Parton energy loss in collinear expansion

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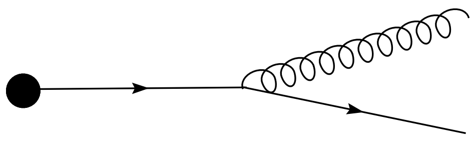
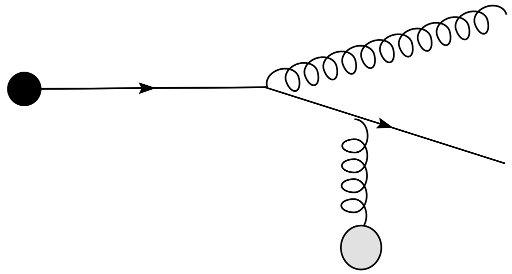
Outline:

- Approaches to the induced gluon emission
- Contradiction between the Guo-Wang-Zhang pQCD higher twist scheme and other approaches
- Connection between the light-cone path integral approach and the Feynman diagram calculation
- Collinear expansion for the $N = 1$ rescattering contribution and Guo-Wang-Zhang expansion.
- Connection between the oscillator approximation and collinear expansion

The radiative parton energy loss



Vacuum emission, N=0



One of the diagrams for induced emission, N=1

In vacuum gluon emission is described by the DGLAP equation

Approaches to the induced gluon emission due to parton multiple scattering

- BDMPS (1995, 1997, 1998) – time-ordered diagram calculations, $N \gg 1$, strong LPM effect, massless partons
- Zakharov (1996), Wiedemann (2000) – light cone path integral (LCPI) approach
- Gyulassy-Levai-Vitev (GLV) (2000) – time-ordered diagram calculations for $N \leq 3$, thin quark-gluon plasma
- Guo-Wang-Zhang (GWZ) (2000, 2003) – pQCD higher twist approach, originally developed for gluon emission from fast quark produced in eA DIS. The number of rescatterings $N = 1$

BDMPS and LCPI are equivalent in the limit of strong LPM effect.
GLV corresponds to the first 3 terms in the opacity expansion of LCPI.
Is GWZ equivalent to the $N = 1$ rescattering contribution in the LCPI?

Contradiction between the LCPI and GWZ approach

- The GWZ induced gluon spectrum for quark produced in eA DIS for the Gaussian nuclear density $n_A(r) \propto \exp\left(-r^2/2R_A^2\right)$

$$\frac{dP_{GWZ}}{dz} \propto \alpha_s^2 n_A(0) R_A P_{Gq}(z) \int \frac{dp^2}{p^4} x f_g(x) \left\{1 - \exp\left[-\frac{p^4 R_A^2}{4E^2 z^2 (1-z)^2}\right]\right\} .$$

where $P_{Gq} = C_F[1 + (1-z)^2]/z$, $x \ll 1$. The GWZ spectrum contains a logarithmic factor since $x f_g(x) \approx \frac{3\alpha_s C_F}{\pi} \ln\left(Q^2/\mu^2\right)$ ($Q^2 \sim p^2$, μ is infrared cutoff).

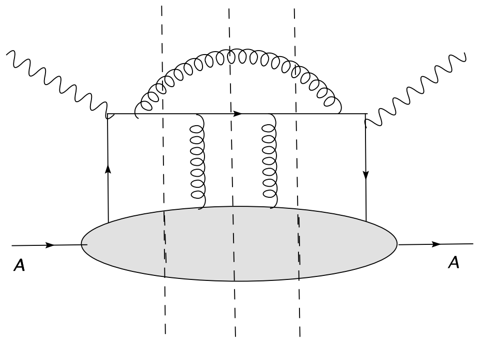
- The LCPI $N = 1$ induced gluon spectrum for massless partons [BGZ (2001)]

$$\frac{dP_{LCPI}}{dz} \propto \alpha_s^3 n_A(0) R_A P_{Gq}(z) \int \frac{dp^2}{p^2(p^2 + \mu^2)} \left\{1 - \exp\left[-\frac{p^4 R_A^2}{4E^2 z^2 (1-z)^2}\right]\right\} .$$

The LCPI spectrum does not have any logarithmic factor which can be interpreted as the nucleon gluon density.

Connection between the LCPI and GWZ formulation

Technically the GWZ approach is very different from the LCPI scheme. For eA DIS GWZ calculate the set of diagrams like



The lower soft parts of the diagrams are expressed via the matrix element

$$\langle A|\bar{\psi}(0)A^+(y_1)A^+(y_2)\psi(y_3)|A\rangle\,.$$

The upper hard parts, H, are calculated perturbatively. The contour integrations over the p^+ momentum components are carried out with the help of the poles in the denominators of the eikonal propagators for fast quarks and gluon ($p^\pm = (p^0 \pm p^3)/\sqrt{2}$)

$$G_{eik}(y) = \frac{i}{(2\pi)^4}\int\limits_{p^->0} dp^+dp^-d\vec{p}_T\,\frac{\exp\left[-i(p^+y^- + p^-y^+ - \vec{p}_T\vec{y}_T)\right]}{2p^-\left(p^+ - \frac{\vec{p}_T^{\,2}+m^2-i0}{2p^-}\right)}\,.$$

- The starting point of the LCPI approach is the S-matrix element written in terms of the wave functions in an external field

$$\langle bc|\hat{S}|a\rangle = i\int dy \lambda \psi_b^*(y)\psi_c^*(y)\psi_a(y)\,.$$

The incoming ($i=a$) and outgoing ($i=b,c$) wave functions are written as

$$\psi_i(y) = \frac{1}{\sqrt{2p_i^-}}\exp[-ip_i^- y^+]\phi_i(y^-,\vec{y}_T)\,.$$

- The y^- dependence of the transverse wave functions ϕ_i is governed by the two-dimensional Schrödinger equation

$$i\frac{\partial\phi_i(y^-,\vec{y}_T)}{\partial y^-} = \hat{H}_i\phi_i(y^-,\vec{y}_T)\,,$$

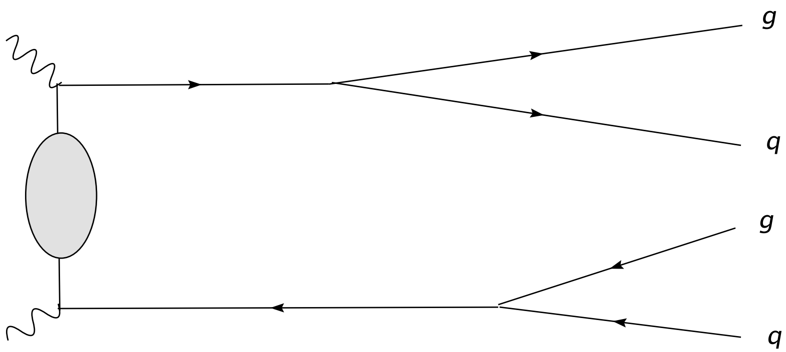
where the Hamiltonian reads

$$\hat{H}_i = \frac{[(\vec{p}_T - g\vec{A}_T)^2 + m_i^2]}{2\mu_i} + gA^+\,, \quad \text{with the Schrödinger “mass”} \mu_i = p^-$$

The y^- evolution of ϕ_i can be written in terms of the Green's function for $\hat{H}_i$

$$\phi(y_2^-, \vec{y}_{T,2}) = \int d\vec{y}_{T,1} K(\vec{y}_{T,2}, y_2^- | \vec{y}_{T,1}, y_1^-) \phi(y_1^-, \vec{y}_{T,1}) \, .$$

Diagrammatically in terms of K ($\rightarrow$) and K^* ($\leftarrow$) the gluon emission in the eA DIS is given by the graph



The integration over y^+ of the frequently oscillating factors can be done in the same way as for the nucleon structure function f_q. As compared to the Collins-Soper formula $f_q = \frac{1}{4\pi} \int dy^- e^{ix_B P^+ y^-} \langle N | \bar{\psi}(-y^-/2) \gamma^+ \psi(y^-/2) | N \rangle$, now

$$e^{ix_B P^+ y^-} \rightarrow e^{ix_B P^+ y^-} K_{q_f}(\infty|\xi) K_g(\infty|\xi) K_{q_i}(\xi|y^-/2) K^*_{q_f}(\infty|\xi') K^*_g(\infty|\xi') K^*_{q_i}(\xi'| - y^-/2).$$

We omit all the transverse coordinates, the vertex spin factors, and the integrations signs. The Green's function K are for partons in the color field of the nucleus.

Feynman diagram treatment and LCPI scheme

The LCPI formula in terms of the transverse Green's functions can be obtained from the pQCD treatment in terms of Feynman diagrams. Indeed,

$$i \int \frac{dk^+ d\vec{k}_T}{(2\pi)^3} \frac{\exp\left[-ik^+ y^- + i\vec{k}_T \vec{y}_T\right]}{k^+ - \frac{\vec{k}_T^2 + m^2}{2k^-} + i0} = K(\vec{y}_T, y^- | 0, 0)\,.$$

Then the eikonal Feynman propagator used in GWZ can be written as

$$G_{eik}(y) = \int_0^\infty \frac{dk^-}{4\pi k^-} e^{-ik^- y^+} K(\vec{y}_T, y^- | 0, 0)\,.$$

The integration in the Feynman diagrams over y^+ gives conservations of the large k^- components in each vertex, and we reproduce exactly the LCPI expression in terms of the transverse Green's functions.

$\Rightarrow$ Conceptually the GWZ pQCD calculations are equivalent to the $N = 1$ rescattering term in the LCPI method.

$\Rightarrow$ The origin of the discrepancy between the GWZ and LCPI gluon spectrum can be only of a technical nature.

Gluon emission in eA DIS in the LCPI scheme

The original formulation of the LCPI approach (and BDMPS, GLV) neglects the quantum nonlocality of the fast parton production. For hard reaction in AA collisions the nonlocality is $\sim 1/Q$, and it can be safely neglected. In DIS the nonlocality is given by the Ioffe length

$$L_I = 1/mx_B \,.$$

If $L_I \ll R_A$, and $L_I \ll L_f$ ($L_f \sim 2z(1-z)E/\epsilon^2$, $\epsilon^2 = m_q^2 z^2 + m_g^2(1-z)$), one can neglect the nonlocality for eA DIS as well. $L_I \ll L_f$ if $M_{gq}^2 \ll Q^2$. It is satisfied for the DGLAP vacuum radiation as well. The rescatterings do not change the parton helicities

$$\Rightarrow \qquad dW_A^{\mu\nu}/dz = AW_N^{\mu\nu}\langle\langle dP/dz\rangle\rangle \,, \qquad \text{(shadowing, EMC are ignored)} \,,$$

$$dP_{LCPI}(r)/dz = \int_{r_3}^{\infty} d\xi n_A(\vec{r}_T, r_3+\xi) d\sigma(z,\xi)/dz \,,$$

$$d\sigma(z,\xi)/dz = \mathrm{Re}\int d\vec{\rho}\psi^*(\vec{\rho},z)\sigma_{q\bar{q}g}(\rho,z)\psi(\vec{\rho},z,\xi) \,, \qquad \text{[BGZ (1997)]}$$

$$\sigma_{q\bar{q}g}(\rho,z) = \frac{9}{8}[\sigma_{q\bar{q}}(\rho) + \sigma_{q\bar{q}}(\rho(1-z))] - \frac{1}{8}\sigma_{q\bar{q}}(\rho z) \,, \qquad \text{[N.N. Nikolaev and BGZ (1994)]}$$

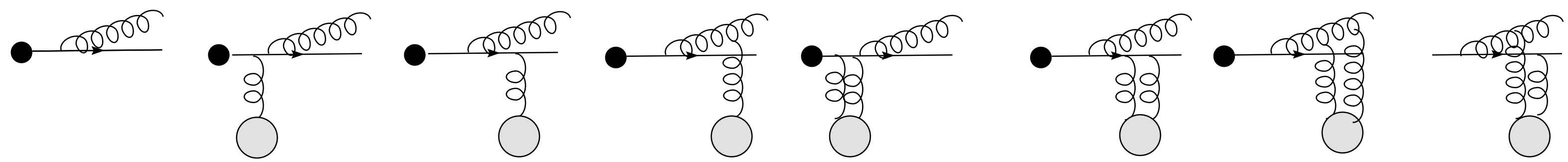
For soft gluon emission $\sigma_{q\bar{q}g}(\rho,z) \approx \frac{9}{4}\sigma_{q\bar{q}}(\rho) = \sigma_{gg}(\rho)$.

In the momentum space at $z \ll 1$ for $N = 1$ we have (for massless partons $\epsilon = 0$)

$$\frac{d\sigma(z,\xi)}{dz} = \frac{3\alpha_s^3 P_{Gq}(z)}{\pi^2} \int d\vec{k}_T d\vec{p}_T \frac{H(\vec{p}_T, \vec{k}_T, z, \xi)}{(\vec{k}_T^2 + \mu^2)^2},$$

$$H(\vec{p}_T, \vec{k}_T, z, \xi) = \left[\frac{\vec{p}_T^2}{(\vec{p}_T^2 + \epsilon^2)^2} - \frac{(\vec{p}_T - \vec{k}_T)\vec{p}_T}{(\vec{p}_T^2 + \epsilon^2)((\vec{p}_T - \vec{k}_T)^2 + \epsilon^2)}\right] \cdot \left[1 - \cos\left(\frac{i(\vec{p}_T^2 + \epsilon^2)\xi}{2Ez(1-z)}\right)\right].$$

It accumulates the contributions to $N = 1$ from the diagrams



Each diagram can be calculated as

$\langle bc|M|a\rangle \propto \int dy^- d\vec{y}_T \phi_b^*(y^-, \vec{y}_T)\phi_c^*(y^-, \vec{y}_T)\phi_a(y^-, \vec{y}_T)$ with the plane wave functions

$\phi(y^-, \vec{y}_T) = \exp\left[-iy^-(\vec{p}_T^2 + m^2)/2p^-) + i\vec{y}_T\vec{p}_T\right]$ with sharp $\vec{p}_T$ change at the points where t-channel gluon kicks fast parton.

The calculated hard parts (at $\epsilon = 0$) agree with that of GWZ.

$\Rightarrow$ The problem maybe connected with the collinear expansion.

Collinear expansion

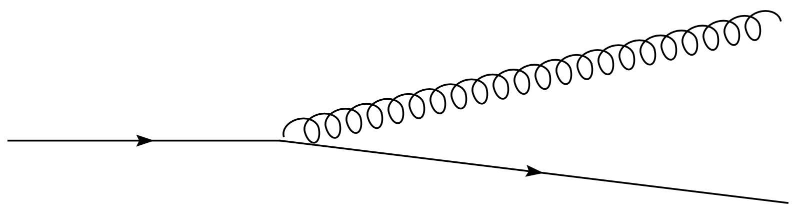
$$H(\vec{k}_T) \approx H(\vec{k}_T=0) + k_T^\alpha \frac{\partial}{\partial k_T^\alpha} H(\vec{k}_T=0) + \frac{k_T^\alpha k_T^\beta}{2} \frac{\partial}{\partial k_T^\alpha} \frac{\partial}{\partial k_T^\beta} H(\vec{k}_T=0)\,,$$

$$\frac{d\sigma(z,\xi)}{dz} \propto \int d\vec{p}_T \left(\frac{\partial}{\partial \vec{k}_T}\right)^2 H(\vec{p}_T,\vec{k}_T=0) \int \frac{d\vec{k}_T k_T^2}{(\vec{k}_T^2+\mu^2)^2} \propto \int d\vec{p}_T \left(\frac{\partial}{\partial \vec{k}_T}\right)^2 H(\vec{p}_T,\vec{k}_T=0) x f_g(x,p^2)\,.$$

But for massless partons ($\epsilon=0$) $\left(\frac{\partial}{\partial \vec{k}_T}\right)^2 H(\vec{p}_T,\vec{k}_T=0)=0$. Indeed, azimuthal averaging gives

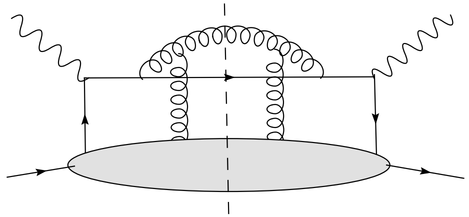
$$\langle H(\vec{p}_T,\vec{k}_T)\rangle = \frac{1}{p_T^2}\theta(k_T-p_T) R(p_T,z,\xi)\,, \qquad R(p,z,\xi) = 1-\cos\left(\frac{\vec{p}_T^2\xi}{2Ez(1-z)}\right).$$

The factor R acts as an infrared regulator at $p_T \lesssim 1/\rho_{dif}$, $\rho_{dif} \sim \sqrt{\xi/Ez(1-z)}$ is the typical separation in the final qg state at the longitudinal scale ξ



GWZ's expansion

For off-diagonal part of the hard part GWZ obtain $\left(\frac{\partial}{\partial \vec{k}_T}\right)^2 H_{off-d}(\vec{p}_T, \vec{k}_T = 0) = 0$. But for the diagonal part they have non-zero result. It comes from the diagram (for $z \ll 1$)



GWZ use for the integral variable the final gluon momentum l_T, then

$$H^{GWZ}_{diag}(\vec{l}_T, \vec{k}_T, z, \xi) \propto \frac{R(\vec{l}_T - \vec{k}_T)}{(\vec{l}_T - \vec{k}_T)^2}\,, \qquad R(\vec{l}_T - \vec{k}_T) = 1 - \cos\left(\frac{i(\vec{l}_T - \vec{k}_T)^2 \xi}{2Ez(1-z)}\right).$$

GWZ DO NOT DIFFERENTIATE the factor R, and obtain

$$\left(\frac{\partial}{\partial \vec{k}_T}\right)^2 \frac{R(\vec{l}_T - \vec{k}_T)}{(\vec{l}_T - \vec{k}_T)^2}\Bigg|_{k=0} = R(\vec{l}_T)\left(\frac{\partial}{\partial \vec{k}_T}\right)^2 \frac{1}{(\vec{l}_T - \vec{k}_T)^2}\Bigg|_{k=0} = \frac{4R(\vec{l}_T)}{l_T^4}\,.$$

However, the neglected terms are important. If one includes them one obtains

$$\left(\frac{\partial}{\partial \vec{k}_T}\right)^2 \int d\vec{l}_T \, \frac{R(\vec{l}_T - \vec{k}_T)}{(\vec{l}_T - \vec{k}_T)^2}\Bigg|_{k=0} \approx \left(\frac{\partial}{\partial \vec{k}_T}\right)^2 \int d\vec{l}_T \, \frac{R(\vec{l}_T)}{l_T^2} = 0\,.$$

Collinear expansion and oscillator approximation

Vanishing the $N = 1$ contribution in the collinear expansion agrees with the BDMPS spectrum for massless partons. Indeed

$$\sigma_{q\bar{q}}(y_T) \propto \int d\vec{b}_T \langle N|[W(\vec{y}_T + \vec{b}_T) - W(\vec{b}_T)]^2|\rangle\,, \qquad W(\vec{y}_T) = \int dy^- A^+(y^-, \vec{y}_T)\,.$$

Collinear expansions is equivalent to the linear approximation

$$A^+(y^-, \vec{y}_T + \vec{b}_T) \approx A^+(y^-, \vec{b}_T) + \vec{y}_T \frac{\partial}{\partial \vec{y}_T} A^+(y^-, \vec{b}_T)\,, \qquad \Rightarrow \qquad \sigma_{q\bar{q}}(y_T) \propto \vec{y}_T^{\,2} \langle F_T^{+\,2}\rangle\,.$$

It corresponds to the oscillator approximation in the LCPI and BDMPS approaches. The gluon spectrum for $\sigma_{q\bar{q}g}(\rho) = C\rho^2$ reads

$$\frac{dP_{BDMPS}}{dz} = \frac{\alpha_s P_{Gq}(z)}{\pi} \ln|\cos\Omega L|\,, \qquad \Omega = \sqrt{-i\frac{Cn}{z(1-z)E}}$$

$$\Rightarrow \quad \frac{dP_{BDMPS}}{dz} \approx \frac{\alpha_s P_{Gq}(z)}{16\pi} \frac{L^4 C^2 n^2}{[z(1-z)E]^2} \quad \Rightarrow \quad \text{It is } N = 2 \text{ rescattering!} \quad \text{[BGZ, (2001)]}$$

Conclusions:

- Conceptually starting point of the GWZ higher twist method is equivalent to evaluation of the $N = 1$ rescattering term in the LCPI approach. The GWZ method cannot give something new.
- The GWZ's hard parts before the collinear expansion agree with that obtained using the perturbative formula $\langle bc|M|a\rangle \propto \int dy^- d\vec{y}_T \phi_b^*(y^-, \vec{y}_T)\phi_c^*(y^-, \vec{y}_T)\phi_a(y^-, \vec{y}_T)$ with the plane wave functions affected by the t-channel gluon kicks.
- The collinear expansion in the GWZ analysis is performed incorrectly: some important terms are not included. Inclusion of the missed terms gives vanishing $N = 1$ gluon spectrum.
- The collinear expansion is equivalent to the oscillator approximation in the LCPI and BDMPS approaches formulated in the impact parameter space. Vanishing $N = 1$ spectrum within the collinear expansion agrees with absence of the $N = 1$ contribution to the spectrum evaluated by BDMPS in the oscillator model.