D Decays on the lattice

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in collaboration with

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Charm physics and recent experiment

1. **Charm spectroscopy**: New states observed, including the suprinsingly narrow "strange states"
   - 0^+ state : $D_0^*(2400)$, $\Gamma = 261 \pm 50\mathrm{MeV}$; $\mathrm{D}_{s0}^*(2317)$, $\Gamma < 4.6$ MeV
   - 1^+ state : $D_1(2420)$, $\Gamma = 20.4 \pm 1.7\mathrm{MeV}$; $D_{s1}(2460)$, $\Gamma < 5.5$ MeV
2. $\mathbf{D} - \bar{\mathbf{D}}$ mixing
3. **BaBar and BELLE** provide high statistic of charm events
   $\rightarrow$ good for studying D-decays
4. **Charm factories**: Cleo-c working at the $\psi(3770)$ resonnance; BES-III starting operating
5. **Super-B**? **Super-Flavour**?

$\downarrow$

Better theoretical estimates needed

Leptonic decays

1. Most direct way to determine the CKM matrix element :

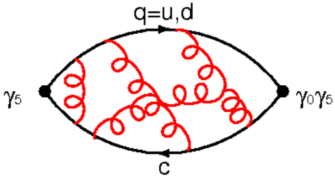
$$\Gamma(D_q^+ \to \ell^+ \nu_\ell) = |V_{cq}|^2 \frac{G_F^2}{8\pi} f_{D_q^+}{}^2 m_{D_q} m_\ell^2 \left(1 - \frac{m_\ell^2}{m_{D_q^+}^2}\right)$$

$$\langle 0|\bar{c}\gamma_\mu\gamma_5 q|D_q^+(p)\rangle = i p_\mu f_{D_q^+}$$

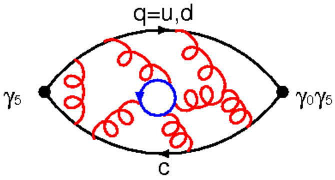
2. Results form **CLEOc**, Assuming $|V_{us}| = |V_{cd}|$:

$$f_{D^+} = 222.6(16.7)^{+2.8}_{-3.4}\ \mathrm{MeV}$$

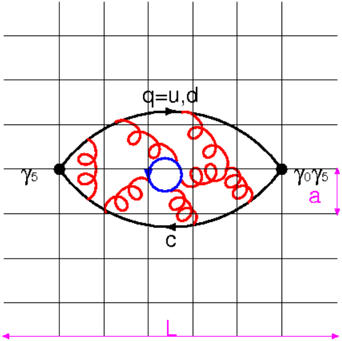
Our calculation



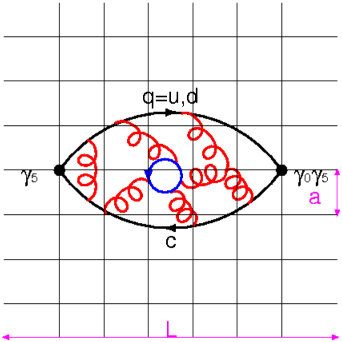
Our calculation



Our calculation



Our calculation



$\rightarrow$ Gauge configurations produced by the QCDSF collaboration

$\rightarrow$Wilson Action, $\mathcal{O}(a)$ improved with $N_F = 2$

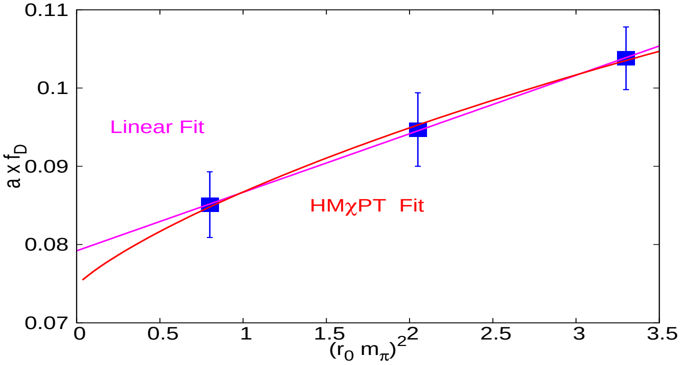
$\rightarrow$ $a \simeq 0.08 fm \quad L \simeq 2 fm$

$\rightarrow$ 3 values of $m_\pi > 390\ \mathrm{MeV}$

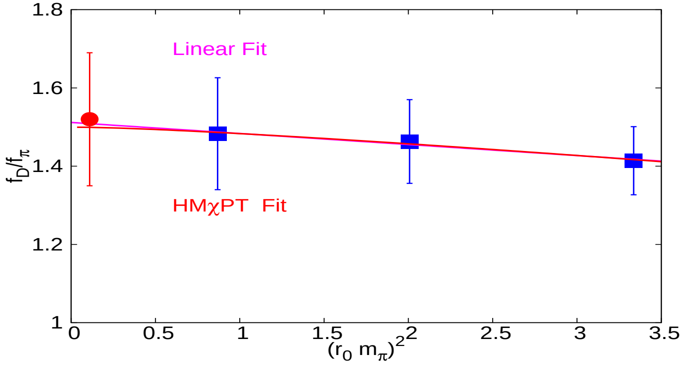
The light u and d quarks are unquenched

$\rightarrow$ No study of D_s - NO PARTIAL QUENCHING

Our results


0.11
0.1
0.09
0.08
0.07
0
0.5
1
1.5
2
2.5
3
3.5
a x f_D
(r_0 m_π)^2
Linear Fit
HMχPT Fit

Our results



Fit used	f_D/f_π
Linear	1.52(17)
HMχPT	1.50(24)

$\longrightarrow f_D/f_\pi = 1.52(17)\begin{pmatrix}+03\\-07\end{pmatrix}$ $f_D = 201(22)\begin{pmatrix}+4\\-9\end{pmatrix}$ MeV

Semileptonic decay $D \to \pi \ell \nu_\ell$

1. Extraction of the CKM matrix element:

$$\frac{d\Gamma}{dq^2}(D \to \pi \ell \nu_\ell) = |V_{cd}|^2 \frac{G_F^2}{192\pi^2 m_D^3} \lambda^{3/2}(q^2) |F_+(q^2)|^2$$

where

$$\langle \pi(k) | V_\mu^{qc} | D(p) \rangle = (p + k - q\frac{m_D^2 + m_P^2}{q^2})_\mu F_+(q^2) + q_\mu \frac{m_D^2 + m_P^2}{q^2} F_0(q^2)$$

and

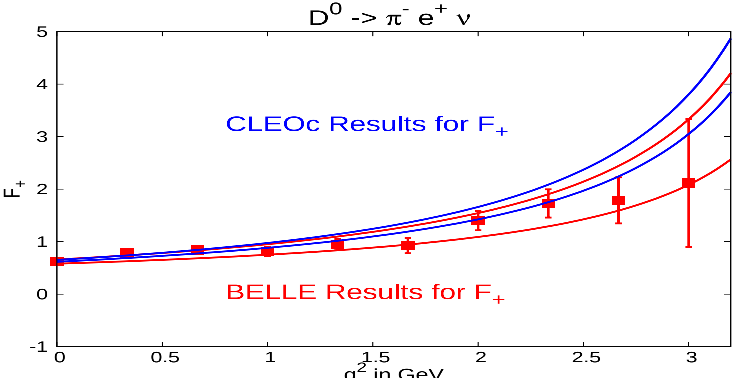
$$F_+\left(q^2 = 0\right) = F_0\left(q^2 = 0\right)$$

2. Results reported by **BELLE** and **CLEO-c**, and those from **BaBar** are expected soon.

Experimental results

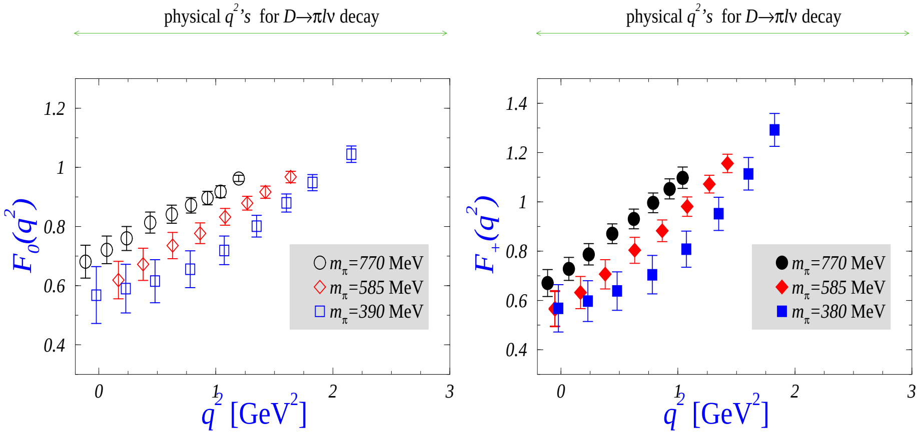
1. Extraction of $F_+(q^2)$

IMPOSING $|V_{us}| = |V_{cd}|$

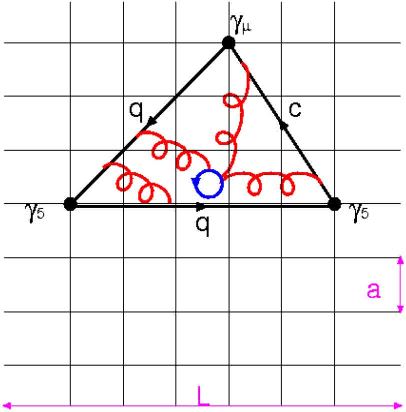


2. Computation from Lattice QCD leading to the extraction of $|V_{cd}|$.

Results


physical q^2's for $D\to\pi l\nu$ decay
$F_0(q^2)$
q^2 [GeV^2]
m_π=770 MeV
m_π=585 MeV
m_π=390 MeV
physical q^2's for $D\to\pi l\nu$ decay
$F_+(q^2)$
q^2 [GeV^2]
m_π=770 MeV
m_π=585 MeV
m_π=380 MeV

Notable Improvements



Double Ratio of 3-point functions

↓

→ Automatic Cancellation of Renormalization factors

→ Cancellation of source terms

Twisted Bound. Cond.

↓

→ Arbitrary q^2's accessible WITHIN the physical kinematical range

$$\frac{F_+(1\mathrm{GeV}^2)}{f_D} = 3.76(54)\ \mathrm{GeV}^{-1} \quad \text{LinearFit}$$

$$4.32(56)\ \mathrm{GeV}^{-1} \quad \mathrm{HM}\chi\mathrm{PT}$$

Summary

- There is room for making substantial progress in D-physics
- We improved the extraction of the D-decay form factors
- From $\mathcal{O}(a)$ improved Wilson action (at $a \approx 0.08$ fm) with $N_F = 2$, we obtain $f_D = 200(22)$ MeV and

$$\frac{F_+(1\mathrm{GeV}^2)}{f_D} = \begin{array}{ll} 3.76(54)\ \mathrm{GeV}^{-1} & \mathrm{LinearFit} \\ 4.32(56)\ \mathrm{GeV}^{-1} & \mathrm{HM}\chi\mathrm{PT} \end{array}$$

- We plan to study saturation of the form factors by their first poles $(f_{D_0^*}, f_{D^*},\ g_{DD^*\pi}, g_{DD_0^*\pi})$
- Employ effective treatment of c-quark à la Fermilab

Computation of f_D and f_π

1. $\langle 0|\bar{c}\gamma_\mu\gamma_5 q|D(p)\rangle = ip_\mu f_D$ is extracted from the fit of two point functions for various $m_q \gg m_{u,d}^{\text{phys.}}$
2. Chiral extrapolation

Linear Fit: Data show a linear behaviour

HMχPT:

$$\Phi_D \equiv f_D\sqrt{m_D} = \Phi_0\left[1 - \frac{1+3g^2}{(4\pi f_0)^2} m_\pi^2 \log m_\pi^2 + c_\Phi m_\pi^2\right]$$

$$f_\pi = f_0\left[1 - \frac{2}{(4\pi f_0)^2} m_\pi^2 \log m_\pi^2 + c_f m_\pi^2\right]$$

g is the strong coupling $\propto g_{DD^*\pi}$ in the chiral limit

Chiral Extrapolation with $N_F = 2$ *

*Valid in the static limit

$$\frac{F_+(E_\pi)}{\sqrt{m_D}} = \frac{(f_D\sqrt{m_D})_\chi\, g}{2f_0(E_P+\Delta)}\left[1+\frac{4g^2}{(4\pi f_0)^2}J_1(m_\pi,E_\pi)-\frac{1+3g^2}{(4\pi f_0)^2}\frac{3}{4}m_\pi^2\log m_\pi^2 + c_+(E_\pi)m_\pi^2\right]$$

$$F_0(E_\pi)\sqrt{m_D} = \frac{(f_D\sqrt{m_D})_\chi}{f_0}\left[1+\frac{1}{(4\pi f_0)^2}\left(\frac{15-27g^2}{12}m_\pi^2\log m_\pi^2 + 2I_2(m_\pi,E_\pi)\right) + c_0 m_\pi^2\right]$$

g: is the strong coupling $\propto g_{DD^*\pi}$ in the chiral limit f_D: D decay constant

f_0: π decay constant in the chiral limit

Improved extraction for *D*-decays

Based on double ratios of triangle diagrams

1. AUTOMATIC CANCELLATION OF RENORMALIZATION FACTOR
2. AUTOMATIC CANCELLATION OF PSEUDOSCALAR DENSITY NORMALIZATIONS

$\downarrow$

Combining various double ratios allows to access F_+ and F_0

see arXiv:0710.1741

Form factors - shape

$\rightarrow$ We want to access q^2's in $(0,(m_D - m_\pi))^2]$ to study the shapes of $F_+(q^2)$ and $F_0(q^2)$

1. Periodic boundary conditions: $\psi(x_i) = \psi(x_i + L_i)$

$$k_i = n_i \frac{2\pi}{L} \geq k_{\min} = \frac{2\pi}{L} \simeq 650 \text{ MeV}$$

   On usual lattices, $k > k_{\min} \Rightarrow q^2 \lesssim 0$

2. Twisted boundary conditions (TwBC): $\psi(x_i) = \psi(x_i + L_i) e^{i\frac{\theta_i}{L_i}}$
   - Uplift the overall quark momenta by $\frac{\theta_i}{L_i}$:

$$k_i = \frac{\theta_i}{L_i}$$

   - No need to redo the Monte Carlo for each θ_i
   - Instead, quark propagators must be re-computed for each θ_i

Understanding the shape of the form factors

1. Beyond pole dominance in F_+:

$$F_+^{\text{pole}}\left(q^2\right) = \frac{f_D^* g_{D^* D\pi}}{q^2 - m_D^*} \qquad \text{where} \quad g_{D^* D\pi} = \frac{2\sqrt{m_D m_D^*}}{f_\pi} g$$

$$F_+\left(q^2\right) - F_+^{\text{pole}}\left(q^2\right) = ???$$

g calculation in progress

Can g be measured experimentally?

2. Beyond pole dominance in F_0

$$F_0^{\text{pole}}\left(q^2\right) = \frac{f_{D0}^* g_{D_0^* D\pi}}{q^2 - m_{D0}^*}$$

Callan-Treiman Theorem:

$$F_0\left(m_D^2\right) = f_D/f_\pi = 1.51(10)$$