Neural Network Determination of Parton Distribution Functions

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The NNPDF Collaboration
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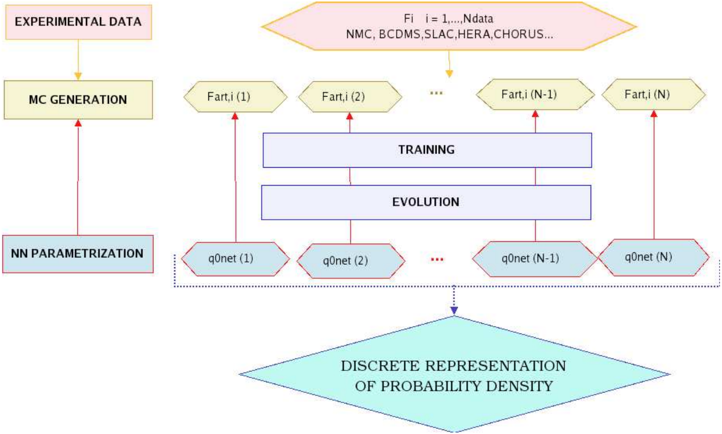
Issue and Standard Approach

- Given a set of data points we must determine a set of functions with error.
- We need an error band in the space of functions, i.e. a probability density $\mathcal{P}[q(x)]$ in the space of PDFs, q(x). For an observable $\mathcal{F}$ depending on PDFs :

$$\langle \mathcal{F}[q(x)] \rangle = \int [\mathcal{D}q] \, \mathcal{F}[q(x)] \mathcal{P}[q(x)]$$

- Standard approach, choose a basis of functions and project PDFs on it: the ∞-dimensional space of function reduces to a finite-dimensional space of parameters.
- Issues:
  - Non trivial propagation of errors: non-gaussian errors and incompatible data.
  - The error associated to the choice of parametrisation is difficult to assess.



$$\langle \mathcal{F}[q(x)] \rangle = \int [\mathcal{D}q]\, \mathcal{F}[q(x)]\mathcal{P}[q(x)] \quad \longrightarrow \quad \langle \mathcal{F}[q(x)] \rangle = \frac{1}{N_{\mathrm{rep}}} \sum_{k=1}^{N_{\mathrm{rep}}} \mathcal{F}[q^{(k)(\mathrm{net})}(x)]$$

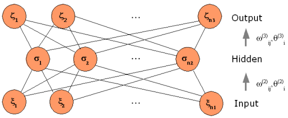
Main Ingredients

1. Monte Carlo determination of errors:

After fitting, the error of an observable depending on PDFs $\longrightarrow$

$$\sigma_{\mathcal{F}[q(x)]} = \sqrt{\langle \mathcal{F}[q(x)]^2 \rangle - \langle \mathcal{F}[q(x)] \rangle^2}$$

2. Neural Networks as redundant and unbiased parametrisation of PDFs:

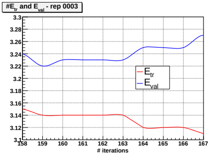


* Each neuron receives input from neurons in preceding layer.
* Activation determined by weights and thresholds according to a non linear function:

$$\xi_i = g\Big(\sum_j \omega_{ij}\xi_j - \theta_i\Big), \qquad g(x) = \frac{1}{1+e^{-x}}$$

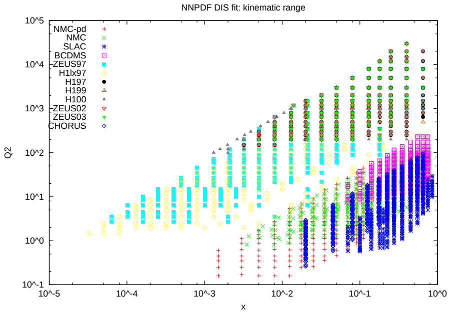
3. Dynamical stopping criterion in order to fit data and not statistical noise.

* Divide data in two sets: training and validation.
* Minimisation is performed only on the training set. The validation χ^2 for the set is computed.
* When the training χ^2 still decreases while the validation χ^2 stops decreasing $\rightarrow$ STOP.



Singlet fit

- NLO fit.
- ZM-VFN treatment of heavy quarks.
- All DIS data included.
- Flavor Assumptions:
  - Symmetric strange sea $s(x) = \bar{s}(x)$
  - Strange sea proportional to non-strange sea $\bar{s}(x) = \frac{C}{2}(\bar{u}(x) + \bar{d}(x))$ (C = 0.5)

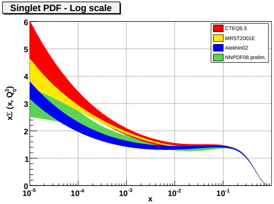


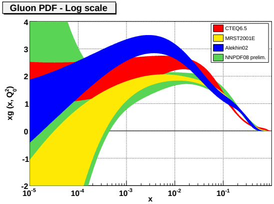
- Parametrization of 4+1 combinations of PDFs at $Q_0^2 = 2$ GeV^2:

Singlet : $\Sigma(x)$	$\longmapsto$	$NN_{\Sigma}(x)$	2-3-2-1 20 pars
Gluon : $g(x)$	$\longmapsto$	$NN_{g}(x)$	2-3-2-1 20 pars
Total valence : $V(x) \equiv u_V(x) + d_V(x)$	$\longmapsto$	$NN_{V}(x)$	2-3-2-1 20 pars
Non-singlet triplet : $T_3(x)$	$\longmapsto$	$NN_{T3}(x)$	2-3-2-1 20 pars
Sea asymmetry : $\Delta_S(x) \equiv \bar{d}(x) - \bar{u}(x)$	$\longmapsto$	$NN_{\Delta}(x)$	2-3-1 13 pars

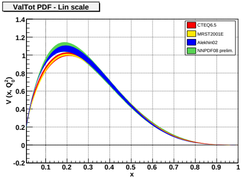
93 parameters

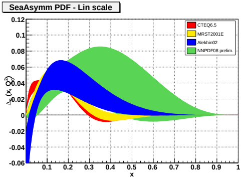
Some Very Preliminary Results

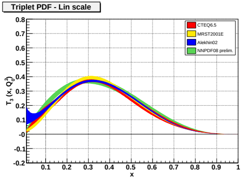




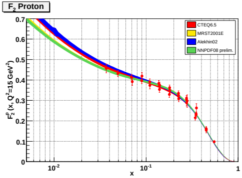
pdfs

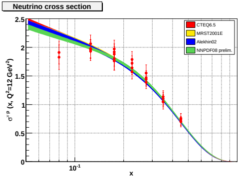


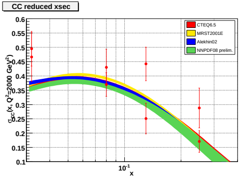




observables







Conclusions

- Standard approaches to PDFs fitting might lead to underestimation of errors associated with parton densities.
- Combination of Monte Carlo techniques and Neural Networks as unbiased interpolating functions has proved to be a fast and robust alternative method.
- A non singlet fit has been published [hep-ph/0701127] and a full DIS fit will be published very soon.