Phenomenology of the rare decay $\bar{B} \to X_s \ell^+ \ell^-$

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In collaboration with E. Lunghi, M. Misiak, D. Wyler, T. Hurth,
based on arXiv:0712.3009 and hep-ph/0512066

Les Rencontres de Moriond, March 9th, 2008

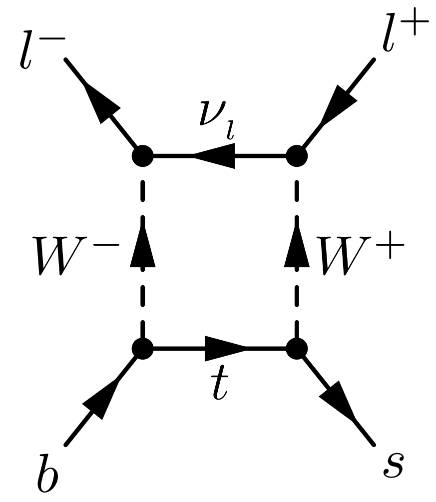
General Features of $\bar{B} \to X_s \ell^+ \ell^-$

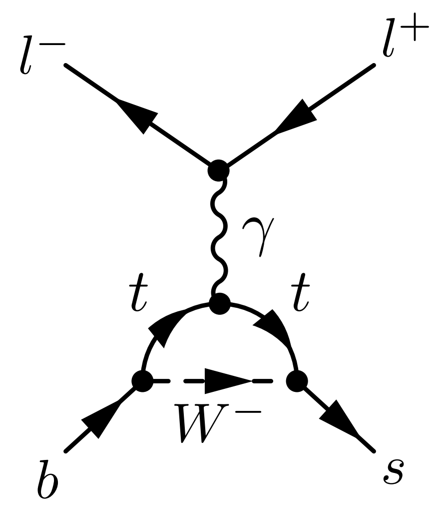
- Rare decay, FCNC process,
  probes SM directly at one-loop level

- Sensitive to new physics
  beyond the SM

- A complementary SM test to $\bar{B} \to X_s \gamma$

- Precision in both experiment
  and theory needed and achievable





General features of $B \to X_s \ell^+ \ell^-$

- Differential decay width: (q^2 : lepton inv. mass; $\hat{s} \equiv q^2/m_b^2$)

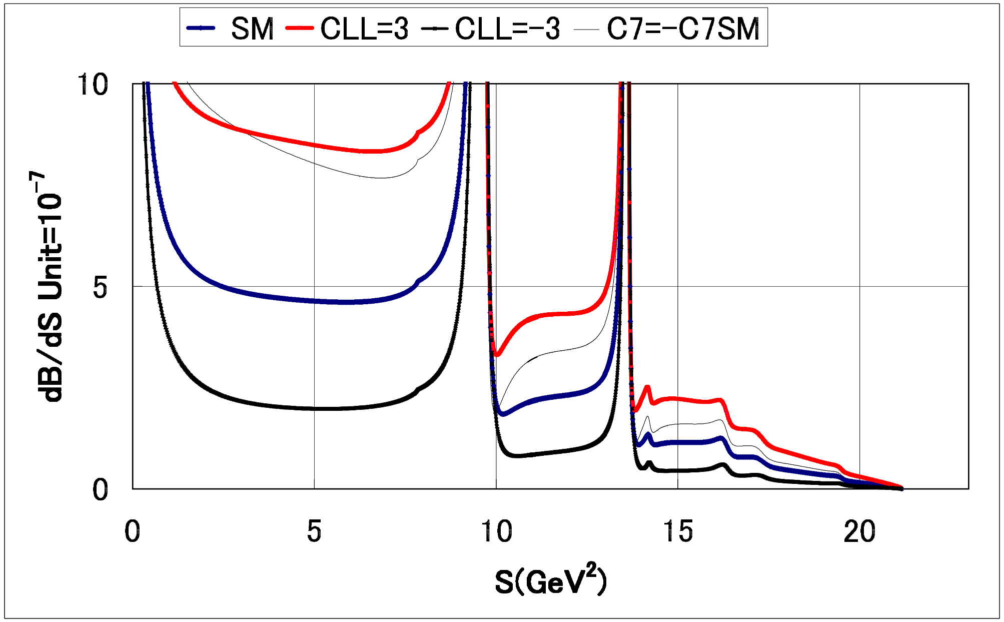
$$\frac{d\Gamma(\bar{B} \to X_s\, l^+ l^-)}{d\hat{s}} = \frac{G_F^2 m_b^5 |V_{ts}^* V_{tb}|^2 \alpha_{em}^2(\mu)(1-\hat{s})^2}{768\pi^5}$$

$$\times\left\{\left(4+\tfrac{8}{\hat{s}}\right)\left|\tilde{C}_7^{eff}\right|^2 + (1+2\hat{s})\left(\left|\tilde{C}_9^{eff}\right|^2 + \left|\tilde{C}_{10}^{eff}\right|^2\right) + 12\,\mathrm{Re}\left(\tilde{C}_7^{eff}\tilde{C}_9^{*\,eff}\right) + \frac{d\Gamma^{brems}}{d\hat{s}}\right\}$$

- Compare to:

$$\Gamma(\bar{B} \to X_s\, \gamma) \propto \left|\tilde{C}_7^{eff}\right|^2$$

- SM size and signs of amplitudes
  - $\tilde{C}_7^{eff} \simeq -0.30$
  - $\tilde{C}_9^{eff} \simeq +4.05$
  - $\tilde{C}_{10}^{eff} \simeq -4.26$

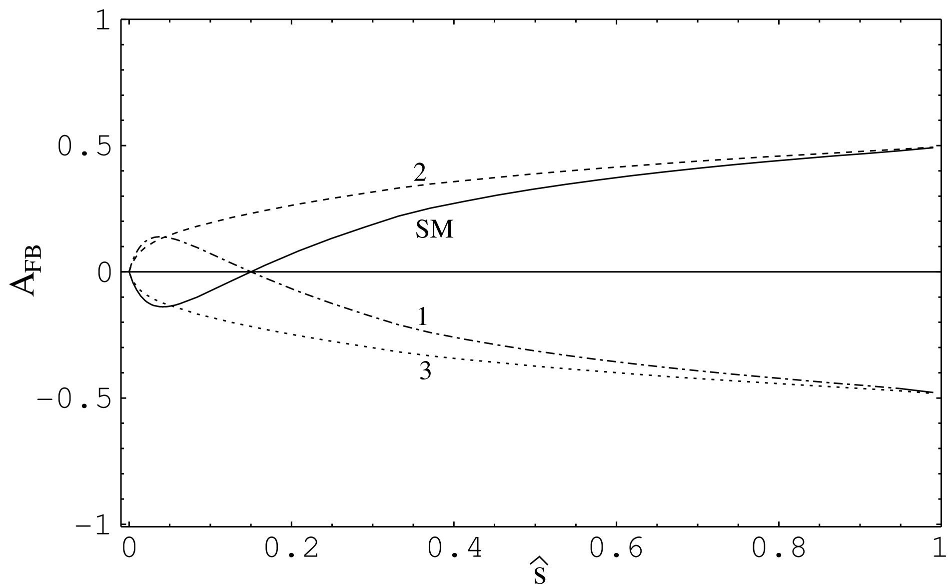


[Akeroyd et. al.]

Forward backward asymmetry

- Forward backward asymmetry:
  - $$\mathscr{A}_{FB}(q^2) \equiv \frac{d\mathrm{BR}_{\ell\ell}/dq^2(\cos\theta_l > 0) - d\mathrm{BR}_{\ell\ell}/dq^2(\cos\theta_l < 0)}{d\mathrm{BR}_{\ell\ell}/dq^2(\cos\theta_l > 0) + d\mathrm{BR}_{\ell\ell}/dq^2(\cos\theta_l < 0)}$$
  - $$\mathscr{A}_{FB}(\hat{s}) = \frac{G_F^2 m_b^5 |V_{ts}^* V_{tb}|^2 \alpha_{em}^2(\mu)(1-\hat{s})^2}{768\pi^5}$$
    $$\times \left\{ -6\,\mathrm{Re}\left(\tilde{C}_{7,FB}^{eff} \tilde{C}_{10,FB}^{*\,eff}\right) - 3\hat{s}\,\mathrm{Re}\left(\tilde{C}_{9,FB}^{eff} \tilde{C}_{10,FB}^{*\,eff}\right) + A_{FB}^{brems} \right\}$$
- Zero of FBA represents SM precision observable (theor. uncertainty $\sim 5\%$)
- A measurement of $d\mathrm{BR}_{\ell\ell}/d\hat{s}$ and $\mathscr{A}_{FB}(\hat{s})$ can provide information on the sign of $\tilde{C}_7^{eff}$, which again will allow to constrain parameter space of new physics models.

[Gambino,Haisch,Misiak]
[Wyler,Misiak,Cho]



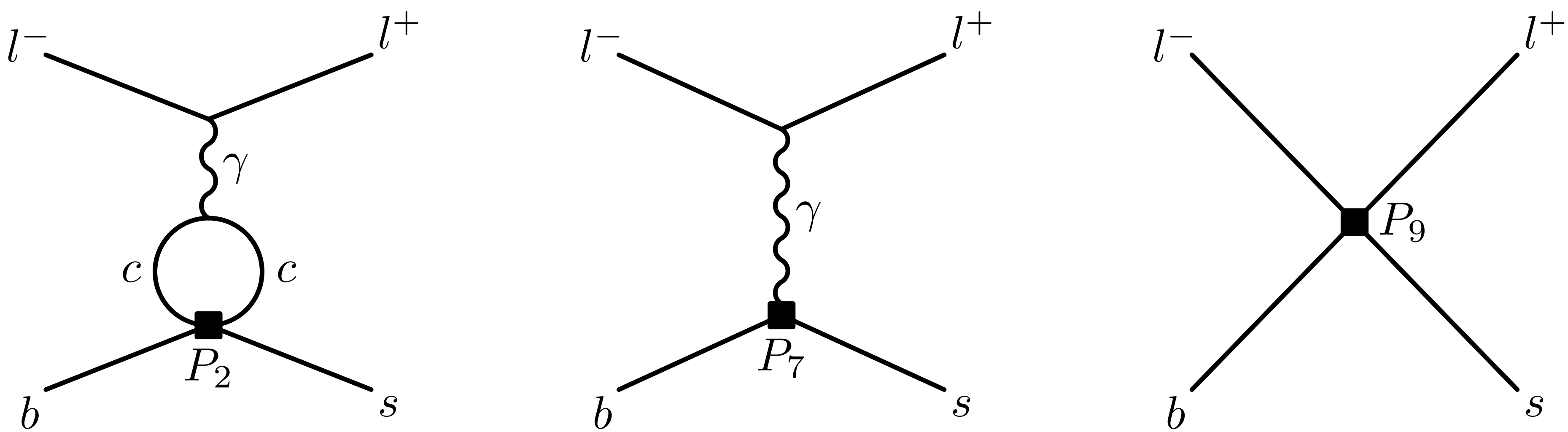
[Ali,Greub,Hiller,Lunghi, $\hat{s} \equiv q^2/m_b^2$]
[1: C10=-C10SM, 2: C7=-C7SM, 3: C(7,10)=-C(7,10)SM]

Effective Lagrangian

$$\mathscr{L}_{eff} = \mathscr{L}_{QCD\times QED}(u,..,b,e,\mu,\tau) + \frac{4G_F}{\sqrt{2}}\, V_{ts}^* V_{tb} \cdot \left[\sum_{i=1}^{10} C_i\, P_i + \underbrace{\sum_{i=3}^{6} C_{iQ}\, P_{iQ} + C_b\, P_b}_{for\ QED\ corrections} \right]$$

$$\begin{aligned} P_1 &= (\bar{s}_L\gamma_\mu T^a c_L)(\bar{c}_L\gamma^\mu T^a b_L), & P_4 &= (\bar{s}_L\gamma_\mu T^a b_L)\textstyle\sum_q(\bar{q}\gamma^\mu T^a q), \\ P_2 &= (\bar{s}_L\gamma_\mu c_L)(\bar{c}_L\gamma^\mu b_L), & P_5 &= (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\sigma b_L)\textstyle\sum_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\sigma q), \\ P_3 &= (\bar{s}_L\gamma_\mu b_L)\textstyle\sum_q(\bar{q}\gamma^\mu q), & P_6 &= (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\sigma T^a b_L)\textstyle\sum_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\sigma T^a q), \end{aligned}$$

$$\begin{aligned} P_7 &= \tfrac{e}{16\pi^2} m_b(\bar{s}_L\sigma^{\mu\nu} b_R)F_{\mu\nu}, & P_9 &= (\bar{s}_L\gamma_\mu b_L)\textstyle\sum_l(\bar{l}\gamma^\mu l), \\ P_8 &= \tfrac{g}{16\pi^2} m_b(\bar{s}_L\sigma^{\mu\nu} T^a b_R)G^a_{\mu\nu}, & P_{10} &= (\bar{s}_L\gamma_\mu b_L)\textstyle\sum_l(\bar{l}\gamma^\mu\gamma_5 l), \end{aligned}$$



Effective Lagrangian

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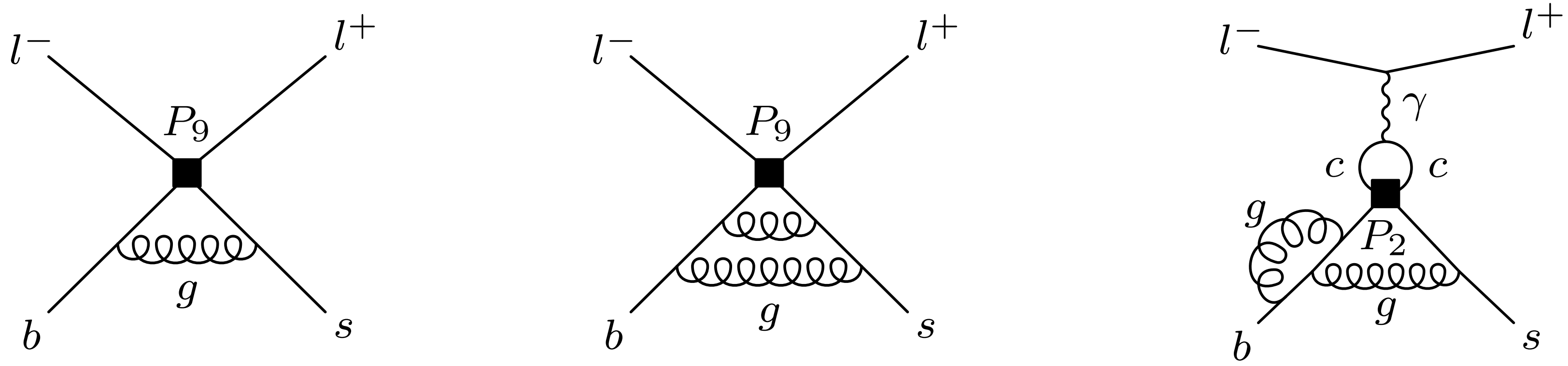
$$\begin{aligned}
P_1 &= (\bar{s}_L\gamma_\mu T^a c_L)(\bar{c}_L\gamma^\mu T^a b_L), & P_4 &= (\bar{s}_L\gamma_\mu T^a b_L)\textstyle\sum_q(\bar{q}\gamma^\mu T^a q),\\
P_2 &= (\bar{s}_L\gamma_\mu c_L)(\bar{c}_L\gamma^\mu b_L), & P_5 &= (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\sigma b_L)\textstyle\sum_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\sigma q),\\
P_3 &= (\bar{s}_L\gamma_\mu b_L)\textstyle\sum_q(\bar{q}\gamma^\mu q), & P_6 &= (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\sigma T^a b_L)\textstyle\sum_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\sigma T^a q),
\end{aligned}$$

$$\begin{aligned}
P_7 &= \tfrac{e}{16\pi^2} m_b(\bar{s}_L\sigma^{\mu\nu} b_R)F_{\mu\nu}, & P_9 &= (\bar{s}_L\gamma_\mu b_L)\textstyle\sum_l(\bar{l}\gamma^\mu l),\\
P_8 &= \tfrac{g}{16\pi^2} m_b(\bar{s}_L\sigma^{\mu\nu} T^a b_R)G^a_{\mu\nu}, & P_{10} &= (\bar{s}_L\gamma_\mu b_L)\textstyle\sum_l(\bar{l}\gamma^\mu\gamma_5 l),
\end{aligned}$$

$$\begin{aligned}
P_{3Q} &= (\bar{s}_L\gamma_\mu b_L)\textstyle\sum_q Q_q(\bar{q}\gamma^\mu q),\\
P_{4Q} &= (\bar{s}_L\gamma_\mu T^a b_L)\textstyle\sum_q Q_q(\bar{q}\gamma^\mu T^a q),\\
P_{5Q} &= (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\sigma b_L)\textstyle\sum_q Q_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\sigma q),\\
P_{6Q} &= (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\sigma T^a b_L)\textstyle\sum_q Q_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\sigma T^a q),\\
P_b &= \tfrac{1}{12}\left[(\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\sigma b_L)(\bar{b}\gamma^\mu\gamma^\nu\gamma^\sigma b) - 4(\bar{s}_L\gamma_\mu b_L)(\bar{b}\gamma^\mu b)\right].
\end{aligned}$$

Perturbative and non-pert. Corrections

- QCD corrections to quark level decay rate are known to NNLO
  [Misiak, Buras, Münz, Bobeth, Urban, Asatrian, Asatryan, Greub, Walker]
  [Bobeth, Gambino, Gorbahn, Haisch, Bieri, Ghinculov, Hurth, Isidori, Yao]
  - Diagrams involved:

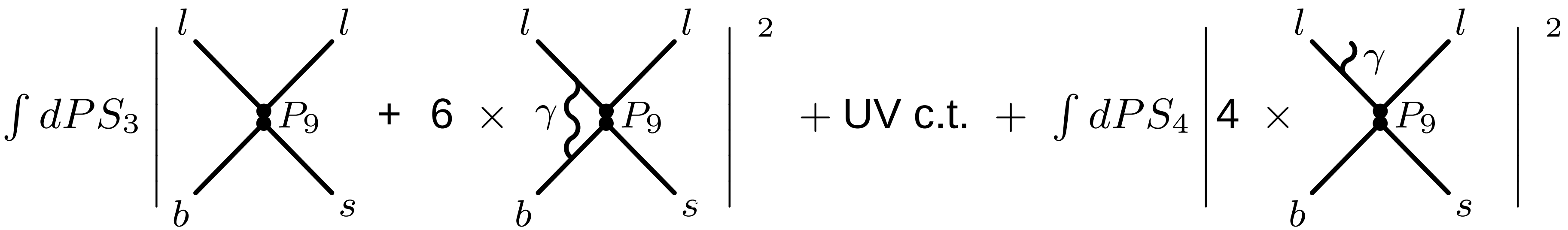


- Order $\mathcal{O}(\alpha_s^2)$ ME of P_9 for high-q^2 region now also known analytically
  [Blokland,Czarnecki,Melnikov,Slusarczyk]
- $1/m_b^2$, $1/m_b^3$ and $1/m_c^2$ non-pert. corrections *[Falk, Luke, Savage, Ali, Hiller, Handoko, Morozumi]* *[Bauer, Burrell, Buchalla, Isidori, Rey]*
  - impact in the low-q^2 region is at the few percent level
  - large impact in the high-q^2 region due to breakdown at the endpoint
- Factorizable $c\bar{c}$ contributions implemented via the KS approach *[Krüger,Sehgal]*

QED Corrections

- NLO QED corrections
  - are expected to be larger than N^3LO QCD corrections.
  - reduce $\pm 4\%$ scale uncertainty due to
    $\alpha_e(m_b) \approx 1/133$ vs. $\alpha_e(m_Z) \approx 1/128$.
  - This $\pm 4\%$ uncertainty is as large as NNLO QCD precision.

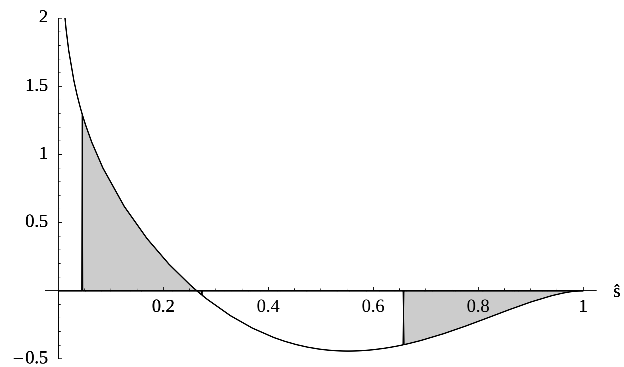
- Calculation of NLO QED corrections is threefold
  - Matching and running calculation;
    checked to agree with [Bobeth, Gambino, Gorbahn, Haisch]
  - Finite corrections, matrix elements of the P_i
  - Phenomenologically most important: IR divergent ME's



NLO QED Matrix Elements

- Contrary to the integrated decay width, the differential decay width is not an infrared safe object with respect to the emission of collinear photons from lepton lines.
- Collinear divergence manifests itself through large EM logarithm

$$\frac{\alpha_e}{4\pi}\left[\log\left(\frac{m_b^2}{m_l^2}\right)\cdot h(\hat{s})+k(\hat{s})\right] \quad \text{with} \quad \int_0^1 d\hat{s}\;\,h(\hat{s})=0.$$



- Plot: Log-enhanced contribution to $|\langle P_9\rangle|^2$
- Relative effect of this log is much larger for high q^2 than for low q^2.
- Include also log-enhanced corrections to $|\langle P_7\rangle|^2$, $|\langle P_{10}\rangle|^2$, $Re\left[\langle P_i\rangle\langle P_j\rangle^*\right]$
- Presence of $\log\left(m_b^2/m_l^2\right)$ depends on exptl. setup (finite detector resolution)
  - Separation of muons and collinear photons is no problem
  - e^-: inside an angle θ_c collinear γ's are included in 4-momentum [Berryhill, Ishikawa]

$$q^2=(p_+ + p_- + p_\gamma)^2 \qquad m_\ell^2 \le (p_\ell+p_\gamma)^2 \le \Lambda^2 \simeq 2E_\ell^2(1-\cos\theta_c) \qquad \Lambda\sim\mathcal{O}(m_\mu)$$

Results: BR in low-q^2-region

- Including all NLO-QED corrections: *[Lunghi, Misiak, Wyler, TH]*

$$BR(\bar{B} \to X_s \mu\mu) = (1.59 \pm 0.08_{scale} \pm 0.06_{m_t} \pm 0.015_{m_b} \pm 0.024_{C,m_c} \pm 0.02_{\alpha_s(M_Z)} \pm 0.015_{CKM} \pm 0.026_{\mathrm{BR}_{sl}}) \cdot 10^{-6} = (1.59 \pm 0.11) \cdot 10^{-6}$$

$$BR(\bar{B} \to X_s ee) = (1.64 \pm 0.08_{scale} \pm 0.06_{m_t} \pm 0.015_{m_b} \pm 0.025_{C,m_c} \pm 0.02_{\alpha_s(M_Z)} \pm 0.015_{CKM} \pm 0.026_{\mathrm{BR}_{sl}}) \cdot 10^{-6} = (1.64 \pm 0.11) \cdot 10^{-6}$$

- Experimental values:
  - $BR(\bar{B} \to X_s ll) = (1.493 \pm 0.504_{stat.}\,{}^{+0.411}_{-0.321}{}_{sys.}) \cdot 10^{-6}$ *[Belle, 152 M evts.]*
  - $BR(\bar{B} \to X_s ll) = (1.8 \pm 0.7_{stat.} \pm 0.5_{sys.}) \cdot 10^{-6}$ *[BaBar, 89 M events]*
  - weighted average: $(1.60 \pm 0.51) \cdot 10^{-6}$

- With reversed sign of $\widetilde{C}_7^{\mathrm{eff}}$
  - $BR(\bar{B} \to X_s \mu\mu) = 3.11 \cdot 10^{-6}$ $\quad$ $BR(\bar{B} \to X_s ee) = 3.19 \cdot 10^{-6}$
    - SM-sign of $\widetilde{C}_7^{\mathrm{eff}}$ is favored
    - Models with positive $\widetilde{C}_7^{\mathrm{eff}}$ require sizable contributions to $\widetilde{C}_9^{\mathrm{eff}}$ and $\widetilde{C}_{10}^{\mathrm{eff}}$

[Gambino,Haisch,Misiak]

BR in high-q^2-region

- Branching ratio integrated over $q^2 > 14.4\,\mathrm{GeV}^2$ *[Hurth,Lunghi,TH]*

$$
\begin{aligned}
\mathcal{B}_{\mu\mu}^{\rm high} &= 2.40\times 10^{-7}\Big(1+\left[{}^{+0.01}_{-0.02}\right]_{\mu_0}+\left[{}^{+0.14}_{-0.06}\right]_{\mu_b}\pm 0.02_{m_t}+\left[{}^{+0.006}_{-0.003}\right]_{C,m_c}\pm 0.05_{m_b} \\
&\quad +\left[{}^{+0.0002}_{-0.001}\right]_{\alpha_s}\pm 0.002_{\rm CKM}\pm 0.02_{{\rm BR}_{sl}}\pm 0.05_{\lambda_2}\pm 0.19_{\rho_1}\pm 0.14_{f_s}\pm 0.02_{f_u}\Big) \\
&= 2.40\times 10^{-7}\,(1^{+0.29}_{-0.26})
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{B}_{ee}^{\rm high} &= 2.09\times 10^{-7}\Big(1+\left[{}^{+0.02}_{-0.04}\right]_{\mu_0}+\left[{}^{+0.16}_{-0.08}\right]_{\mu_b}\pm 0.02_{m_t}+\left[{}^{+0.005}_{-0.0009}\right]_{C,m_c}\pm 0.05_{m_b} \\
&\quad +\left[{}^{+0.0003}_{-0.002}\right]_{\alpha_s}\pm 0.002_{\rm CKM}\pm 0.02_{{\rm BR}_{sl}}\pm 0.05_{\lambda_2}\pm 0.22_{\rho_1}\pm 0.16_{f_s}\pm 0.02_{f_u}\Big) \\
&= 2.09\times 10^{-7}\,(1^{+0.32}_{-0.30})
\end{aligned}
$$

- Largest uncertainties from poorly known $O(1/m_b^3)$ power corrections
- Impact of EM logs is -8% (μ) and -20% (e), that of KS reson. -10% (μ) and -12% (e)
- Experimental values:
  - $BR(\bar{B}\to X_s ll) = (4.18 \pm 1.17_{stat.}\,{}^{+0.61}_{-0.68}{}_{sys.})\cdot 10^{-7}$ *[Belle, 152 M evts.]*
  - $BR(\bar{B}\to X_s ll) = (5 \pm 2.5_{stat.}\,{}^{+0.8}_{-0.7}{}_{sys.})\cdot 10^{-7}$ *[BaBar, 89 M events]*

The ratio $\mathcal{R}(s_0)$

- Introduction of the ratio $$\mathcal{R}(s_0) = \frac{\int_{\hat{s}_0}^{1} \mathrm{d}\hat{s}\ \ d\Gamma(\bar{B} \to X_s \ell^+ \ell^-)/d\hat{s}}{\int_{\hat{s}_0}^{1} \mathrm{d}\hat{s}\ \ d\Gamma(\bar{B}^0 \to X_u \ell \nu)/d\hat{s}}$$ [Ligeti,Tackmann]
  - Normalize to semileptonic $\bar{B}^0 \to X_u \ell \nu$ rate with the same cut
  - Impact of non-perturbative $1/m_b^2$ and $1/m_b^3$ power corrections drastically reduced
- For lower integration limit $s_0 = 14.4\,\mathrm{GeV}^2$ one obtains

$$\begin{aligned}\mathcal{R}(s_0)_{\mu\mu} &= 2.29 \times 10^{-3}\Big(1 \pm 0.04_{\rm scale} \pm 0.02_{m_t} \pm 0.01_{C,m_c} \pm 0.006_{m_b} \pm 0.005_{\alpha_s} \\ &\qquad \pm 0.09_{\rm CKM} \pm 0.003_{\lambda_2} \pm 0.05_{\rho_1} \pm 0.03_{f_u^0+f_s} \pm 0.05_{f_u^0-f_s}\Big) \\ &= 2.29 \times 10^{-3}(1 \pm 0.13)\end{aligned}$$

$$\begin{aligned}\mathcal{R}(s_0)_{ee} &= 1.94 \times 10^{-3}\Big(1 \pm 0.06_{\rm scale} \pm 0.02_{m_t} \pm 0.02_{C,m_c} \pm 0.004_{m_b} \pm 0.006_{\alpha_s} \\ &\qquad \pm 0.09_{\rm CKM} \pm 0.01_{\lambda_2} \pm 0.09_{\rho_1} \pm 0.05_{f_u^0+f_s} \pm 0.05_{f_u^0-f_s}\Big) \\ &= 1.94 \times 10^{-3}(1 \pm 0.16)\end{aligned}$$

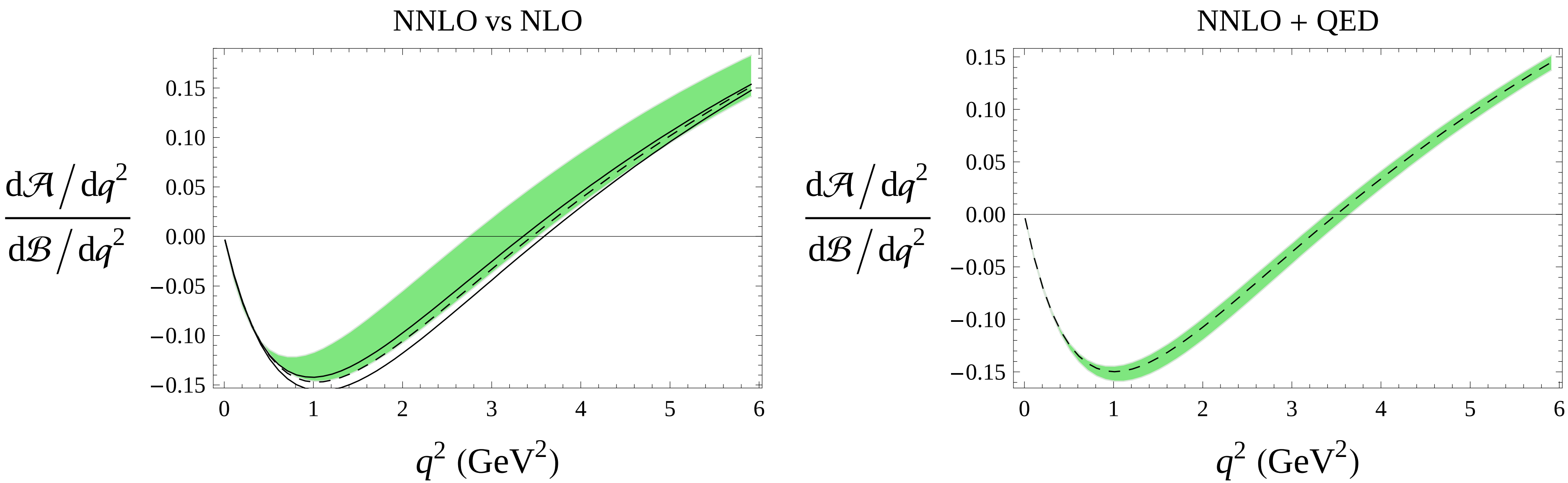
- Uncertainties from poorly known $O(1/m_b^3)$ power corrections are under control
- Largest source of error is V_{ub}
- Impact of EM logs is -9% (μ) and -23% (e), that of KS reson. -11% (μ) and -12% (e)

Zero of FBA and integrated FBA

[Hurth,Lunghi,TH]

$$(q_0^2)_{\mu\mu} = \left[\,3.50 \pm 0.10_{\rm scale} \pm 0.002_{m_t} \pm 0.04_{m_c,C} \pm 0.05_{m_b} \pm 0.03_{\alpha_s(M_Z)} \pm 0.001_{\lambda_1} \pm 0.01_{\lambda_2}\,\right] \mathrm{GeV}^2$$
$$= (3.50 \pm 0.12)\, \mathrm{GeV}^2$$

$$(q_0^2)_{ee} = \left[\,3.38 \pm 0.09_{\rm scale} \pm 0.002_{m_t} \pm 0.04_{m_c,C} \pm 0.04_{m_b} \pm 0.03_{\alpha_s(M_Z)} \pm 0.002_{\lambda_1} \pm 0.01_{\lambda_2}\,\right] \mathrm{GeV}^2$$
$$= (3.38 \pm 0.11)\, \mathrm{GeV}^2$$



- Integrated FBA: Pronounced sensitivity to NP
- Bin 1 ($q^2 \in [1, 3.5]\,\mathrm{GeV}^2$) bin 2 ($q^2 \in [3.5, 6]\,\mathrm{GeV}^2$) low-$\hat{s}$ ($q^2 \in [1, 6]\,\mathrm{GeV}^2$)

$$(\bar{\mathcal{A}}_{\mu\mu})_{\rm bin1} = [\,-9.09 \pm 0.91\,]\,\% \qquad (\bar{\mathcal{A}}_{\mu\mu})_{\rm bin2} = [\,+7.80 \pm 0.76\,]\,\% \qquad (\bar{\mathcal{A}}_{\mu\mu})_{\rm low} = [\,-1.50 \pm 0.90\,]\,\%$$
$$(\bar{\mathcal{A}}_{ee})_{\rm bin1} = [\,-8.14 \pm 0.87\,]\,\% \qquad (\bar{\mathcal{A}}_{ee})_{\rm bin2} = [\,+8.27 \pm 0.69\,]\,\% \qquad (\bar{\mathcal{A}}_{ee})_{\rm low} = [\,-0.86 \pm 0.85\,]\,\%$$

Third independent observable

[Lee,Ligeti,Stewart,Tackmann]

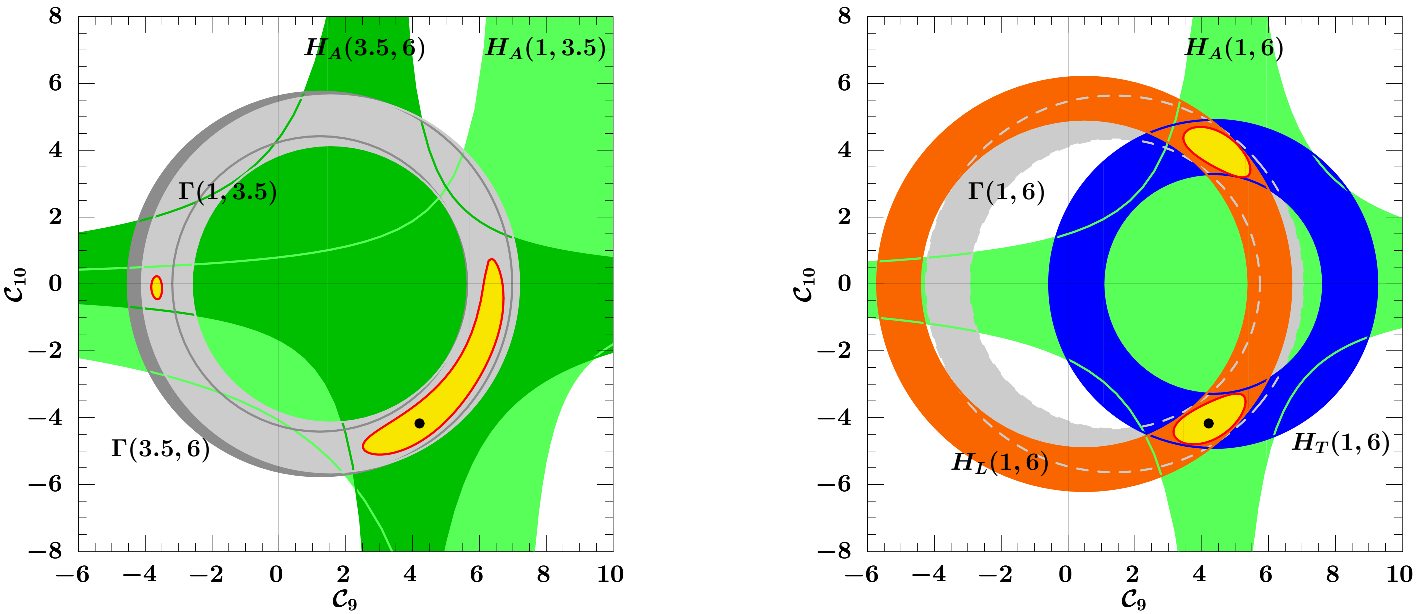
- Recent proposal: 3rd independent combination of Wilson Coefficients: ($z = \cos\theta$)

$$\frac{d^2\Gamma}{dq^2\,dz} = 3/8\,\left[(1+z^2)\,H_T(q^2) + 2\,z\,H_A(q^2) + 2\,(1-z^2)\,H_L(q^2)\right]$$

Note: $$\frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2)\,, \qquad \frac{dA_{\rm FB}}{dq^2} = 3/4\,H_A(q^2)$$

- Current data extrapolated to 1ab^{-1}.

  Size of C_7 (< 0) taken from $\bar{B} \to X_s\,\gamma$. Constraints in C_9-C_{10} plane:



Summary and Outlook

- $\bar{B} \to X_s \ell^+ \ell^-$ serves as a precision test for the SM and is a sensitive probe for new physics
- Higher order perturbative corrections are indispensable for obtaining precision data to test the SM
- Together with $\bar{B} \to X_s \gamma$, all relevant WC's can be predicted within the SM and constraints on NP can be set.
- FCNC precision observables serve as an important ingredient for synergy and complementarity between flavour and collider physics

Backup slides

Numerical inputs

$\alpha_s(M_z) = 0.1189 \pm 0.0010$	$m_e = 0.51099892$ MeV
$\alpha_e(M_z) = 1/127.918$	$m_\mu = 105.658369$ MeV
$s_W^2 \equiv \sin^2\theta_W = 0.2312$	$m_\tau = 1.77699$ GeV
$\|V_{ts}V_{tb}/V_{cb}\|^2 = 0.962 \pm 0.002$	$m_c(m_c) = (1.224 \pm 0.017 \pm 0.054)$ GeV
$\|V_{ts}V_{tb}/V_{ub}\|^2 = (1.28 \pm 0.12) \times 10^2$	$m_b^{1S} = (4.68 \pm 0.03)$ GeV
$BR(B \to X_c e\bar{\nu})_{\rm exp} = 0.1061 \pm 0.0017$	$m_{t,\rm pole} = (170.9 \pm 1.8)$ GeV
$M_Z = 91.1876$ GeV	$m_B = 5.2794$ GeV
$M_W = 80.426$ GeV	$C = 0.58 \pm 0.01$
$\lambda_2^{\rm eff} = (0.12 \pm 0.02)$ GeV^2	$\rho_1 = (0.06 \pm 0.06)$ GeV^3
$\lambda_1^{\rm eff} = (-0.243 \pm 0.055)$ GeV^2	$f_u^0 + f_s = (0 \pm 0.2)$ GeV^3
$f_u^0 - f_s = (0 \pm 0.04)$ GeV^3	$f_u^\pm = (0 \pm 0.4)$ GeV^3

Organizing the Expansion

- Amplitude: $\kappa = \alpha_e(\mu_b)/\alpha_s(\mu_b)$

$$\begin{aligned} \mathscr{A} &= \kappa \left[\mathscr{A}_{LO} + \widetilde{\alpha}_{\rm s}\, \mathscr{A}_{NLO} + \widetilde{\alpha}_{\rm s}^2\, \mathscr{A}_{NNLO} + \mathcal{O}(\widetilde{\alpha}_{\rm s}^3)\right] \\ &+ \kappa^2 \left[\mathscr{A}_{LO}^{em} + \widetilde{\alpha}_{\rm s}\, \mathscr{A}_{NLO}^{em} + \widetilde{\alpha}_{\rm s}^2\, \mathscr{A}_{NNLO}^{em} + \mathcal{O}(\widetilde{\alpha}_{\rm s}^3)\right] + \mathcal{O}(\kappa^3) \end{aligned}$$

- Decay width:

$$\begin{aligned} |\mathscr{A}|^2 &= \kappa^2 \left[\mathscr{A}_{LO}^2 + 2\,\widetilde{\alpha}_{\rm s}\, \mathscr{A}_{LO}\mathscr{A}_{NLO} + \widetilde{\alpha}_{\rm s}^2\, \mathscr{A}_{NLO}^2\right] \quad \Leftarrow \text{QCD, NLO} \\ \text{QCD, NNLO} \Rightarrow &+ \kappa^2 \left[2\,\widetilde{\alpha}_{\rm s}^2\, \mathscr{A}_{LO}\mathscr{A}_{NNLO} + 2\,\widetilde{\alpha}_{\rm s}^3\, \mathscr{A}_{NLO}\mathscr{A}_{NNLO} + \ldots\right] \\ \text{QED} \Rightarrow &+ \kappa^3 \left[2\,\mathscr{A}_{LO}\mathscr{A}_{LO}^{em} + 2\,\widetilde{\alpha}_{\rm s}\left(\mathscr{A}_{NLO}\mathscr{A}_{LO}^{em} + \mathscr{A}_{LO}\mathscr{A}_{NLO}^{em}\right)\right. \\ &\quad + 2\,\widetilde{\alpha}_{\rm s}^2\left(\mathscr{A}_{NLO}\mathscr{A}_{NLO}^{em} + \mathscr{A}_{NNLO}\mathscr{A}_{LO}^{em} + \mathscr{A}_{LO}\mathscr{A}_{NNLO}^{em}\right) \\ &\quad \left. + 2\,\widetilde{\alpha}_{\rm s}^3\left(\mathscr{A}_{NLO}\mathscr{A}_{NNLO}^{em} + \mathscr{A}_{NNLO}\mathscr{A}_{NLO}^{em}\right) + \ldots\right] \end{aligned}$$

- Accidentially: $\mathscr{A}_{LO} \sim \widetilde{\alpha}_{\rm s}\, \mathscr{A}_{NLO}$ and $A_{LO}^{em} \sim \widetilde{\alpha}_{\rm s}\, \mathscr{A}_{NLO}^{em}$

$\Rightarrow$ quite high terms in the expansion remain numerically important

Definition of observables

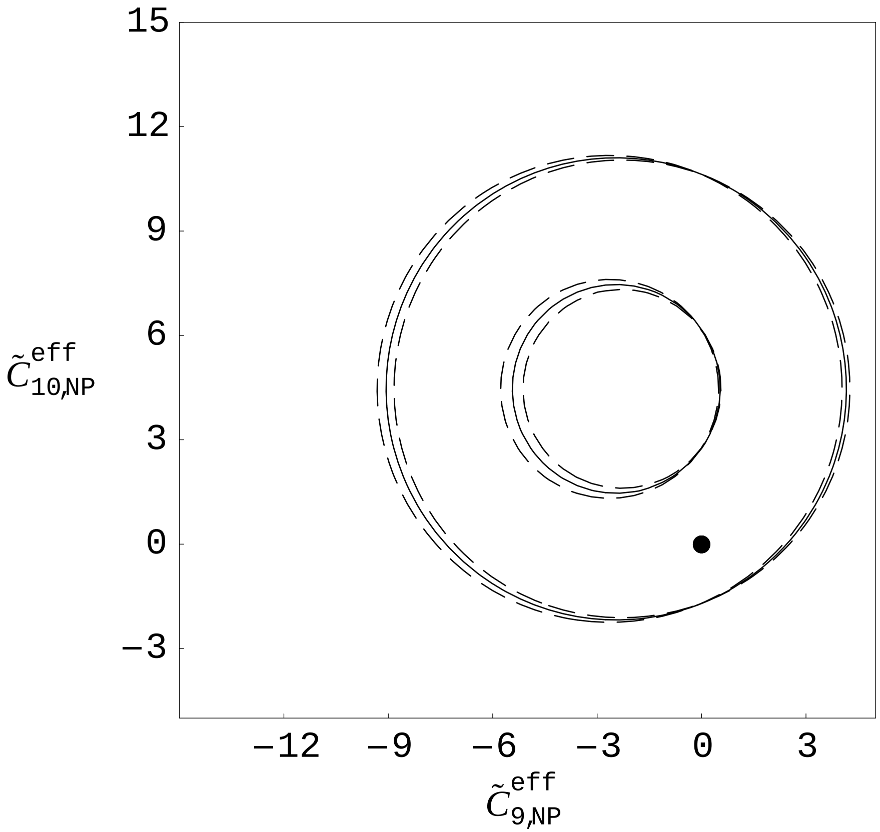
- $$\frac{d\mathcal{B}(\bar{B} \to X_s \ell^+ \ell^-)}{d\hat{s}} = BR^{exp.}_{b \to c\, e\, \nu} \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{1}{C} \frac{d\Gamma(\bar{B} \to X_s \ell^+ \ell^-)/d\hat{s}}{\Gamma(\bar{B} \to X_u e \bar{\nu})}$$
- Normalize the differential decay width and bare FBA to the semilep. $\bar{B} \to X_u e \bar{\nu}$ rate
  - Removes pre-factor $m^5_{b,pole}$ and avoids phase space factors involving $m_{c,pole}$
  - $$C = \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\Gamma(\bar{B} \to X_c e \bar{\nu})}{\Gamma(\bar{B} \to X_u e \bar{\nu})} = 0.58 \pm 0.01$$ *[Bauer,Ligeti,Luke,Manohar,Trott]*
  - We assume 100 % correlation between the errors on C and m_c
- $$\frac{d\mathcal{A}_{\rm FB}(\bar{B} \to X_s \ell^+ \ell^-)}{d\hat{s}} = BR^{exp.}_{b \to c\, e\, \nu} \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{1}{C} \frac{d\Gamma^{\rm FB}(\bar{B} \to X_s \ell^+ \ell^-)/d\hat{s}}{\Gamma(\bar{B} \to X_u e \bar{\nu})}$$

$$\frac{d\Gamma^{\rm FB}(\bar{B} \to X_s \ell^+ \ell^-)}{d\hat{s}} = \int_{-1}^{1} {\rm d}z\, {\rm sgn}(z)\, \frac{{\rm d}^2\Gamma(\bar{B} \to X_s \ell^+ \ell^-)}{{\rm d}\hat{s}\, {\rm d}z} \qquad (z = \cos\theta_\ell)$$

Definition of observables

- Total Forward backward asymmetry: $[\frac{d\mathcal{A}_{\rm FB}(\bar{B}\to X_s\ell^+\ell^-)}{d\hat{s}}]\,/\,[\frac{d\mathcal{B}(\bar{B}\to X_s\ell^+\ell^-)}{d\hat{s}}]$
  - Each of the brackets is normalized to Γ_u and gets fully expanded in the couplings, but no overall expansion is done:
  - No expansion of badly converging $d\Gamma(\bar{B}\to X_s\ell^+\ell^-)/d\hat{s}$
  - Closest to experiment
  - Small scheme dependence on b quark mass due to removal of $m_{b,pole}^5$-factor
  - Stable, well-converging perturbative expansion
  - Each for the square brackets alone is made renormalon-free and is observable
- Introduction of the ratio $\mathcal{R}(s_0) = \dfrac{\int_{\hat{s}_0}^1 \mathrm{d}\hat{s}\, \frac{d\Gamma(\bar{B}\to X_s\ell^+\ell^-)}{d\hat{s}}}{\int_{\hat{s}_0}^1 \mathrm{d}\hat{s}\, \frac{d\Gamma(\bar{B}^0\to X_u\ell\nu)}{d\hat{s}}}$ *[Ligeti,Tackmann]*
  - Normalize to semileptonic $\bar{B}^0\to X_u\ell\nu$ rate with the same cut
  - Impact of non-perturbative $1/m_b^2$ and $1/m_b^3$ power corrections drastically reduced

More on sign of $\widetilde{C}_7^{\text{eff}}$



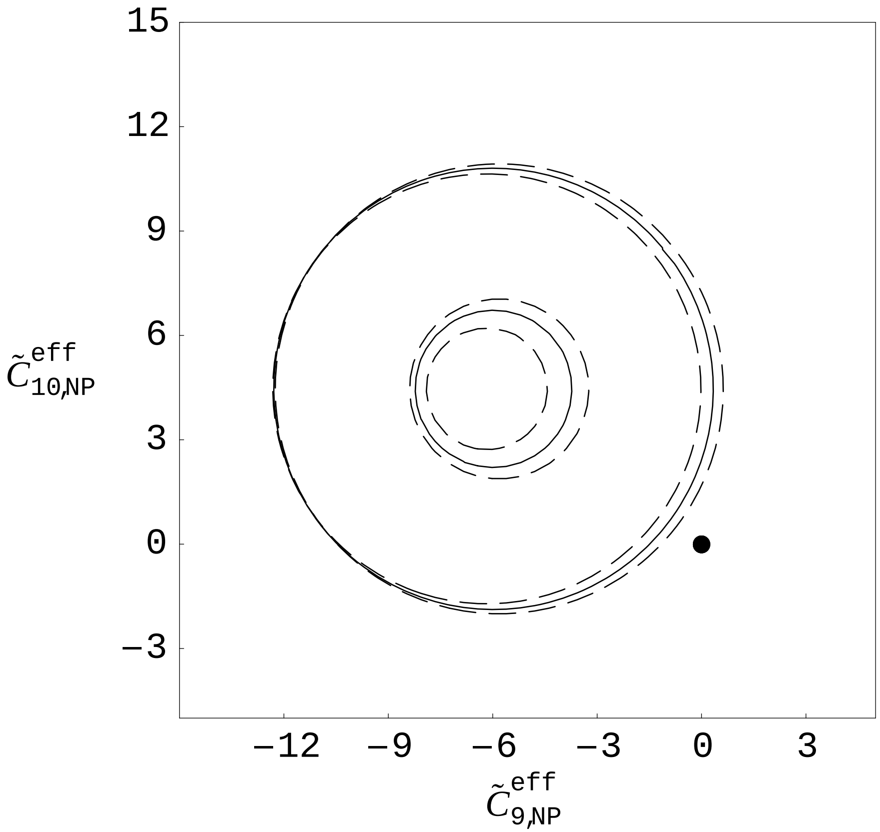
Model-independent constraints on additive new physics contributions to $\widetilde{C}^{\text{eff}}_{9,10}$ at 90% C.L.

SM-like sign of $\widetilde{C}_7^{\text{eff}}$

[Gambino,Haisch,Misiak]

- Extract bounds on $|\widetilde{C}_7^{eff}|$ from $\mathcal{B}(\bar{B} \to X_s\gamma) = (3.52 \pm 0.70) \cdot 10^{-4}$
- Use bounds on low q^2-region from $\mathcal{B}(\bar{B} \to X_s ll) = (1.60 \pm 0.90) \cdot 10^{-6}$ to extract allowed region for $\widetilde{C}^{\text{eff}}_{9,10}$
- Regions outside the rings are excluded
- Dot at the origin indicates the SM case for $\widetilde{C}^{\text{eff}}_{9,10}$.

More on sign of $\widetilde{C}_7^{\rm eff}$



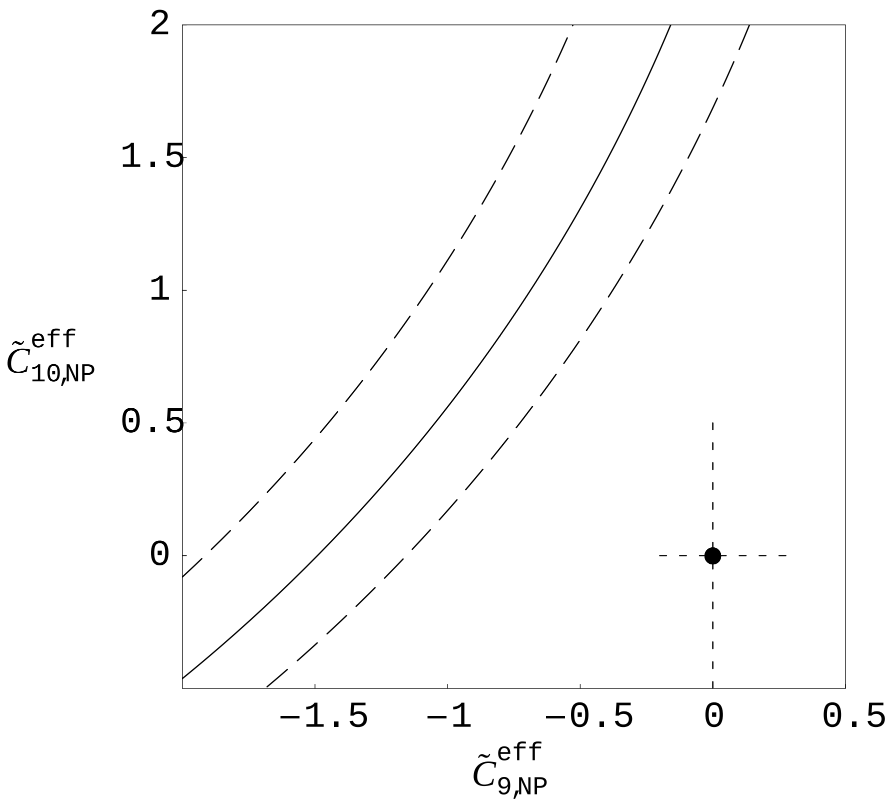
Model-independent constraints on additive new physics contributions to $\widetilde{C}_{9,10}^{\rm eff}$ at 90% C.L.

opposite sign of $\widetilde{C}_7^{\rm eff}$

[Gambino,Haisch,Misiak]

- Extract bounds on $|\widetilde{C}_7^{eff}|$ from $\mathcal{B}(\bar{B} \to X_s \gamma) = (3.52 \pm 0.70) \cdot 10^{-4}$
- Use bounds on low q^2-region from $\mathcal{B}(\bar{B} \to X_s ll) = (1.60 \pm 0.90) \cdot 10^{-6}$ to extract allowed region for $\widetilde{C}_{9,10}^{\rm eff}$
- Regions outside the rings are excluded
- Dot at the origin indicates the SM case for $\widetilde{C}_{9,10}^{\rm eff}$.

More on sign of $\widetilde{C}_7^{\text{eff}}$



Enlarged surroundings of the origin.

opposite sign of $\widetilde{C}_7^{\text{eff}}$

[Gambino,Haisch,Misiak]

- Dashed cross:

  Maximal MFV MSSM contributions to $\widetilde{C}_{9,10}^{\text{eff}}$

  [Ali,Lunghi,Greub,Hiller]

- Dashed cross too small to reach border of allowed region
- Extensions of SM with reversed sign of $\widetilde{C}_7^{\text{eff}}$ but only small corrections to $\widetilde{C}_{9,10}^{\text{eff}}$ are disfavored
- Models with positive $\widetilde{C}_7^{\text{eff}}$ require sizable contributions to $\widetilde{C}_9^{\text{eff}}$ and $\widetilde{C}_{10}^{\text{eff}}$

Numerical values of couplings at $\mu = \mu_b$

- Numerical values for $\widetilde{\alpha}_{\rm s}(\mu_b)$ and $\kappa(\mu_b)$ with $\mu_b = 5$ GeV
  - $\widetilde{\alpha}_{\rm s}(\mu_b) \;=\; 0.0170$
  - $\kappa(\mu_b) \;=\; 0.0354$
  - $\widetilde{\alpha}_{\rm em}(\mu_b)\ln(m_b^2/m_e^2) \;=\; 0.011$