RENCONTRES DE MORIOND - COSMOLOGY

Expectation of lepton hierarchy and absolute neutrino masses in a microscopic cosmological model

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1- Geometry with time-space symmetry (1)

- **Objectives:** ***To understand origin of the mass hierarchy of leptons and to predict absolute masses of neutrinos.***

- **Motivations:**

1/ Need to search ***for a link*** between: **Curvature ≡Mass**

i.e. between: ***General Relativity*** **(GR)** >< ***Quantum Mechanics*** **(QM),**

Namely: Einstein Gravitational Equation >< Klein-Gordon-Fock Equation.

→ To formulate a "microscopic" cosmological model to describe masses and mass hierarchy of leptons.

2/ Need to search for a link between:

The number of dimensions >< The number of lepton generations.

→ Extending 4D space-time to a symmetrical 6D={3D,3D} time-space to cover all 06 leptons: 03 charged leptons + 03 neutrinos.

*** My approach** is a semi-phenomenological.

1- Geometry with time-space symmetry (2)

- **Our study was motivated by:**
  - **Extra-dimension (ED) dynamics**: Kaluza and Klein.
  - **Semi-classical approach to Quant Mechanics*:*** de Broglie & Bohm. (Evidence of **violation of Bell inequalities** *allows only* non-local hidden parameters).
  - **Time-like Extra-Dimensions (EDs)**:
- ***Anti-de Sitter*** geometry:

  AdS/CFT : Maldacena [1];

  Hierarchy problem : Randall & Sundrum [2]:
- ***Induced matter*** models:

  For Cosmology (added 01 time-like ED): Wesson [3]

  For Particle Physics (added 01 time-like ED): Koch [4].

→ **My presentation, somehow following induced matter model, but added 02 time-like EDs is based on recent studies:** Vo Van Thuan [5-7].

1- Geometry with time-space symmetry (3)

- Considering an ideal ***6D flat time-space*** $\{t_1, t_2, t_3 | x_1, x_2, x_3\}$ consisting of orthonormal sub-spaces 3D-time and 3D-space:

$$\boldsymbol{dS^2 = dt_k^2 - dx_l^2} \text{ ; summation: } k, l = 1 \div 3. \qquad (1)$$

- Our physics works on its ***symmetrical "light-cone"***:

$$\boldsymbol{d\vec{k}^2 = d\vec{l}^2} \quad \text{or} \quad dt_k{}^2 = dx_l{}^2\text{; summation: } k, l = 1 \div 3 \qquad (2)$$

Natural units ($\hbar = c = 1$) used unless it needs an explicit quantum dimension.

- **Introducing a 6D isotropic plane wave equation**:

$$\frac{\partial^2 \psi_0}{\partial t_k^2} = \frac{\partial^2 \psi_0}{\partial x_l^2} \quad ; \qquad (3)$$

- Where $\boldsymbol{\psi}_0(\boldsymbol{t_k}, \boldsymbol{x_l})$ is a harmonic correlation of dt and dx, containing only linear variables $\{\boldsymbol{t_k}, \boldsymbol{x_l}\}$, serving a primitive source of quantum fluctuations in space-time. All chaos of displacements dt and dx can form averaged time-like and space-like potentials V_T and V_X.

1- Geometry with time-space symmetry (4)

- Suggesting that the global potentials, originally, accelerating linear space-time into curved time-space of **a symmetrical bi-spherical geometry** $(\boldsymbol{\psi}, \boldsymbol{\theta}, \boldsymbol{\varphi})$ which describes 3D spinning $\vec{\tau}$ and $\vec{s}$ in symmetrical orthonormal Type equation here.subspaces of 3D-time and 3D-space: $\vec{\tau}$ is a pseudo-spin and $\vec{s}$ is spin.

- **For a kinetic state** (curved rotation + linear translation): $\{t_{3,} x_3\}$ are accepted as longitudinal central axes of a **bi-cylindrical geometry** $(\boldsymbol{\psi}, \boldsymbol{\varphi}, y_l)$;

The curved coordinates for 3D-space: $\{x_j\} \equiv \{x_1, x_2, z\}$ with $dz^2 = dx_n{}^2 + dx_3{}^2$;

Similarly, for 3D-time there are $\{t_i\} \equiv \{t_1, t_2, t\}$ with $dt^2 = dt_0{}^2 + dt_3{}^2$; where combining t_3 with a rescaled parameter t_0 by an orthogonal relationship.

→ t_0 and x_n are local affine parameters in 3D-time and 3D-space, respectively.

EDs turn into dynamical ones depending on other space-time dimensions:

$\boldsymbol{\psi} = \boldsymbol{\psi}(\boldsymbol{t_0}, \boldsymbol{t_3}, \boldsymbol{x_n}, \boldsymbol{x_l})$ and $\boldsymbol{\varphi} = \boldsymbol{\Omega t} - \boldsymbol{k_j x_j} = \boldsymbol{\Omega_0 t_0} + \boldsymbol{\Omega_3}\, \boldsymbol{t_3} - \boldsymbol{k_n x_n} - \boldsymbol{k_l x_l}$; (4)

where: $\boldsymbol{\Omega t} = \boldsymbol{\Omega_0 t_0} + \boldsymbol{\Omega_3}\, \boldsymbol{t_3}$ and $\boldsymbol{k_j x_j} = \boldsymbol{k_1 x_1} + \boldsymbol{k_2 x_2} + \boldsymbol{k_z}\mathrm{z}$; $\boldsymbol{k_z}\mathrm{z} = \boldsymbol{k_n x_n} + \boldsymbol{k_3 x_3}$.

1- Geometry with time-space symmetry (5)

→In particular, observation of an individual lepton ($\boldsymbol{\tau_n}$, $\boldsymbol{s_n} = \pm 1/2$), leads to a symmetrical bi-cylindrical curvature:

$$\boldsymbol{d\Sigma_S}^{\mathbf{2}} = \boldsymbol{dt^2 - d\lambda^2} =$$

$$= [d\psi(t_0, t_k)^2 + \psi(t_0, t_k)^2 d\varphi(t_0, t_k)^2 + dt_k{}^2] -$$

$$-[d\psi(x_n, x_l)^2 + \psi(x_n, x_l)^2 d\varphi(x_n, x_l)^2 + dx_l{}^2]. \qquad (5)$$

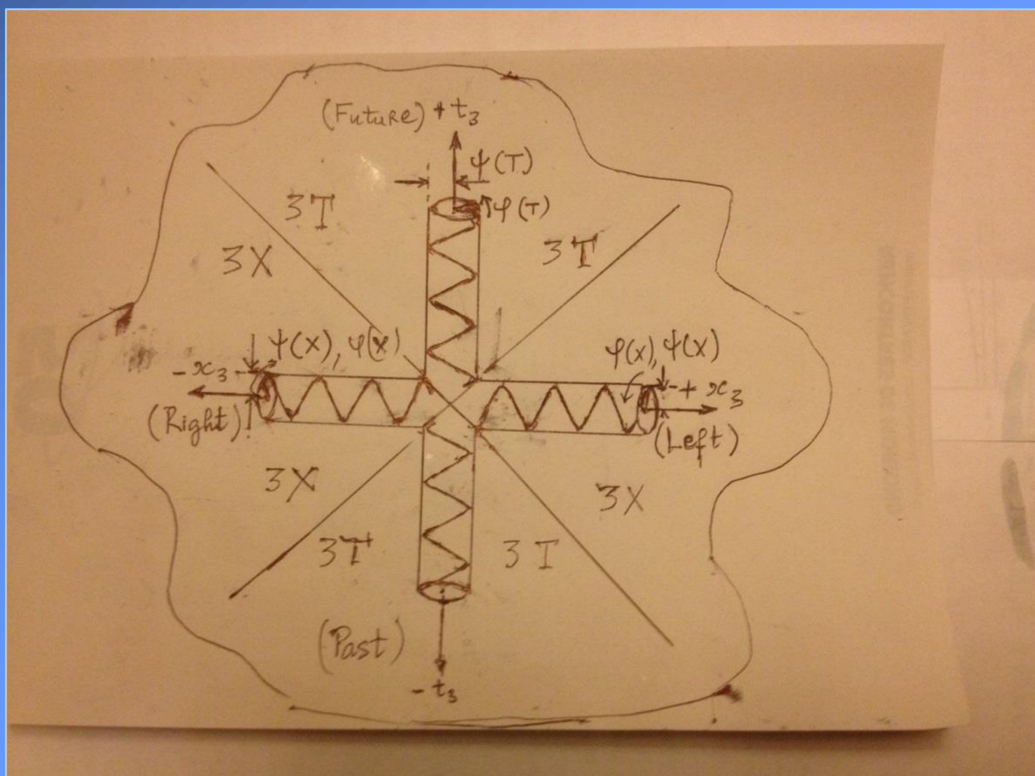


Fig.1.1. Symmetrical bi-cylindrical geometry

1- Geometry with time-space symmetry (6)

- **D**ue to interaction of Higgs-like potential V_T the time-space symmetry is spontaneously broken, forming energy-momentum, and leading to an **asymmetrical bi-cylinder:**

$$d\Sigma_A{}^2 \equiv (ds_0{}^2 + ds_{ev}{}^2) - (d\sigma_{ev}{}^2 + d\sigma_L{}^2) =$$

$$= [d\psi(t_0)^2 + \psi(t_0)^2 d\varphi(t_0)^2 + dt_3{}^2] -$$

$$-[d\psi(x_n)^2 + \psi(x_n)^2 d\varphi(x_n)^2 + dx_3{}^2]\,. \qquad (6)$$

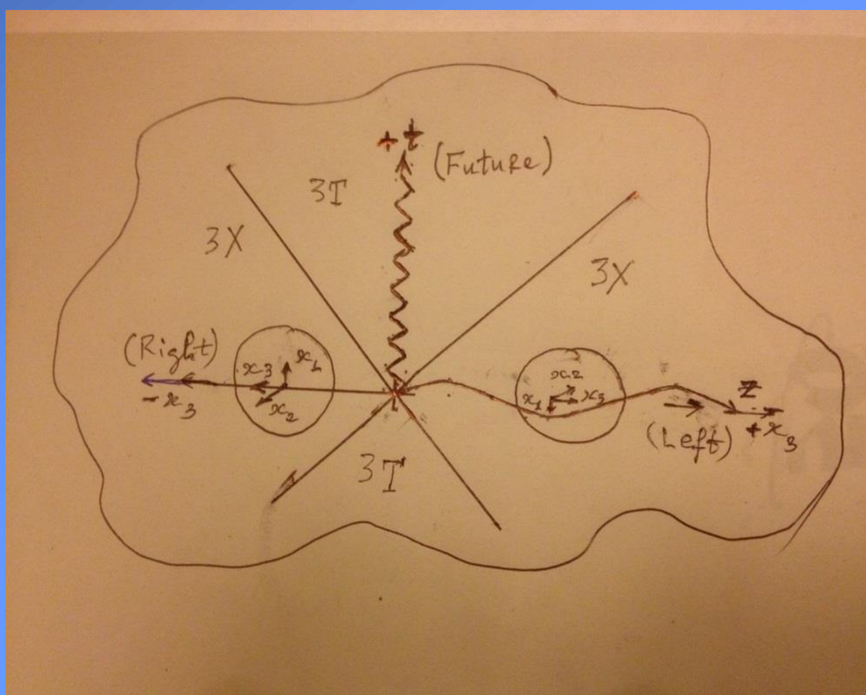


Fig.1.2. Asymmetrical bi-cylindrical geometry:

-Time-like curvature strong and almost absolute;

-Space-like curvatures: odd-term is weak (PNC) and even term is not absolute (quasi-curvature)

1- Geometry with time-space symmetry (7)

❑ $$d\Sigma_A{}^2 \equiv (ds_0{}^2 + ds_{ev}{}^2) - (d\sigma_{ev}{}^2 + d\sigma_L{}^2) =$$
$$= [d\psi(t_0)^2 + \psi(t_0)^2 d\varphi(t_0)^2 + dt_3{}^2] -$$
$$-[d\psi(x_n)^2 + \psi(x_n)^2 d\varphi(x_n)^2 + dx_3{}^2]\,. \qquad (6^*)$$

Where:

- ds_0 as an odd-term is a normal interval;
- $d\sigma_{ev}$ is 3D-space pseudo-interval characterizing P-*even contribution of spinning* $\vec{s}$;
- $d\sigma_L$ *is P-odd contribution* of intrinsic space-like curvature s_L // x_l (*left-handed helicity*);

A local proper $x_n{}^L$ will describes a weak curvature in 3D-space instead of x_n.

- $ds_{ev} = d\sigma_{CPV}$ is time-like even-term introduced to describe CP-violation (a pseudo-interval).

Therefore, even-terms are not true intervals: due to spin-flipping in a 3D-subspace.

Obviously, for charged leptons (and for us): $d\sigma_{CPV} \ll d\sigma_L \ll d\sigma_{ev} \ll ds_0$.

As $d\sigma_{CPV}$ is often ignored → 3D-time cylinder turns exact and the subluminal physics described by EDs (ψ and φ) is evolved time t, as a linear variable.

For neutrinos, assuming: $d\sigma_{ev} \ll ds_0 << d\sigma_{CPV} \ll d\sigma_L$.→ $d\sigma_{ev}, ds_0$ can be ignored.

2- Quantum mechanical (QM) solution from Einstein gravitation (1)

- Applying **Geometry (5) of bi-cylindrical curvatures**, we will work with **the gravitational equation in vacuum** (T_i^m=0) as:

$$R_i^m - \frac{1}{2}\delta_i^m R = -\delta_i^m \Lambda \; ; \quad (7)$$

- When $\Lambda = 0$, the {3T,3X} equation (7) may separate into two independent 3D sub-equations. It leads to a symmetrical representation:

$$R_\alpha^\beta(T) - \frac{1}{2}\delta_\alpha^\beta R(T) = R_\gamma^\sigma(X) - \frac{1}{2}\delta_\gamma^\sigma R(X). \qquad (8)$$

Where Ricci tensors with $\alpha, \beta \in 3D$-time and ones with $\gamma, \sigma \in 3D$-space.

- As $\psi = \psi(y)$ and $\varphi = \varphi(y)$ are functional, the Hubble law of the cosmological expansion is applied for bi-cylinder (5) of microscopic space-time:

$$\frac{\partial\psi}{\partial y} = v_y = H_y\psi \quad \text{and therefore} \quad \left[\frac{\partial y}{\partial\psi}\right] = \frac{1}{H_y\psi} \; ; \quad (9)$$

Where H_y is the "micro-Hubble constant", then the expansion rate v_y increases proportional to the "micro-scale factor" ψ;

$y \equiv \{t, z\} \in \{t, x_j\} \equiv \{t_0, t_3, x_n, x_l\} \in \{t_i, x_j\}$;

$y_3 \equiv \{t_3, x_3\}$ being implicitly embedded in 3D-time: $t_3 \in \{t_k\}$ and in 3D-space: $x_3 \in \{x_l\}$, respectively.

→In general, it is mathematically possible converting: $y \leftrightarrow iy$.

2- QM solution from Einstein gravitation (2)

- **Christoffel symbols**: by applying (9) following are found valid:

$$\Gamma^{\psi}_{\varphi\varphi} = -\frac{g^{\psi\psi}}{2}\frac{\partial g_{\varphi\varphi}}{\partial\psi} = -\frac{1}{H_y}\frac{\partial\psi}{\partial y}\,; \quad \Gamma^{\varphi}_{\psi\varphi} = \Gamma^{\varphi}_{\varphi\psi} = \frac{g^{\varphi\varphi}}{2}\frac{\partial g_{\varphi\varphi}}{\partial\psi} = \frac{1}{\psi^2 H_y}\frac{\partial\psi}{\partial y}$$

$$\Gamma^{3}_{\varphi\varphi} = -\frac{g^{33}}{2}\frac{\partial g_{\varphi\varphi}}{\partial y_3} = -\frac{1}{2}\frac{\partial(\psi^2)}{\partial y_3} = -\psi\frac{\partial\psi}{\partial y_3}\,; \quad \Gamma^{\varphi}_{3\varphi} = \Gamma^{\varphi}_{\varphi 3} = \frac{g^{\varphi\varphi}}{2}\frac{\partial g_{\varphi\varphi}}{\partial y_3} = \frac{1}{2\psi^2}\frac{\partial(\psi^2)}{\partial y_3} = \frac{1}{\psi}\frac{\partial\psi}{\partial y_3} \quad . \text{(C1)}$$

- **Ricci tensors** for the bi-3D cylindrical geometry (5):

$$R_{\psi\psi} = -\frac{\partial\Gamma^{\varphi}_{\varphi\psi}}{\partial y}\left[\frac{\partial y}{\partial\psi}\right] - \Gamma^{\varphi}_{\psi\varphi}\Gamma^{\varphi}_{\psi\varphi} = -\frac{1}{\psi^3 {H_y}^2}\frac{\partial^2\psi}{\partial y^2} + \frac{1}{\psi^4 {H_y}^2}\left(\frac{\partial\psi}{\partial y}\right)^2\,; \quad R_{33} = -\frac{\partial\Gamma^{\varphi}_{\varphi 3}}{\partial y_3} - \Gamma^{\varphi}_{3\varphi}\Gamma^{\varphi}_{3\varphi} = -\frac{1}{\psi}\frac{\partial^2\psi}{\partial {y_3}^2}\,;$$

$$R_{\varphi\varphi} = \frac{\partial\Gamma^{\psi}_{\varphi\varphi}}{\partial y}\left[\frac{\partial y}{\partial\psi}\right] + \frac{\partial\Gamma^{3}_{\varphi\varphi}}{\partial y_3} - \Gamma^{\varphi}_{\varphi\psi}\Gamma^{\psi}_{\varphi\varphi} - \Gamma^{\varphi}_{\varphi 3}\Gamma^{3}_{\varphi\varphi} = -\frac{1}{\psi {H_y}^2}\frac{\partial^2\psi}{\partial y^2} + \frac{1}{\psi^2 {H_y}^2}\left(\frac{\partial\psi}{\partial y}\right)^2 - \psi\frac{\partial^2\psi}{\partial {y_3}^2}\,.$$

Obviously, $\boldsymbol{R = g^{im}R_{im} = \delta^m_i R^m_i = -\frac{2}{\psi^3 {H_y}^2}\frac{\partial^2\psi}{\partial y^2} + \frac{2}{\psi^4 {H_y}^2}\left(\frac{\partial\psi}{\partial y}\right)^2 - \frac{2}{\psi}\frac{\partial^2\psi}{\partial {y_3}^2}}$. (R1)

Its space-time representation reads: $R = \delta^{\sigma}_{\gamma} R^{\sigma}_{\gamma}(X) - \delta^{\beta}_{\alpha} R^{\beta}_{\alpha}(T)$, (R2)

- **GR equation (8) leads to its sub-equations**:

$\boldsymbol{R^3_3 = 0}$; (10) *and* $\boldsymbol{R^{\psi}_{\psi} = 0}$; (11)

and a trivial sub-equation: $R^{\varphi}_{\varphi} = R^{\psi}_{\psi} + R^3_3 = 0$.

→ **Two independent sub-equations (10) and (11) will be considered**.

2- QM solution from Einstein gravitation (3)

❑ The meaning of sub-equation (10):

$$R_3^3 = -\frac{1}{\psi}\frac{\partial^2\psi}{\partial {y_3}^2} = R_3^3(X) - R_3^3(T) = -\frac{1}{\psi}\left[\frac{\partial^2\psi}{\partial {x_3}^2} - \frac{\partial^2\psi}{\partial {t_3}^2}\right] = 0; \qquad (12)$$

It defines conservation of linear translation (CLT) of Eq. (3) : $-\frac{\partial^2\psi}{\partial {t_3}^2} + \frac{\partial^2\psi}{\partial {x_3}^2} = \mathbf{0}$. (13)

Equation (13) has both kinds of exponential $\psi = \psi_0 e^{\pm(\omega_3 t_3 - k_3 x_3)}$ and wave-like solutions $\psi = \psi_0 e^{\mp i(\omega_3 t_3 - k_3 x_3)}$ which leads to a Lorentz-like condition for compensating longitudinal fluctuations:

$$\left({\omega_3}^2 - {k_3}^2\right)\psi = 0. \qquad (14)$$

❑ The meaning of sub-equation (11):

$$R_\psi^\psi = -\frac{1}{\psi^3 {H_y}^2}\frac{\partial^2\psi}{\partial y^2} + \frac{1}{\psi^4 {H_y}^2}\left(\frac{\partial\psi}{\partial y}\right)^2 = R_\psi^\psi(X) - R_\psi^\psi(T) = 0\ ; \qquad (15)$$

- For real y it leads to: $\frac{\partial^2\psi}{\partial y^2} - \left(\frac{\partial\varphi}{\partial y}\right)^2 \psi = 0;$ (16)

Which has an exponential solution: $\psi = \boldsymbol{\psi_0} e^{H_y \cdot y} = \boldsymbol{\psi_0} e^{\varphi} = \boldsymbol{\psi_0} e^{\Omega t - k_j x_j}$.

- For imaginary y it leads to a wave-like solution: $-\frac{\partial^2\psi}{\partial y^2} - \left(\frac{\partial\varphi}{\partial y}\right)^2 \psi = 0.$ (17)

2- QM solution from Einstein gravitation (4)

- In an explicit time-space symmetrical representation, accounting Lorentz-like condition (14), it reads:

$$\frac{\partial^2\psi}{\partial t^2} - \left(\frac{\partial\varphi}{\partial t_0}\right)^2 \psi = \frac{\partial^2\psi}{\partial {x_j}^2} - \left(\frac{\partial\varphi}{\partial x_n}\right)^2 \psi \,, \quad (18)$$

where due to the local orthogonality: $\frac{\partial^2\psi}{\partial t^2} = \frac{\partial^2\psi}{\partial {t_0}^2} + \frac{\partial^2\psi}{\partial {t_3}^2}$ and $\frac{\partial^2\psi}{\partial {x_j}^2} = \frac{\partial^2\psi}{\partial {x_n}^2} + \frac{\partial^2\psi}{\partial {x_l}^2}$.

- For a homogeneity condition → Eq.(18) is getting a symmetrical equation of bi-geodesic acceleration of deviation ψ as in [8]:

$$\frac{\partial^2\psi}{\partial {t_0}^2} - \left(\frac{\partial\varphi}{\partial t_0}\right)^2 \psi = \frac{\partial^2\psi}{\partial {x_n}^2} - \left(\frac{\partial\varphi}{\partial x_n}\right)^2 \psi \,. \quad (19)$$

Due to 3D local geodesic conditions, both sides in (19) are independent and lead to de Sitter-like exponential sub-solutions able to describe Hubble-like expansion in microscopic 3D-time or 3D-space, correspondingly. Those conditions will be applied in a so-called "microscopic" cosmological model for lepton mass hierarchy as in [7].

- Accordingly, with Lorentz-like condition (14), the wave-like solution (17) reads:

$$-\frac{\partial^2\psi}{\partial t^2} + \frac{\partial^2\psi}{\partial {x_j}^2} = \left[\left(\frac{\partial\varphi}{\partial t_0}\right)^2 - \left(\frac{\partial\varphi}{\partial x_n}\right)^2\right]\psi. \quad (20)$$

2- QM solution from Einstein gravitation (5)

- Assuming qualitatively that the acceleration term in 3D-time is enhanced due to interaction with a Higgs-like potential, producing **a strong time-like polarization** $\boldsymbol{V_T P} \Rightarrow \mathbf{P}^+$ and leaving **a week space-like Parity non-conservation (PNC):**

$$(V_T\boldsymbol{P})^2 = \left[V_T\left(\frac{\partial\varphi}{\partial t_0^-}+\frac{\partial\varphi}{\partial t_0^+}\right)\right]^2\psi \equiv [\boldsymbol{f_e}(\chi+\boldsymbol{\phi_0})]^2\psi \Rightarrow (P^+)^2=\left(\frac{\partial\varphi}{\partial t_0^+}\right)^2\psi \equiv (\boldsymbol{f_e\phi_0})^2\psi = m_0^2\psi\ ; \qquad (21)$$

where χ is Higgs field and ϕ_0 is Higgs vacuum; $\boldsymbol{f_e}$ is Higgs-lepton interaction constant .

It makes $\boldsymbol{d\sigma_{CPV}} \ll \boldsymbol{d\sigma_L} \ll \boldsymbol{d\sigma_{ev}} \ll \boldsymbol{ds_0}$ **in Geometry (6)** . Then:

→ **Transformation from 6D time-space to 4D space-time** is performed.

- Finally, the symmetrical geodesics (18) turns to 4D asymmetrical equation:

$$\frac{\partial^2\psi}{\partial t^2}-\frac{\partial^2\psi}{\partial x_j{}^2} = \left[[\boldsymbol{\Lambda_T}-\left(\frac{\partial\varphi}{\partial x_n}\right)^2_{even}-\boldsymbol{\Lambda_L}\right]\boldsymbol{\psi}; \qquad (22)$$

where : Effective strong potentials $\mathbf{V}_T$ of a time-like "cosmological constant" $\boldsymbol{\Lambda_T}$ with a residue of P-odd component $\boldsymbol{\Lambda_L}$ fulfilled breaking symmetry: $[\boldsymbol{\Lambda_T}-\boldsymbol{\Lambda_L}]\boldsymbol{\psi} \equiv \left[\left(\frac{\partial\varphi}{\partial t_0^+}\right)^2-\left(\frac{\partial\varphi}{\partial x_n^L}\right)^2\right]\boldsymbol{\psi}.$

- Accordingly, the wave-like Equation (20) turns to an asymmetrical representation:

$$-\frac{\partial^2\psi}{\partial t^2}+\frac{\partial^2\psi}{\partial x_j{}^2} = \left[\left(\frac{\partial\varphi}{\partial t_0^+}\right)^2 - B_e(\boldsymbol{k_n}.\,\mu_e)^2_{even}-\left(\frac{\partial\varphi}{\partial x_n^L}\right)^2\right]\boldsymbol{\psi}; \qquad (23)$$

where B_e is a calibration scale factor and μ_e is magnetic dipole moment of charged lepton; its orientation is in correlation with spin vector $\vec{s}$ and being P-even.

2- QM solution from Einstein gravitation (6)

- **For formulation of quantum mechanical equations** rescaling the **wave-like solution** (23) with Planck constant and the functional parameter $\boldsymbol{\psi}$ with Compton length, then adopting the quantum operators: $\frac{\partial}{\partial t} \rightarrow \boldsymbol{i}.\hbar\frac{\partial}{\partial t} = \widehat{\boldsymbol{E}}$ and $\frac{\partial}{\partial x_j} \rightarrow -\boldsymbol{i}.\hbar\frac{\partial}{\partial x_j} = \widehat{\boldsymbol{p}}_j$

→ Equation (23) leads to **the basic quantum equation - a generalized Klein-Gordon-Fock equation (KGF)** with a wave-like function $\boldsymbol{\psi} \equiv \boldsymbol{\psi_w}$:

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} + \hbar^2\frac{\partial^2\psi}{\partial {x_j}^2} - (\hbar\boldsymbol{\Omega})^2\boldsymbol{\psi} = \mathbf{0} \quad ; \qquad (24)$$

The Fourier transformation leads to a momentum representation ($\boldsymbol{\psi} \rightarrow \boldsymbol{\psi} \equiv \boldsymbol{\psi_p}$):

$$E^2\boldsymbol{\psi_p} - \boldsymbol{p_j}^2\boldsymbol{\psi_p} = \boldsymbol{m}^2\boldsymbol{\psi_p}; \qquad (25)$$

where : $\boldsymbol{m}^2 = \boldsymbol{m}_0^2 - \boldsymbol{\delta m}^2 = \boldsymbol{m}_0^2 - \boldsymbol{m}_S^2 - \boldsymbol{m}_L^2$

- $\boldsymbol{m_0} = \hbar\Omega_0$ is the conventional rest mass, defined by $\boldsymbol{\Lambda}_T$; $\boldsymbol{m_S}$ as a P-even contribution links with an external rotational curvature in 3D-space which can be compensated due to 3D local geodesic conditions (19); $\boldsymbol{m_L} \ll \boldsymbol{m_S}$ is a tiny mass factor generated by $\boldsymbol{\Lambda}_L$, related to a P-odd effect of parity non-conservation (PNC).

- For depolarized fields, applying (19) and ignoring $\boldsymbol{\Lambda}_L$, i.e. $\boldsymbol{m} \rightarrow \boldsymbol{m_0}$, and $x_j \rightarrow x_l$, Equation (24) is identical to the traditional ***Klein-Gordon-Fock equation***:

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} + \hbar^2\frac{\partial^2\psi}{\partial {x_l}^2} - \boldsymbol{m_0}^2\boldsymbol{\psi} = \mathbf{0} \quad ; \qquad (26)$$

2- QM solution from Einstein gravitation (7)

Outputs of the solution:

- Mathematical transformation from the geodesic equation (22) with an exponential solution to a wave-like solution (23) is performed by transformation of variables: $t \to -it$ and $x_j \to ix_j$ in similar to that of quantum dynamic operators. This is not only a mathematical formalism, but also a significant physical operation, equivalent to transformation from external to internal investigation. Indeed, for the phase $\varphi = \Omega t - k_j x_j = const$ in the internal phase continuum: the phase velocity is superluminal, i.e. $v_{phase} = \frac{dx_j}{dt} = \frac{\Omega}{k_j} > c$. It is equivalent to converting the role of space vs. time in the internal superluminal frame comparing with the external subluminal space-time.
- It leads to a conclusion that quantum mechanics is a special technics for describing the superluminal microscopic gravitational waves in microscopic phase continuum carrying energy-momentum in the corresponding subluminal macroscopic space-time.
- In a duality to solution (23) of Einstein general relativity (7) the geodesic solution (22) describes the particle as a material point in the superluminal time-space which in a homogeneity condition leads to Hubble expansion in the microscopic time-space. In analogue to the standard model (SM) of macroscopic cosmology, the proposed microscopic cosmological model (MCM) is able to solve the mass hierarchy problem of elementary particles (*see next section*).

3- Microscopic cosmological model for lepton hierarchy (1)

- In 4D space-time assuming that all leptons, as a material points, are to involve in a common basic time-like cylindrical geodesic evolution with a internal 1D circular curvature of the time-like circle $\boldsymbol{S_1}(\varphi^+)$, where $\varphi^+$is azimuth rotation in the plane $\{t_1, t_2\}$ about t_3 and its sign “+” means a fixed time-like polarization from the past to the future;
- Developing more generalized 3D spherical system, described by **nautical angles** $\{\varphi^+, \theta_T, \gamma_T\}$, where θ_T is a zenith in the plane $\{t_1, t_3\}$ and γ_T is another zenith in the orthogonal plane $\{t_2, t_3\}$.

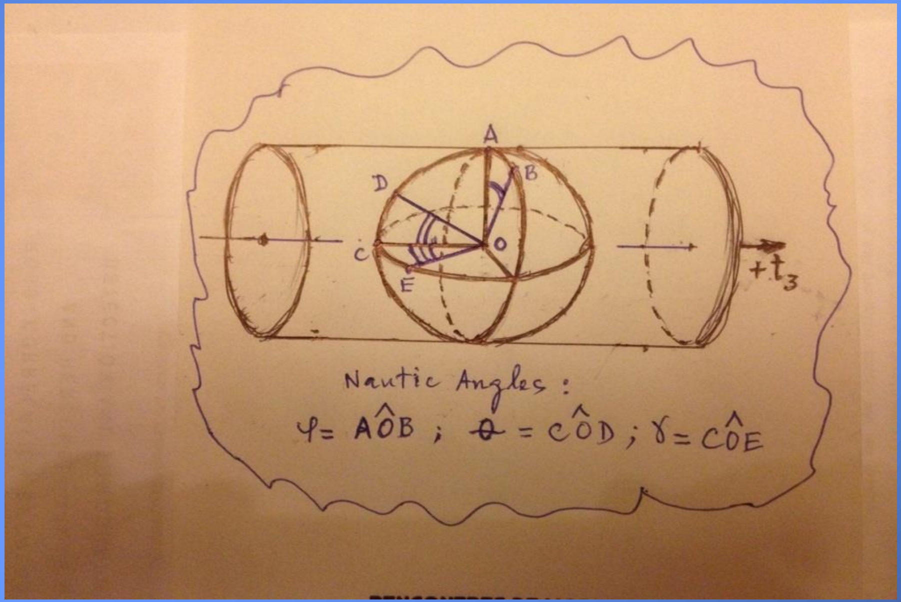


Fig 3.1. Nautical angles to a time-like cylinder.

3- Microscopic cosmological model for lepton hierarchy (1)

- For *n*-hyper spherical surfaces their highest order curvatures C_n is inversely proportional to *n*-power of time-like radius:

$$C_n \sim \psi^{-n} \quad ; \quad n = 1,3 \quad ; \qquad (27)$$

→ In according to general relativity, the energy density ρ_n correlates with its scalar curvature and the density ρ_1 of lightest lepton as:

$$\rho_n = \frac{\epsilon_0}{\psi^n} = \frac{\epsilon_0}{\psi} \frac{1}{\psi^{n-1}} = \rho_1 \frac{1}{\psi^{n-1}} \quad ; \qquad (28)$$

Where the factor ϵ_0 is assumed a universal lepton energy factor

(universal, because all 3 generations are involved in cylindrical condition).

3- Micro-cosmological model for lepton hierarchy (2)

- Electron oscillating on a fixed line-segment of the time-like amplitude $\boldsymbol{\Phi}$, formulating 1D proper (or comoving) “volume”: $\boldsymbol{V_1(\varphi^+) = \Phi = \psi T}$;

  where $\boldsymbol{T}$ is the 1D time-like Lagrange radius.

- For instance, $\boldsymbol{\Phi}$ plays a role of the time-like micro Hubble radius and the wave function $\boldsymbol{\psi}$ plays a role of the time-like scale factor. They are probably changeable during the expansion of the Universe (!).

- The mass of electron defined by 1D Lagrange “volume” will be:

$$\boldsymbol{m_1 = \rho_1 V_1 = \rho_1 \Phi = \frac{\epsilon_0}{\psi}\psi T = \epsilon_0 T} \quad ; \quad (29)$$

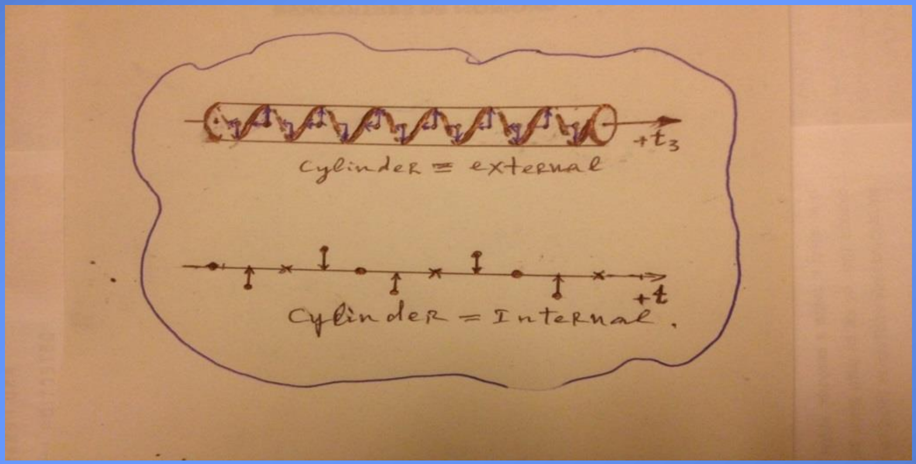


Fig 3.2. Linearization of time axis of electron

For muon and tauon except the basic time-like cylindrical curved evolution φ^+, it needs to add ED curvatures made by evolution in simplest configurations of hyper-spherical “surfaces”:

i/ $\boldsymbol{S_1}\ (\theta_T)$ *and* $\boldsymbol{S_1}\ (\gamma_T)$ or

ii/ $\boldsymbol{S_2}(\theta_T, \gamma_T)$.

3- Micro-cosmological model for lepton hierarchy (3)

- The "comoving volumes" $V_n(\Phi)$ with fixed Φ are calculated as:

$$V_n(\Phi) = \int_0^{\Phi} S_{n-1}(v)dv = S_{n-1}(\Phi)\int_0^{\Phi} dv = S_{n-1}\Phi = V_1 S_{n-1} \qquad (30)$$

- For homogeneity condition the simplest "2D-rotational comoving volume" is:

$$V_2(\varphi^+, "\theta_T + \gamma_T") = V_1(\varphi^+)[S_1(\theta_T) + S_1(\gamma_T)] = \Phi.2S_1 = 4\pi\Phi^2$$

- Accordingly, the lepton mass of 2D time-like curved particle (muon) is:

$$m_2 = \rho_2 V_2 = \rho_1 \frac{1}{\psi}\Phi.2S_1 = \frac{\epsilon_0}{\psi^2}4\pi\Phi^2 = \epsilon_0 4\pi T^2; \qquad (31)$$

- The next simplest 3D-rotational comoving volume is:

$$V_3(\varphi^+, "\theta_T * \gamma_T") = V_1(\varphi^+)S_2(\theta_T, \gamma_T) = \Phi.S_2 = 4\pi\Phi^3$$

- Accordingly, the lepton mass of 3D time-like curved particle (tauon) is:

$$m_3 = \rho_3 V_3 = \rho_1 \frac{1}{\psi^2}\Phi.S_2 = \frac{\epsilon_0}{\psi^3}4\pi\Phi^3 = \epsilon_0 4\pi T^3; \qquad (32)$$

In principle, we could use the precise experimental data of electron and muon masses to determine ϵ_0 and T in according to (29) and (31) as two free parameters, and then to calculate the tauon mass by (32), as a prediction.

3- Micro-cosmological model for lepton hierarchy (4)

Assuming a qualitative scenario for estimation of Lagrange radius T:

- During the Big-Bang inflation, we suggest, the following a ***scenario similar to the standard cosmological model***: micro factor ψ increases exponentially (time-like Hubble constant $H_T = \sqrt{\Lambda_T} = \Omega_0$ =7.764*10^{20} sec^{-1} and the instant of inflation $\Delta t_1 = 1.926 * 10^{-20}$ sec after 1 sec from the Big-Bang). For the next time-life of the Universe 13.7 Bill. years assuming: $\psi \sim t^{1/2}$ for radiation dominant era and $\sim t^{2/3}$ for matter dominant era.

→ The time-like Lagrange radius T decreases from $T_0 = \frac{\Phi}{\psi_0}$ =1 for Δt_1 then steps up to the present value $T = \frac{\Phi}{\psi} \approx 16.5$.

- For leptons born after the inflation era, assuming following anthropic principle (very *qualitatively)* that the Hubble radius of any quantum fluctuations should adapt the contemporary value Φ, while the scale factor ψ being governed by a contemporary chaotic Higgs-like potential in such a way, that is to meet the contemporary time-like Lagrange radius T (for today, T =16.5).
- Using $T = 16.5$, and the lepton energy factor $\epsilon_0 = 31.0\ keV$ calibrated to m_e, we come to mass ratios of all three charged lepton generations:

$m_e : m_\mu : m_\tau = m_1 : m_2 : m_3$=1:207.4:3421.5=**0.511:106.0:1748.4** (*MeV*); (33)

3- Micro-cosmological model for lepton hierarchy (5)

The result (as for the 1rst order of approximation) is resumed in the Table 1:

n-Lepton	1-electron	2-muon	3-tau lepton
Density, ρ_n	$\frac{\epsilon_0}{\psi}$	$\frac{\epsilon_0}{\psi^2}$	$\frac{\epsilon_0}{\psi^3}$
Comoving volume, $\boldsymbol{V_n}$	$\boldsymbol{\Phi}$	$4\pi\boldsymbol{\Phi}^2$	$4\pi\boldsymbol{\Phi}^3$
Formulas of mass, $\boldsymbol{m_n}$	$\epsilon_0 T$	$\epsilon_0 4\pi T^2$	$\epsilon_0 4\pi T^3$
Calculated mass ratio $\boldsymbol{T \approx 16.5}$; $\boldsymbol{\epsilon_0 = 31.0}\ keV$	**1**	**207.4**	**3421.5**
Experimental leton mass, $\boldsymbol{m_n}$ (MeV) [8]	0.510998928(11)	105.6583715(35)	1776.82(16)
Calculated lepton mass, $\boldsymbol{m_n}$ (MeV)	**0.511***	**106.0**	**1748.4**

*) *Same value* $\boldsymbol{m_e}$ *for calibration.*

❑ Outputs of the model:

- ➢ The deviation from masses of muon and tau-lepton < +1% and - 2%.
- ➢ Lepton masses increase during cosmological expansion with different rates. It would open a unique window for experimental verification by measuring a time-variation of the mass ratios. In particular, $\boldsymbol{R_{21} = m_\mu : m_e = 4\pi T}$ **and the increasing rate is** $\boldsymbol{T \sim t^{1/3}}$. Let take **a duration of data sampling** $\boldsymbol{\Delta t_s = 10}$ **years**, then mass ratio increases by $\boldsymbol{\Delta R_{21} = \frac{4\pi T}{3} \frac{\Delta t_s}{13.7.10^9} = 5.10^{-8}}$. It needs to improve precision of both experimental data of muon and electron masses by two orders more before going on for comparison and observation of any time-variation of their ratio.

4- Absolute masses of neutrinos (1)

Experimental status [9]:

- Direct measurements in single beta decays are far from the expected masses (<2.2 eV) for neutrino with given lepton number (electron neutrino).

- Double beta decay searches is approaching to the finest upper limits of absolute masses (<0.2 eV) with electron neutrino as well.

- Neutrino oscillations give only square differences of neutrino masses with the record precisions of the masses:

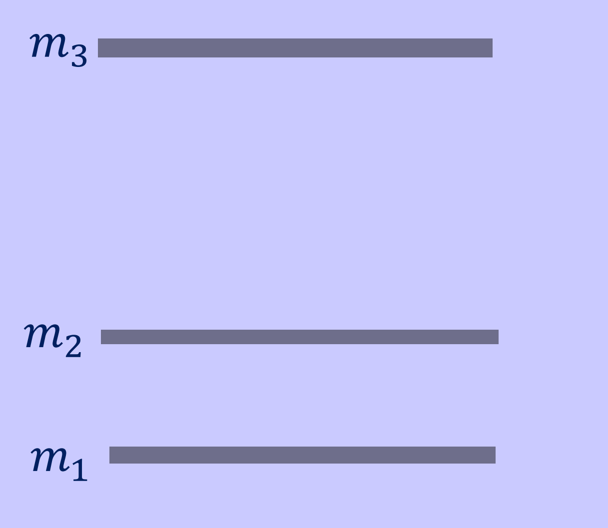
$$\Delta {m_{21}}^2 = 7.50 \text{ x } 10^{-5} eV^2 \quad (2.3\%)$$
$$\Delta {m_{31}}^2 = 2.46 \text{ x } 10^{-3} eV^2 \quad (1.9\%)$$
$$|\Delta {m_{32}}^2| = 2.45 \text{ x } 10^{-3} eV^2 \quad (1.9\%)$$

→ The squared oscillation angles can show the relative probability of each oscillation channel. In this work we consider the mass eigenstates and discuss on the absolute masses of m_1, m_2, m_3; but not their mixing eigenstates with given lepton numbers (v_e, v_μ, v_τ).

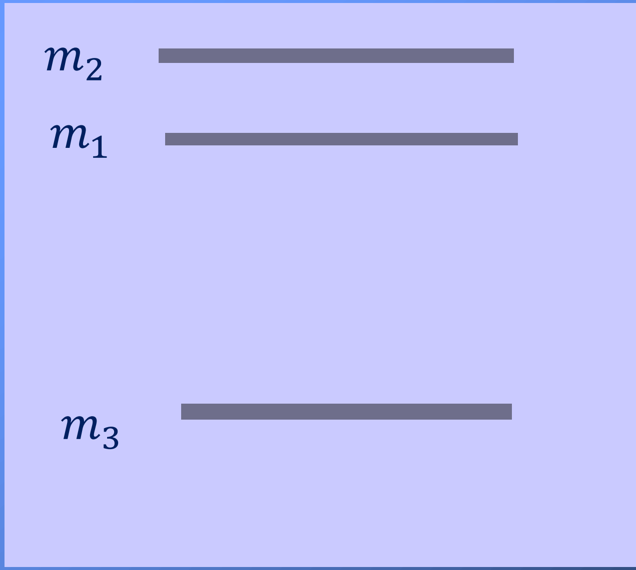
4- Absolute masses of neutrinos (2)

Neutrino masses of three generations: What is their Ordering? Fig 4.1 (a and b).

- In analogue to the charged leptons we accept the normal ordering: m_1 being the lightest neutrino with a basic space-like cylindrical curvature; m_2 has additional S_1 curvatures and m_3 being heaviest neutrino has an additional S_2 curvature.



a. Normal ordering.



b. Inverted ordering

→ according to the normal ordering, i.e.1→2→3

and $|m_3|>>|m_1|$, then $\Delta m_{31}{}^2 \approx m_3{}^2$; if $|m_2|>>|m_1|$, then $\Delta m_{21}{}^2 \approx m_2{}^2$.

4- Absolute masses of neutrinos (3)

- For neutrinos breaking symmetry makes $d\sigma_{ev} \ll ds_0 \ll d\sigma_{CPV} \ll d\sigma_L$ in Geometry (6).

Then, assuming: neutrinos are free in 3D-time ($\Lambda_T = 0$) and weakly curved in 3D-space, replacing $\Lambda_L \rightarrow \Lambda_\nu \sim \Lambda_L$; and as $\Lambda_L \neq 0$, but still very small: $t \rightarrow t_3 \equiv t$ and $x_j \approx x_l$.

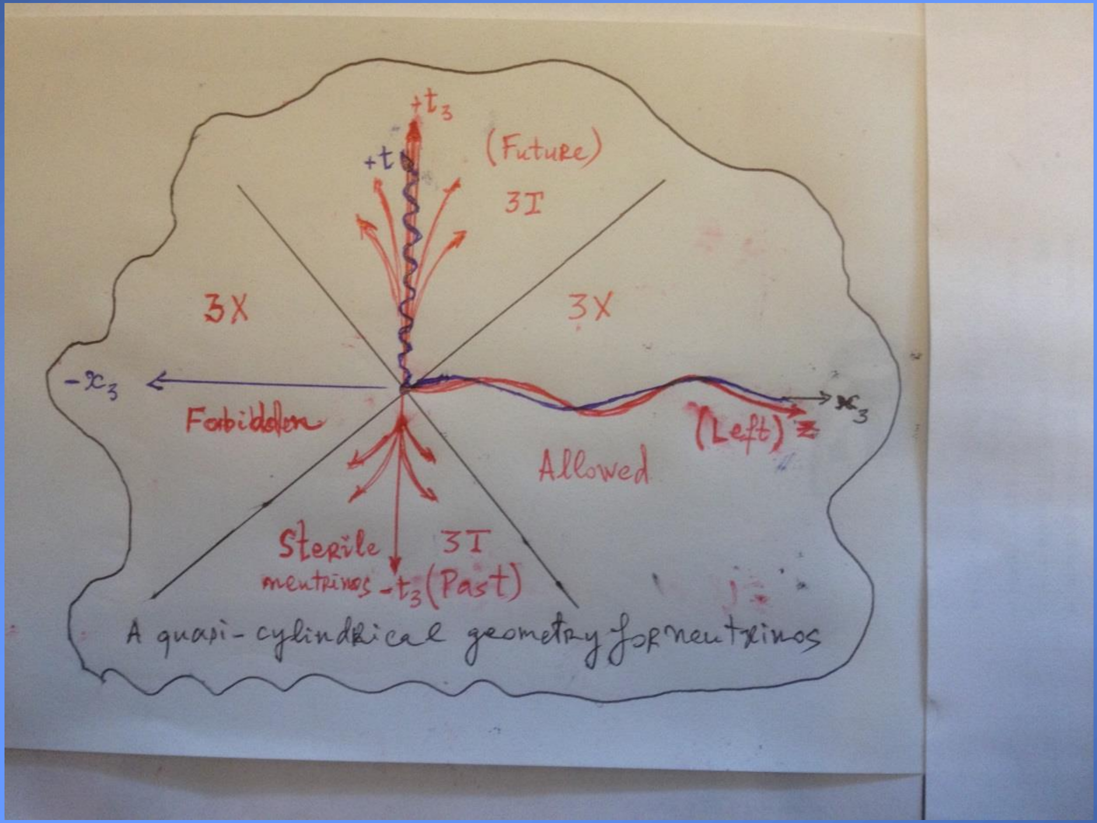


Fig 4.2. A cylindrical geometry for neutrino:
-Space follows a strict cylinder;
-But time evolution is almost open to a flat 3D-time (unobservable)

4- Absolute masses of neutrinos (4)

- Then: neutrinos are free in 3D-time ($\boldsymbol{\Lambda_T = 0}$) and weakly curved in 3D-space, replacing $\boldsymbol{\Lambda_L}$→ $\boldsymbol{\Lambda_v}$~ $\boldsymbol{\Lambda_L}$; and as $\boldsymbol{\Lambda_L \neq 0}$, but still very small: t→$t_3 \equiv t$ and $x_j \approx x_l$.

In analogue to the wave-solution (23) of charged leptons, an equation of neutrino reads:

$$-\frac{\partial^2 \psi_v}{\partial t^2} + \frac{\partial^2 \psi_v}{\partial {x_l}^2} = \left[\mathrm{B}_v (\Omega_0 . d_v)^2 - \left(\frac{\partial \varphi}{\partial x_n^L} \right)^2 \right] \psi_v ; \qquad (34)$$

There super-weak CP violation term with calibration scale factor $\mathbf{B}_v$ is too tiny and often ignored due to electrical dipole moment $\boldsymbol{d_v}$.

Rescaled (34) with Planck constant leads to an equation of time-like lepton with a tiny imaginary mass $\boldsymbol{i.m_v}$:

$$-\hbar^2 \frac{\partial^2 \psi_v}{\partial t^2} + \hbar^2 \frac{\partial^2 \psi_v}{\partial {x_l}^2} = -{m_v}^2 \psi_v ; \qquad (35)$$

→Such a lepton has an almost robust helicity, as Majorana neutrinos.

In practice, because neutrino mass is too small and they are observed as moving with a speed of light.

Equations (34) - (35) appear as a law of transmission of microscopic gravitational waves, carrying out a very weak space-like curvature characterized by function $\boldsymbol{\psi_v}$.

4- Absolute masses of neutrinos (5)

- In analogue to the charged lepton model, extending the space-like curvature of neutrinos to higher orders than the cylindrical one, we can estimate the masses of all three neutrino generations:

$$m_1 = \epsilon_v X_v; \quad m_2 = \epsilon_v 4\pi X_v^2; \quad m_3 = \epsilon_v 4\pi X_v^3; \qquad (36)$$

Where $X_v = \Phi_v/\psi_v$, is the micro space-like Lagrange radius.

- Based on the two "experimental masses" of neutrino-2 and neutrino-3:

$$m_3 = 4.96 * 10^{-2} \text{ eV}; \ m_2 = 8.66 * 10^{-3} \text{eV} ; \qquad (37)$$

we define two parameters:

$$X_v = 5.728 \text{ ; and } \epsilon_v = 2.10 * 10^{-5} \text{ eV}; \qquad (38)$$

- Consequently, we are able to calculate the mass m_1 of the lightest neutrino-1:

$$m_1 = \epsilon_v X_v = 1.20 * 10^{-4} \text{eV}; \qquad (39)$$

For alternative, determining: $\epsilon_v^* = \frac{G_F m_e^2}{\alpha} \epsilon_0 = 1.27 * 10^{-5} \text{eV}$; (40)

→ There is found ϵ_v^* is of order of ϵ_v within a factor of 2, which would be fixed prior for calculating the Lagrange radius $X_v = 6.77$ from "experimental mass" m_3.

4- Absolute masses of neutrinos (6)

The result is resumed in the Table 2:

Neutrino (n)	neutrino (1)	neutrino (2)	neutrino (3)
Density, ρ_v	$\frac{\epsilon_v}{\psi_v}$	$\frac{\epsilon_v}{{\psi_v}^2}$	$\frac{\epsilon_v}{{\psi_v}^3}$
Comoving volume, V_v	Φ_v	$4\pi{\Phi_v}^2$	$4\pi{\Phi_v}^3$
Formulas of mass, m_n	$\epsilon_v X_v$	$\epsilon_v 4\pi {X_v}^2$	$\epsilon_v 4\pi {X_v}^3$
Oscillation squared masses, (eV^2) **: [9]	$\Delta m^2_{31} - \Delta m^2_{32} =$ $= (1.0 \mp 7.0)10^{-5} =$ =Δm^2_{21}* ɔ Δm^2_{21}	$\Delta m^2_{21} =$ $7.50 * 10^{-5}$ (∓2.3%).	$\Delta m^2_{31} =$ $2.46 * 10^{-3}$ (∓1.9%).
Absolute masses (eV):	?	$8.66 * 10^{-3}$ (∓1.2%)	$4.96 * 10^{-2}$(∓1.0%)
a/ Calculated masses, m_n (eV): $X_v = 5.728$ ϵ_v=2.10*10^{-5} eV	$1.20 * 10^{-4}$	8.66 $* 10^{-3}$ (*) *Calibration*	4.96$10^{-2}$ (*) *Calibration*
b/ Alternative, m_n (eV): $X_v = 6.774$ ϵ_v^* =1.27*10^{-5} eV	$8.60 * 10^{-5}$	7.32 $* 10^{-3}$	4.96$10^{-2}$ (*) *Calibration*
Δm(a-b) %	33%	15.5%	(*)

4- Absolute masses of neutrinos (7)

Out puts of the model:

- Neutrinos with mass eigenvalues can not travel with v<c: their helicity is almost fixed strictly, while the electrical properties are not conserved (due to CPV term), which is the appearance of Majorana neutrinos.

- The fact that electron and neutrino energy factors are well correlated as: $\epsilon_{\nu}^{*} = \frac{G_F m_e^2}{\alpha} \epsilon_0$ in an applicable time-space symmetry shows up an argument that charged leptons and neutrinos may be time-space partners.

- The absolute mass values of all 3 generations fit the normal ordering of hierarchy (not to the inverted ordering).

- It needs to improve the precision of experimental values of differences of squared neutrino masses by roughly one order better to confirm the normal ordering and then to prove the predicted absolute mass m_1 of the lightest neutrino.

5- Conclusion

- **The quantum Klein-Gordon- Fock equation is a wave-like solution of Einstein gravitational equation in vacuum (Consistency between Quantum mechanics and General relativity)**

- Applying the model with a maximal time-like dimension (3D) in a time-space symmetry:

By extending curvature to additional 2D and 3D time-like or space-like hyper-spherical configurations:

- ***mass ratios of charged leptons and neutrinos*** are estimated satisfactory.
- **Absolute masses of neutrinos follow the normal ordering**.
- It would serve a ***solution to problems of number "3" of lepton generations and their mass hierarchy.***

- Proposing improvement of precision of oscillation experiments **for confirming the normal ordering and observing tiny absolute neutrino masses**.

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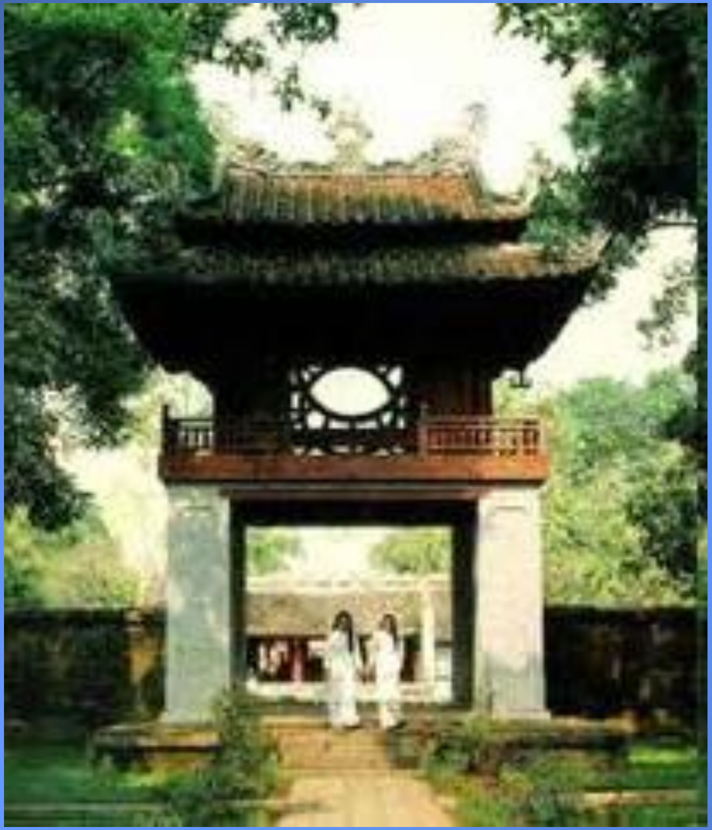

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