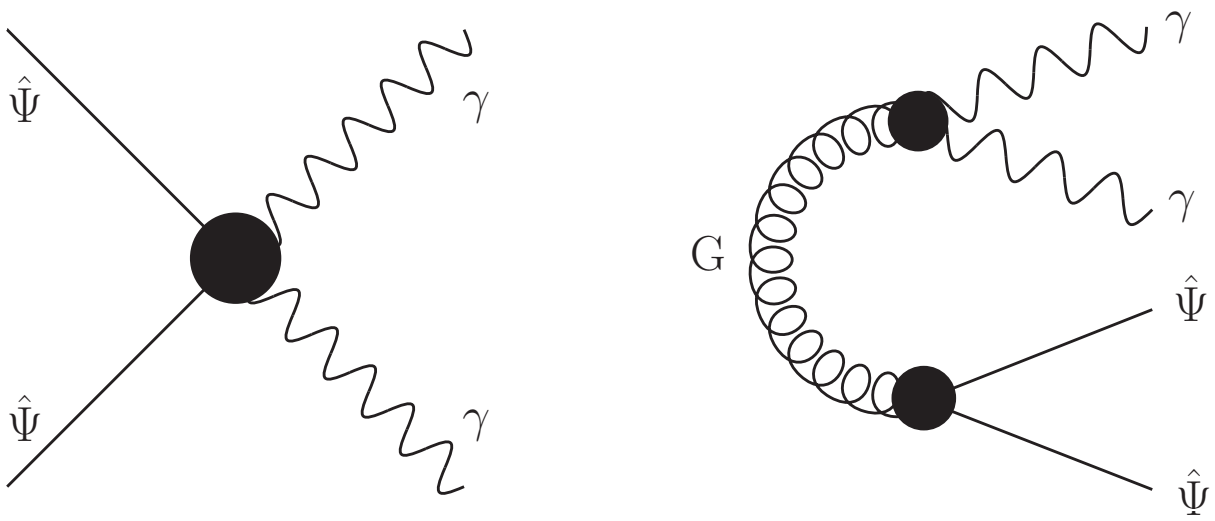


Figure: Diagrams of vacuum decay



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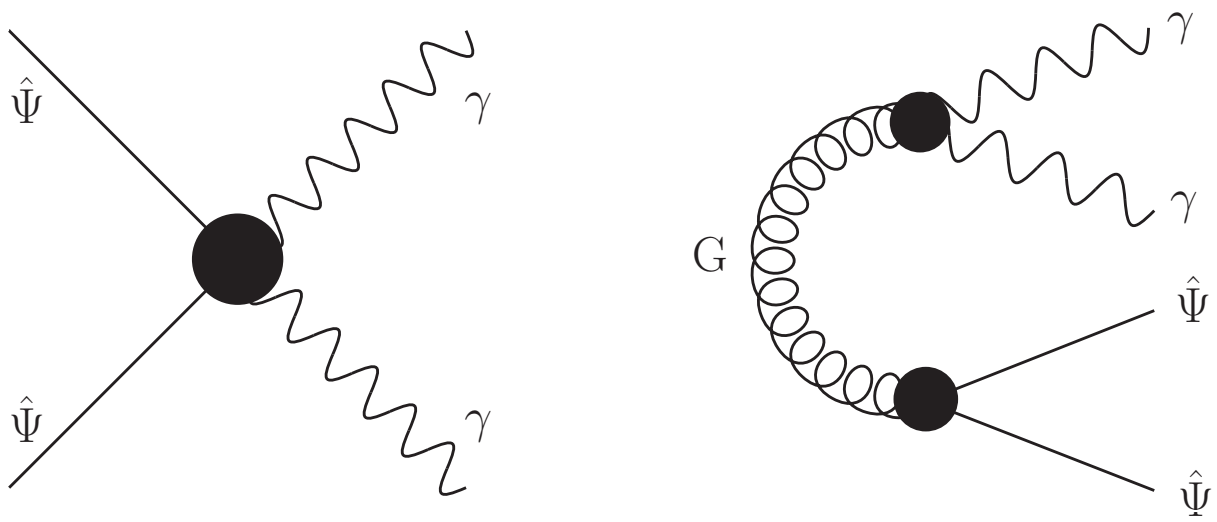

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$$\Psi^2 F_{\mu\nu} F^{\mu\nu} \to \frac{c_s^2}{M_{Pl}^2} \hat{\Psi}^2 F_{\mu\nu} F^{\mu\nu} \qquad \Psi \leftrightarrow h_{\mu\nu} \leftrightarrow \mathcal{A}_\mu$$

$$S_\Psi = \frac{M_{Pl}^2}{8\pi} \left(-\frac{\dot{\Psi}^2}{c_s^2} + \ldots \right) \Longrightarrow \Psi \sim \frac{c_s}{M_{Pl}} \hat{\Psi} \qquad E_{max} \sim c_s \Lambda_p \to 0 \text{ as } c_s \to 0$$

Constraints

We do not want to overproduce cosmic X-rays and gamma ray bursts.

$$F \sim \Gamma \cdot t_0 \lesssim \frac{10 \text{ keV}}{s \cdot cm^2 \cdot E_{ph} \cdot sr} \qquad t_0 \sim H_0^{-1}$$

$$\boxed{\Lambda_p \lesssim 10 \text{ TeV}} \qquad \boxed{c_s^2 \lesssim 10^{-20}}$$

NB In HL with tachyons $|c_s^2| \lesssim 10^{-60} \Longrightarrow \Lambda_p \lesssim 10^{-17}$ eV

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Energies of produced particles $\quad c_s \cdot \Lambda_p \lesssim 1$ keV

NB Vacuum $\to e^+ + e^- + 2$ ghosts is forbidden.

NB Proton decay $p \to n + e^+ + \nu_e +$ ghosts does not go.

Conclusions

- Projectable Hořava–Lifshitz gravity is a viable modification of gravity.
- Reduces to GR plus Dark Matter ('Fluid') in the low energy limit.
- Sound speed squared of DM $c_s^2 \lesssim 10^{-20}$.
- Strong coupling scale $\Lambda_p \lesssim 10$ TeV.
- Problem 1: existence of strong coupling obscures

  UV quantum properties of HL gravity (original motivation is lost).
- Problem 2: How does Dark Matter in HL gravity clusters?

MERCI! GRAZIE!