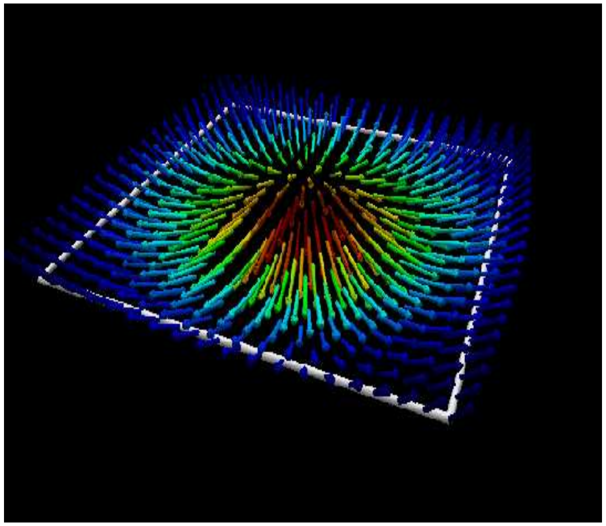
Holographic QCD and Baryons

QCD-Moriond, March 8 - 15, 2008



Based on hep-th/0701276, arXiv:0705.2632, arXiv:0710.4615
with M. Rho, H.-U. Yee, and P. Yi

Deog Ki Hong (Pusan Nat'l U.)

I. Holographic QCD

- If you look at vector mesons,

$$\rho(770), \quad \rho^{(1)}(1450), \ \rho^{(2)}(1700), \cdots , \tag{1}$$

$$\omega(782), \quad \omega^{(1)}(1420), \ \omega^{(2)}(1650), \cdots . \tag{2}$$

- They might be obtained by KK reduction from 5D vector fields,

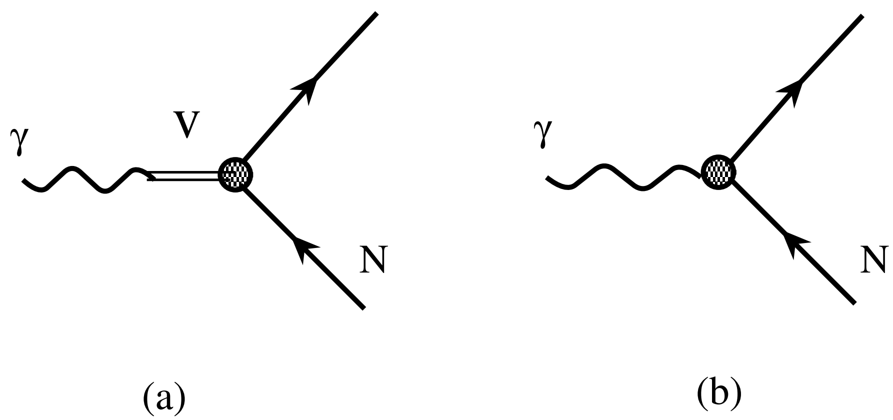
$$A_\mu(x,z) = \sum_{n=0}^{\infty} f_n(z) A_\mu^{(n)}(x), \quad D_z f_n(z) = -m_n^2 f_n(z) . \tag{3}$$

$\longrightarrow$ Holography!

- In string theory people discovered such holography between $\mathcal{N} = 4$ SYM in 4D and type IIB closed string on $AdS_5 \times S^5$ in the large N_C and large $g^2 N_C$ limit at low energy: AdS/CFT correspondence.

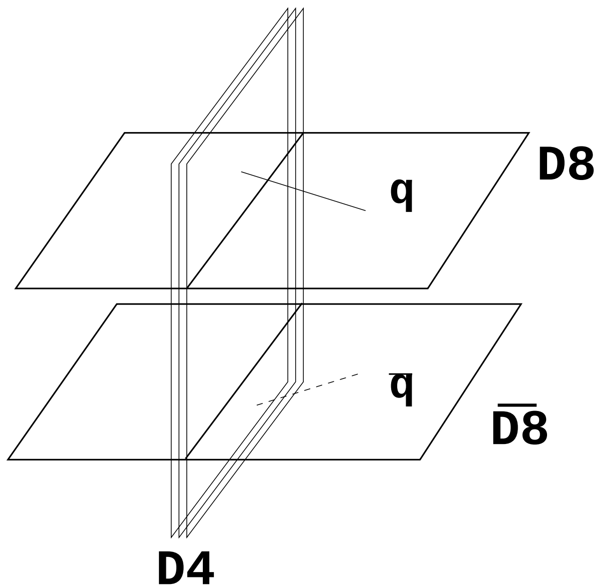
- Holographic QCD is an attempt to find such a gravity dual for QCD, the theory with mesons and baryons as dynamical variables.

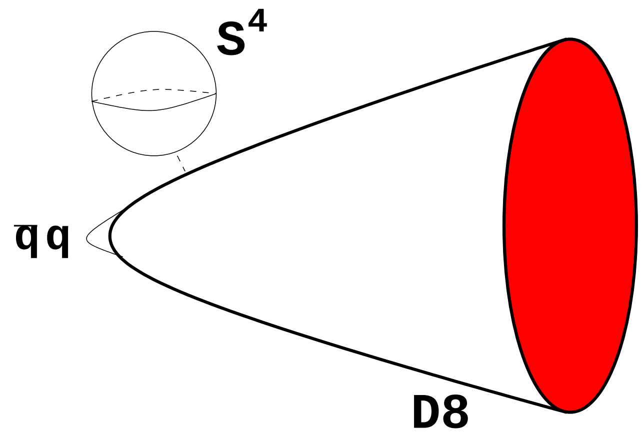
- One of the consequences of holographic QCD is vector meson dominance, where whole tower of vectors contribute. → New VMD.



- If the new VMD is verified experimentally, it will indicate strongly that QCD has a hidden symmetry, which is best described in a five dimensional spacetime with a warped factor. How the 5D is curved at the hadronic scale can be inferred from the spectra of vector mesons and the form factors of hadrons: Bottom-up approach

- Most successful top-down approach is Sakai-Sugimoto model (2004).





$$ds^2 = \left(\frac{U}{R}\right)^{3/2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + f(U)d\tau^2\right) + \left(\frac{R}{U}\right)^{3/2}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

with $R^3 = \pi g_s N_c l_s^3$ and $f(U) = 1 - U_{KK}^3/U^3$.

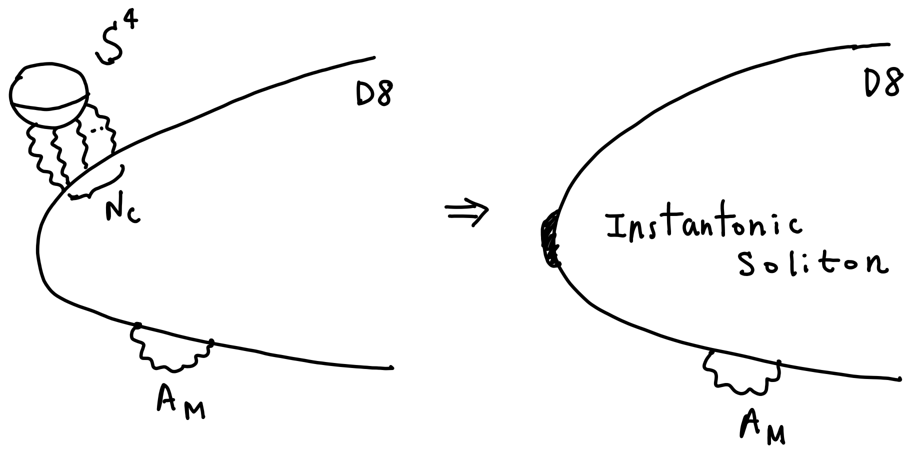
- Effective action on D8 is a $U(N_F)$ gauge theory,

$$S_{D8} = -\mu_8 \int \mathrm{d}^9 x\, e^{-\phi} \sqrt{-\det\left(g_{MN} + 2\pi\alpha' F_{MN}\right)} + \mu_8 \int \sum C_{p+1} \wedge \mathrm{Tr}\, e^{2\pi\alpha' F} ,$$

- Either top-down or bottom-up holographic QCD is a 5D flavor gauge theory, describing mesons (pions and vector mesons or others), which are low energy excitations of open string.

II. Baryons in hQCD

- What are baryons in hQCD? It must be solitons.
- In SS model, D4 brane wrapping S^4 is the baryon vertex (Witten).



- In the SS model the DBI action tends to shrink the solitons but the Coulomb repulsion stabilizes them and the radius of soliton (Rho+Yee+Yi+DKH, Hata+Sakai+Sugimoto+Yamato)

$$\rho_{baryon} \sim \frac{9.6}{M_{KK}\sqrt{g_{YM}^2 N_c}}, \tag{4}$$

where $M_{KK} \simeq 1$ GeV is the UV cut-off of SS model.

- At low energy the baryons are point-like and described by

$$\int d^4x dw \left[-i\bar{\mathcal{B}}\gamma^m D_m \mathcal{B} - i m_b(w) \bar{\mathcal{B}}\mathcal{B} + g_5(w) \frac{\rho^2_{baryon}}{e^2(w)} \bar{\mathcal{B}}\gamma^{mn} F_{mn} \mathcal{B} \right] - \int d^4x dw \frac{1}{4e^2(w)} \operatorname{tr} F_{mn} F^{mn} + \cdots , \tag{5}$$

- The spinor sources YM fields

$$\nabla^2 A^a_m = 2 g_5(0) \rho^2_{baryon} \bar{\eta}^a_{mn} \partial_n \delta^{(4)}(x) , \tag{6}$$

whose solution is compared with the 't Hooft ansatz

$$A^a_m = -\bar{\eta}^a_{mn} \partial_n \log \left(1 + \frac{\rho^2}{r^2 + w^2} \right) \simeq -\rho^2 \bar{\eta}^a_{mn} \partial_n \frac{1}{r^2 + w^2} , \tag{7}$$

- Including the quantum fluctuations to match the long-range instanton tale (Adkins+Nappi+Witten),

$$g_5(0) = \frac{2\pi^2}{3} \tag{8}$$

- The Lagrangian is unique up to operators with two derivatives in the large N_c and large $\lambda = g_s^2 N_c$ and valid for $E < M_{KK}$.
- Though the coefficient of the Pauli term might be model dependent, the fact that it contains only the nonabelian part of the flavor symmetry is model-independent!
  $\longrightarrow$ The $U(1)$ coupling does not have the Pauli term.
- One immediate consequence of this is that the Pauli form factor

$$F_2^p(q^2) = -F_2^n(q^2) + \text{h.o.}. \tag{9}$$

- Especially for instance $\mu_{\text{an}}^p + \mu_{\text{an}}^n = 0$, which is very close to the experimental value, $(\mu_{\text{an}}^p + \mu_{\text{an}}^n)_{\text{exp}} = 1.79\mu_N - 1.91\mu_N = -0.12\mu_N$

III. Phenomenology: Static properties of baryons

- Once you are given the holographic action, you can get various couplings of baryons after KK reduction.
- Vector couplings of baryons,

$$\begin{aligned} g_{\rm min}^{(n)} &= \int_{-w_{max}}^{w_{max}} dw \, |f_L(w)|^2 \, \psi_{(n)}(w) \,, \\ g_{\rm mag}^{(n)} &= 2C\int_w dw \left(\frac{g_5(w)U(w)}{g_5(0)U_{KK}M_{KK}} \right) |f_L(w)|^2 \, \partial_w \psi_{(n)}(w) \,. \end{aligned} \quad (10)$$

- For SS model in the large N_c,

$$C = \frac{6}{\pi^2}\frac{\lambda N_c}{108\pi^3}(\rho_{baryon}M_{KK})^2 \simeq 0.18 N_c \,. \quad (11)$$

For bottom-up, C can be fixed by the anomalous magnetic moment.

- The axial coupling for the SS model with $\lambda N_c = 50$

$$g_A \approx 1.30 - 1.31, \quad g_A^{\text{exp}} = 1.2670 \pm 0.0035 \tag{12}$$

- **The ρNN and ωNN coupling constants:**

1. In the large λ limit

$$|g_{\omega^{(k)}NN}| \simeq N_c \times |g_{\rho^{(k)}NN}| \tag{13}$$

2. For $\lambda N_c = 50$ in the SS model the couplings get corrections from the subleading Pauli term

$$g_{\rho NN} \approx 3.6, \quad g_{\omega NN} \approx 12.6 \tag{14}$$

Thus the relation (13) is modified to

$$\mathcal{R} \equiv \frac{g_{\omega NN}}{3 g_{\rho NN}} \approx 1.2 \tag{15}$$

$$g_{\rho NN}^{\text{emp}} \approx 4.2 - 6.5, \;\; \mathcal{R} \approx 1.1 - 1.5. \tag{16}$$

- Magnetic moments:

$$\frac{\mu^{an}_{proton}}{e_{EM}} = \frac{0.18 N_c}{M_{KK}}, \qquad \frac{\mu^{an}_{neutron}}{e_{EM}} = -\frac{0.18 N_c}{M_{KK}}. \tag{17}$$

- With the shift $N_c \to N_c + 2$ and $m_B \simeq M_{KK}$

$$\mu_p = 1 + 1.08 \left(\frac{N_c + 2}{3}\right) \simeq 2.8, \; \mu_n = -1.08 \left(\frac{N_c + 2}{3}\right) \simeq -1.8$$

The experimental values, $\mu_p = 2.79 \mu_N$ and $\mu_n = -1.91 \mu_N$.

- Form factors:

$$\langle p'|\, J^\mu(x)\, |p\rangle = e^{iqx}\, \bar{u}(p')\, \mathcal{O}^\mu(p,p')\, u(p), \quad q = p' - p \tag{18}$$

$$\mathcal{O}^\mu = \gamma^\mu \left[\frac{1}{2}F_1^S(q^2) + F_1^a(q^2)\tau^a\right] + \frac{\gamma^{\mu\nu}}{2m_B} q_\nu \left[F_2^S(q^2) + F_2^a(q^2)\tau^a\right], \tag{19}$$

- By AdS/CFT correspondence we compute the matrix elements.
- We need to identify the dual bulk fields of external current.

$\longrightarrow$ Bulk photon fields.

- Both $U(1)_B$ and $SU(2)_I$ contribute to form factors, since

$$Q_{\rm em} = \frac{1}{2}B + I_3\,. \tag{20}$$

- We now write the (nonnormalizable) photon field as

$$A_\mu(x,w) = \int_q A_\mu(q) A(q,w)\, e^{iqx} \,, \tag{21}$$

with boundary conditions that $A(q,w)=1$ and $\partial_w A(q,w)=0$ at the UV boundary, $w=\pm w_{\max}$ and the (normalizable) bulk baryon field as

$$\mathcal{B}(w,x) = \int_p \left[f_L(w) u_L(p) + f_R(w) u_R(p)\right] e^{ipx} \,. \tag{22}$$

- From the AdS/CFT correspondence one can read off the form factors from the 5D action.

- Dirac form factor $F_1(Q^2) = F_{1\text{min}}\, Q_{\text{em}} + F_{1\text{mag}}\, I_3$ with $(Q^2 \equiv -q^2)$

$$F_{1\text{min}}(Q^2) = \int_{-w_{max}}^{w_{max}} dw\, |f_L(w)|^2\, A(q,w)\,, \quad (23)$$

$$F_{1\text{mag}}(Q^2) = 2\int_{-w_{max}}^{w_{max}} dw\, \kappa(w)\, |f_L(w)|^2\, \partial_w A(q,w)\,, \quad (24)$$

where $\kappa(w) = 0.18 N_c/M_{KK}$.

- Pauli form factor is given as $F_2(Q^2) = F_2^3(Q^2)\, I_3$ with

$$F_2^3(Q^2) = 4\, m_N \int_{-w_{max}}^{w_{max}} dw\, \kappa(w) f_L^*(w) f_R(w)\, A(q,w)\,, \quad (25)$$

New Vector Meson Dominance

- If we expand the nonnormalizable photon field in terms of the normalizable vector meson ψ_{2k+1} of mass m_{2k+1} as

$$A(q,w) = \sum_k \frac{g_{v^{(k)}} \psi_{(2k+1)}(w)}{Q^2 + m_{2k+1}^2} \,, \tag{26}$$

- EM form factors then take the form

$$F_1(Q^2) = \sum_{k=1}^{\infty} \left(g_{V,min}^{(k)} Q_{\text{em}} + \frac{5}{3} g_{V,mag}^{(k)} \, \tau^3 \right) \frac{\zeta_k m_{2k+1}^2}{Q^2 + m_{2k+1}^2} \,, \tag{27}$$

$$F_2(Q^2) = \sum_{k=1}^{\infty} \frac{g_2^{(k)} \zeta_k m_{2k+1}^2}{Q^2 + m_{2k+1}^2} \,, \tag{28}$$

where

$$g^{(k)}_{V,min} = \int_{-w_{max}}^{w_{max}} dw \, |f_L(w)|^2 \, \psi_{(2k+1)}(w) \tag{29}$$

$$g^{(k)}_{V,mag} = 2 \int_{-w_{max}}^{w_{max}} dw \, \kappa(w) \, |f_L(w)|^2 \, \partial_w \psi_{(2k+1)}(w) \, ,$$

$$g^{(k)}_{2} = 4 m_N \int_{-w_{max}}^{w_{max}} dw \, \kappa(w) f^*_L(w) f_R(w) \psi_{(2k+1)}(w) \, . \tag{30}$$

- The charge and magnetic moment sum rules are saturated for protons (and similarly for neutrons) within a few %:

$$F^p_1(0) \equiv 1 \simeq \sum_{k=1}^{4} \left(g^{(k)}_{V,min} + \frac{5}{6} g^{(k)}_{V,mag} \right) \zeta_k = 1.04,$$

$$F^p_2(0) \equiv \mu_p - 1 \simeq \frac{5}{6} \sum_{k=1}^{4} g^{(k)}_2 \zeta_k = 1.66 \, . \tag{31}$$

- Sachs form factors $G_M = F_1 + F_2$ and $G_E = F_1 - \frac{Q^2}{4m_B^2} F_2$:

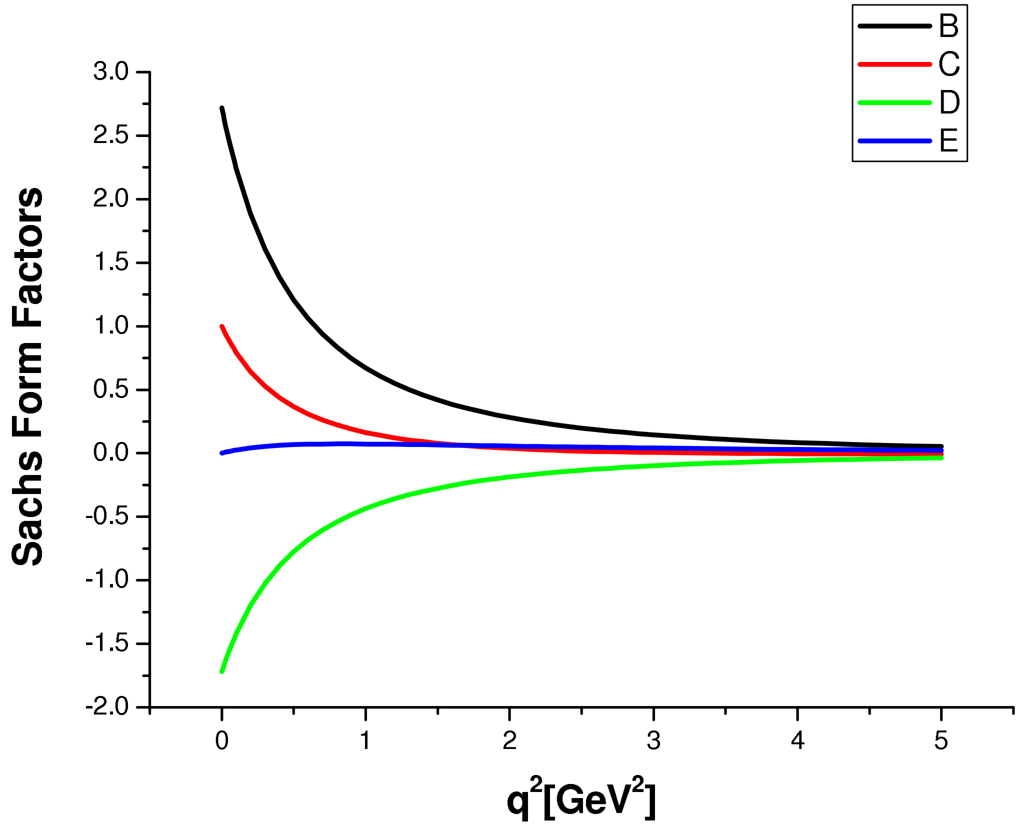
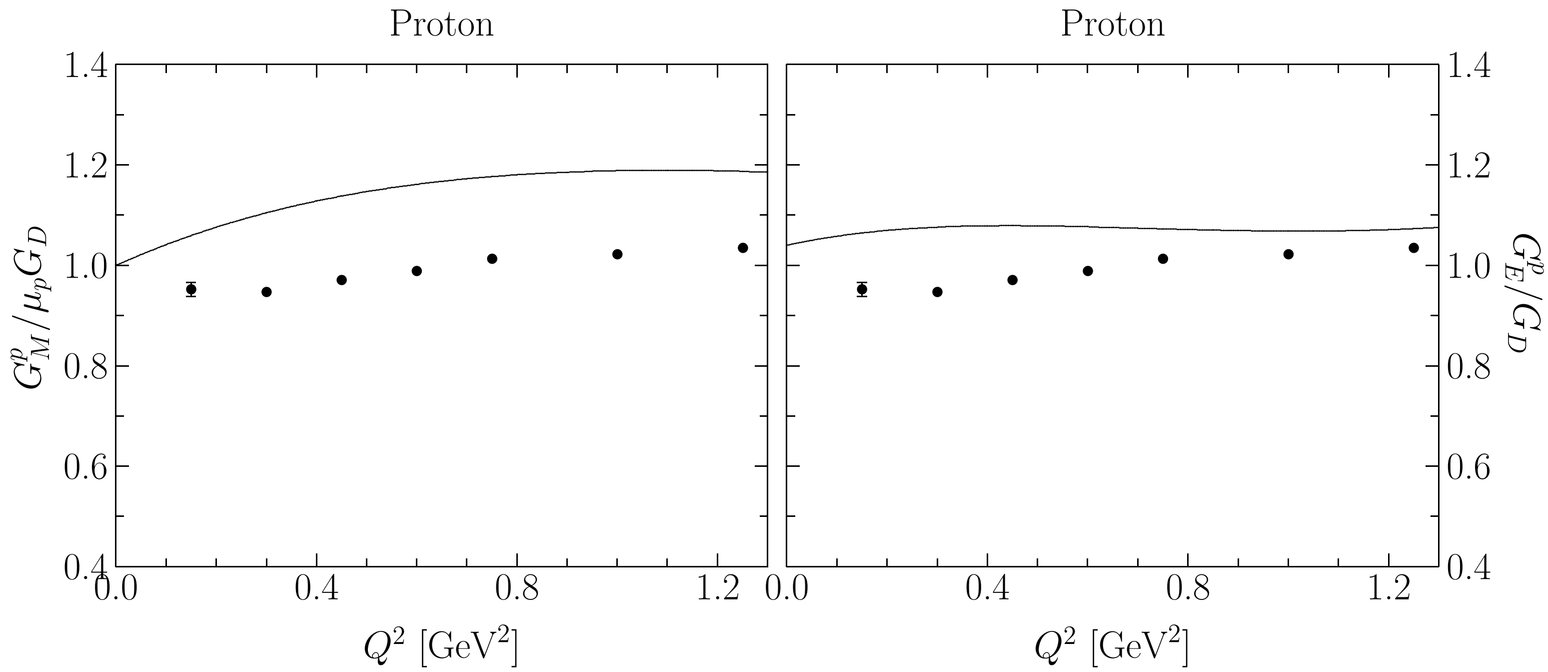


그림 1: The Sachs form factors: B=G_M^p, C=G_E^p, D=G_M^n, and E=G_E^n

- To see how well our form factors fit the experimental data (R. C. Walker *et al.* (1994) and Jones et al (2000)), we plot the ratio with the dipole form factors, $G_D = 1/(1+q^2/0.71)^2$ and the ratio of the magnetic and electric form factors: (Rho+Yee+Yi+DKH, arXiv:0710.4615)



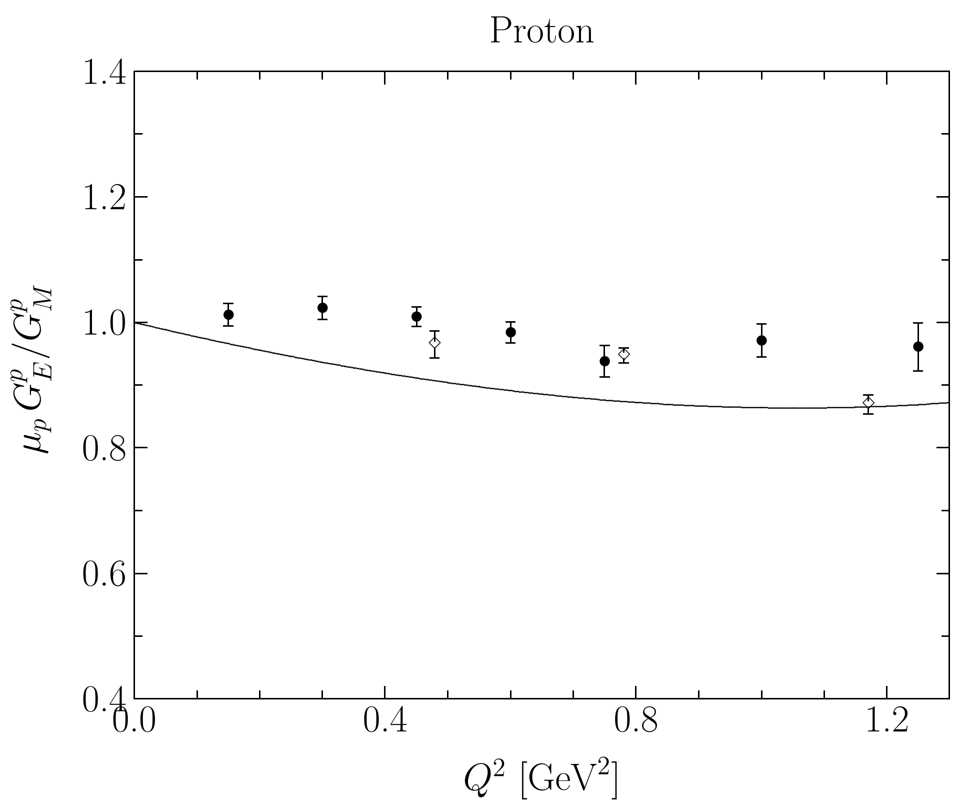


그림 2: The open cicles are the polarization measurements at JLab and the filled circles are the data taken from Walker et al. (1994). The solid line is the prediction in the SS model.

IV. Conclusion and Outlook

- Baryons are realized as 4D instanton solitions in holographic QCD, which are made of pions and whole towers of vector mesons.
- The effective chiral Lagrangian for baryons is uniquely determined up to the Pauli term.
- New VMD is a key feature of holgraphic QCD: Form factors, $\cdots$.
- As a model hQCD gives relations to low energy parameters of hadrons. Low energy parameters of hadrons are determined by a few parameters in 5D up to $1/N_C$:
  1. Mass spectrum. Magnetic moments of baryons and g_A: $g_A \sim \mu_{\rm an}$
  2. Various couplings with vector mesons: $g_{\omega NN} \approx N_c\, g_{\rho NN}, \cdots$.
- Furthermore, it has model-independent predictions, insensitive to $1/N_c$ corr: New sum rules due to the instanton nature of baryons:

$$\mu^p_{\rm an} + \mu^n_{\rm an} = 0, \quad d_n + d_p = 0. \tag{32}$$