Second order viscous hydrodynamics from AdS/CFT

Michał P. Heller
heller@th.if.uj.edu.pl

Department of Theory of Complex Systems
Institute of Physics, Jagiellonian University

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[R. Janik, MPH], [+ A. Buchel, P. Benincasa]

Motivation

- heavy ion collisions @ RHIC - strongly coupled quark-gluon plasma (QGP)
- fully dynamical process - need for a new tool
- idea: exchange

  QCD in favor of $\mathcal{N} = 4$ SYM

  and use the gravity dual
- there are differences
  - SUSY
  - conformal symmetry at the quantum level
  - no confinement...
- ... but not very important at high temperature

Hydrodynamics

- RHIC suggests that QGP behaves as an

  almost perfect fluid

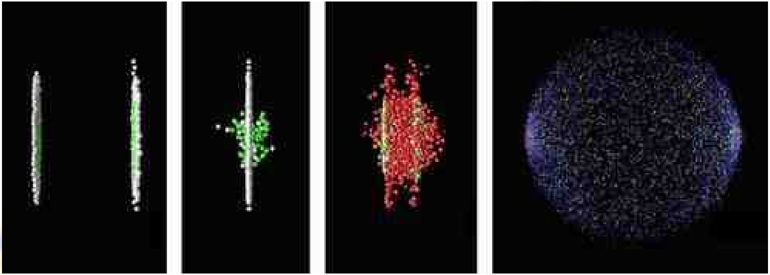
- this raises the question

  what are the transport coefficients of this medium?

- they are encoded in QGP energy-momentum tensor
- let's study then

  the dynamics of energy momentum tensors of gauge theories with gravity duals

Bjorken Hydrodynamics



- one-dimensional expansion along the collision axis x^1
- natural coordinates
  - proper time τ and rapidity y
  - $x^0 = \tau \cosh y$, $x^1 = \tau \sinh y$
- boost invariance (no rapidity dependence)

Boost-invariant dynamics of $\mathcal{N} = 4$ SYM

- hydro = long-wavelength theory (gradient expansion)
- causal theory - second order viscous hydrodynamics
- Bjorken expansion

$T^{\mu\nu}$ is expressed in terms of $T^{00} = \epsilon(\tau)$

- equations of motion ($\nabla_\mu T^{\mu\nu} = 0$) fix the large-$\tau$ expansion

$$\epsilon(\tau) = \left(\frac{N_c^2}{2\pi^2}\right) \frac{1}{\tau^{4/3}} \left\{ 1 - \frac{2\eta_0}{\tau^{2/3}} + \frac{3}{2}(1-2r)\eta_0^2 \frac{1}{\tau^{4/3}} + \cdots \right\}$$

where η_0 - shear viscosity, $\tau_\pi = r\,\tau_\pi^{(\mathrm{Boltzmann})}$

- let's AdS/CFT to get the constants

Gauge-gravity duality is an equivalence between

$\mathcal{N} = 4$ **Supersymmetric Yang-Mills in** $\mathbb{R}^{1,3}$

- strong coupling
- non-perturbative results
- gauge theory operators

Superstrings in curved AdS$_5\times$S^5 10D spacetime

- (super)gravity regime
- classical behavior
- supergravity fields

AdS/CFT dictionary relates **energy-momentum tensor** of $\mathcal{N} = 4$ SYM to 5D **AdS metric**

Gravity dual counterpart

- to extract $T_{\mu\nu}$ we start with the metric

$$\mathrm{d}s^2_{AdS} = \frac{\tilde{g}_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}z^2}{z^2}$$

  and take the limit

$$T_{\mu\nu} = \lim_{z\to 0} \frac{g_{\mu\nu}}{z^4}$$

- large-τ (gradient) expansion has a natural counterpart

$$g_{\mu\nu}(\tau, z) = g^{(0)}_{\mu\nu}\left(\frac{z}{\tau^{1/3}}\right) + \frac{1}{\tau^{2/3}} g^{(1)}_{\mu\nu}\left(\frac{z}{\tau^{1/3}}\right) + \frac{1}{\tau^{4/3}} g^{(2)}_{\mu\nu}\left(\frac{z}{\tau^{1/3}}\right) + \ldots$$

- the metric is the solution of Einstein equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - 6 g_{\mu\nu} = 0$$

  order by order in gradient $\left(\frac{1}{\tau^{2/3}}\right)$ expansion

- the constants η_0 and r are fixed by requiring

regularity of curvature invariants (e.g. $R_{\mu\nu\delta\alpha}R^{\mu\nu\delta\alpha}$)

Main result:

- gauge theory dynamics from AdS/CFT

$$\epsilon(\tau) = \left(\frac{N_c^2}{2\pi^2}\right) \frac{1}{\tau^{4/3}}\left\{1 - \frac{2\eta_0}{\tau^{2/3}} + \frac{3}{2}(1-2r)\eta_0^2 \frac{1}{\tau^{4/3}} + \cdots\right\}$$

Asymptotic behavior:

- perfect fluid behavior from nonsingularity
- temperature cools down as $\frac{1}{\tau^{1/3}}$

Viscosity:

- $\eta_0 = \frac{1}{2^{1/2}3^{3/4}}$
- it saturates the bound $\left(\frac{\eta}{s} = \frac{1}{4\pi}\right)$

Relaxation time:

- $\tau_\pi = r\,\tau_\pi^{(Boltzmann)}$, where $r = \frac{1}{3}(1 - \ln 2) \approx 0.1$

Conclusions

What's good:

- dynamics @ strong coupling
- agrees with near-equilibrium methods

What's bad:

- singularities in the bulk
  (possible signatures of plasma instabilities ?)

Future applications:

- Bjorken expansion of other conformal plasmas
  (A.Buchel, MPH, R.Myers, S.Vazquez - in preparation)
- other symmetric flows of $\mathcal{N} = 4$ SYM
  (work in progress)