Hierarchical Markovian algorithm in QCD evolution

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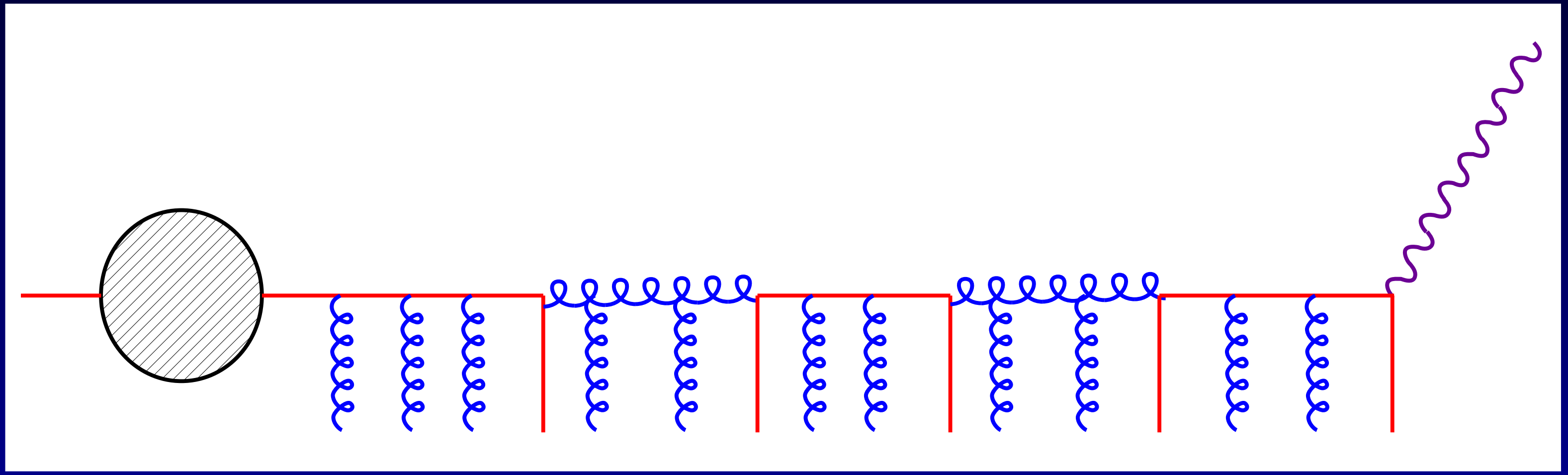
Introduction

Let's look on the process of deep inelastic scattering



To describe it we introduce some set of variables:

x - fraction of parton longitudinal momentum

t - evolution variable (for example virtuality)

... - there could be some more variables necessary like k_T

Evolution equation

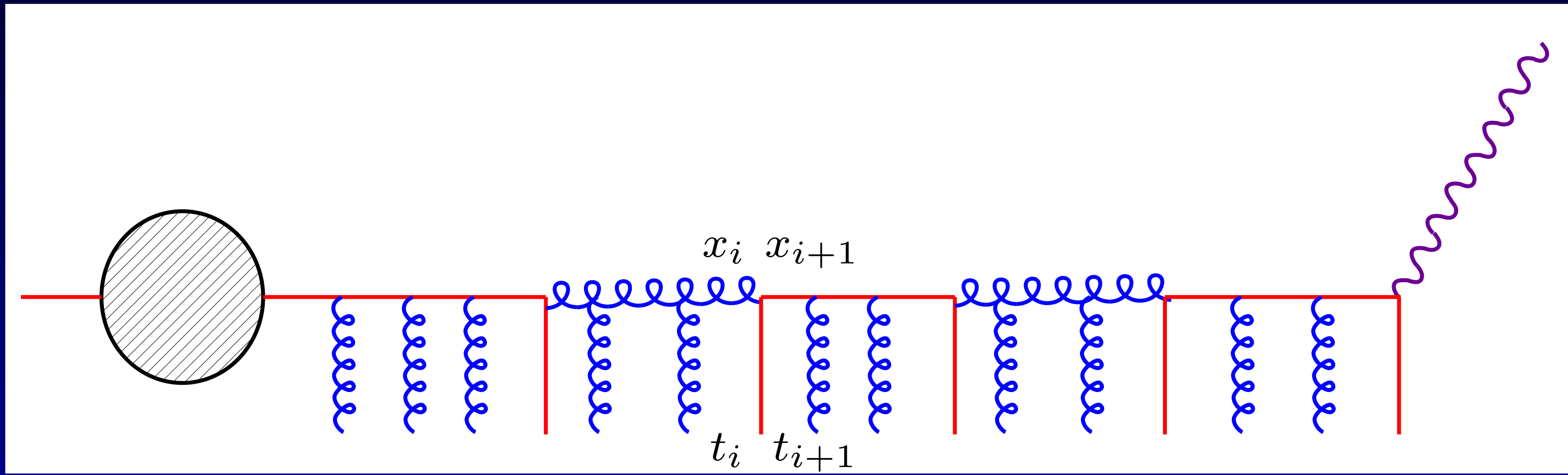
General form of evolution equation:

$$\frac{\partial}{\partial t} D_k(t,x) = \sum_j \int_x^1 \frac{dz}{z} P_{kj}(t,z) D_j(t,x/z)$$

The specific type of equation is defined by the splitting function P_{kj}

$$P_{kj}(t,z) = -P^{\delta}_{kj}(t)\delta_{jk}\delta(1-z) + P^{\Theta}_{kj}(t,z)$$

Standard Markovian algorithm

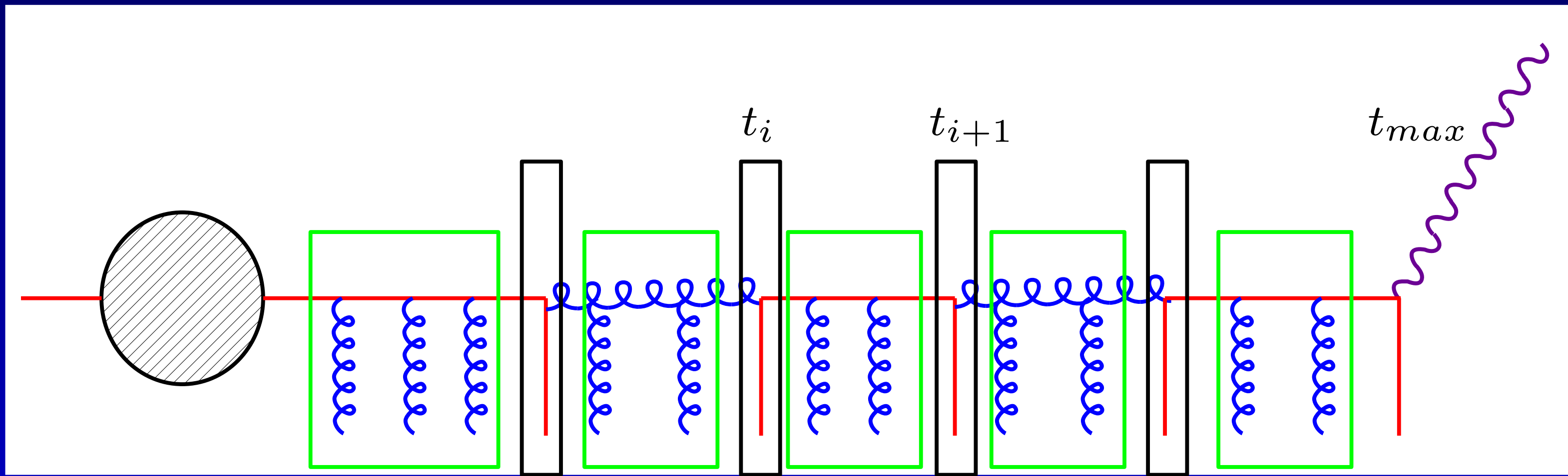


$$p(t_i, x_i | t_1, x_1, ..., t_{i-1}, x_{i-1}) = p(t_i, x_i | t_{i-1}, x_{i-1})$$

Hierarchical solution

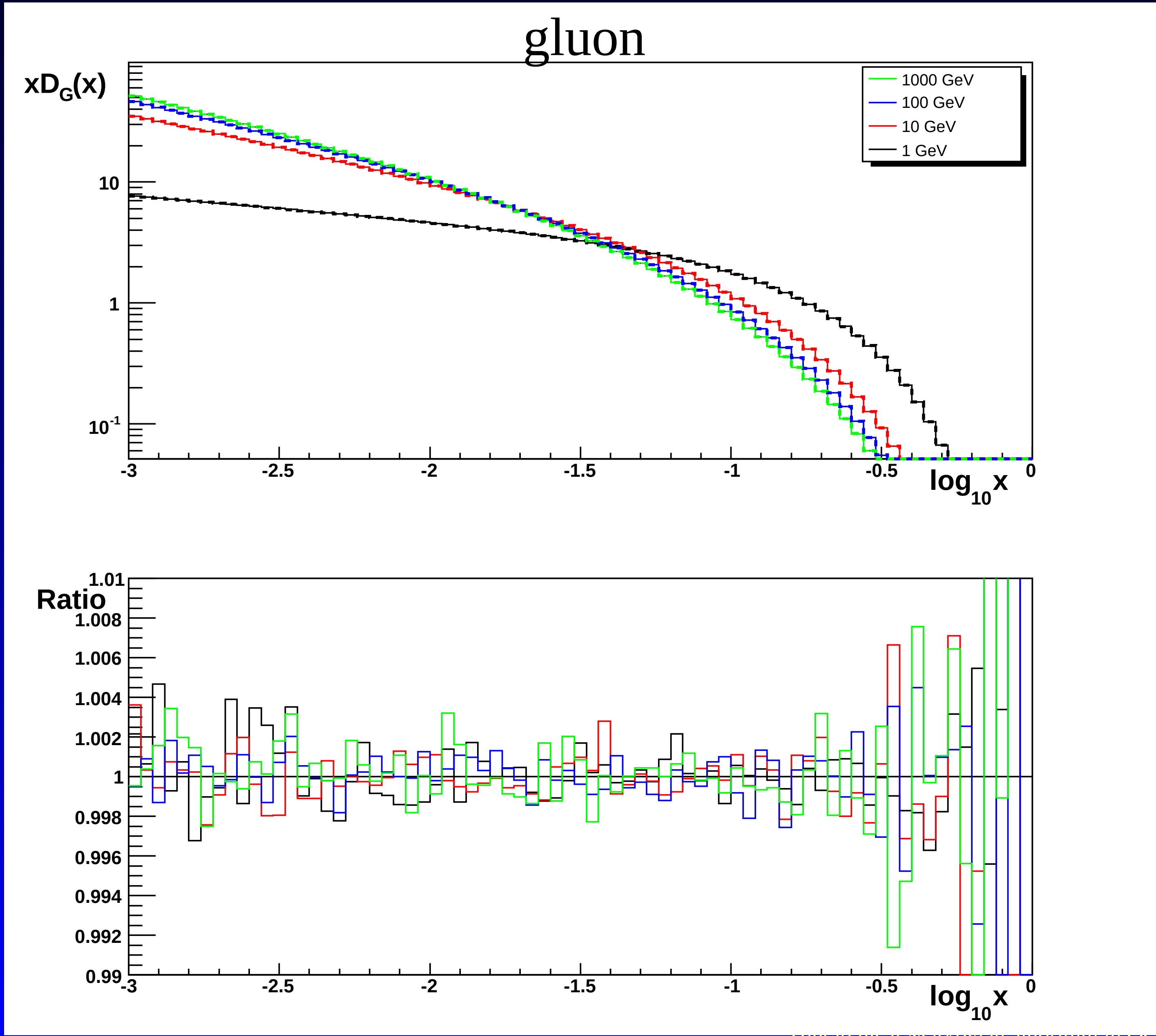
What is the aim of hierarchical solution?

- separate flavour changing emissions from the diagonal ones (bremsstrahlung)

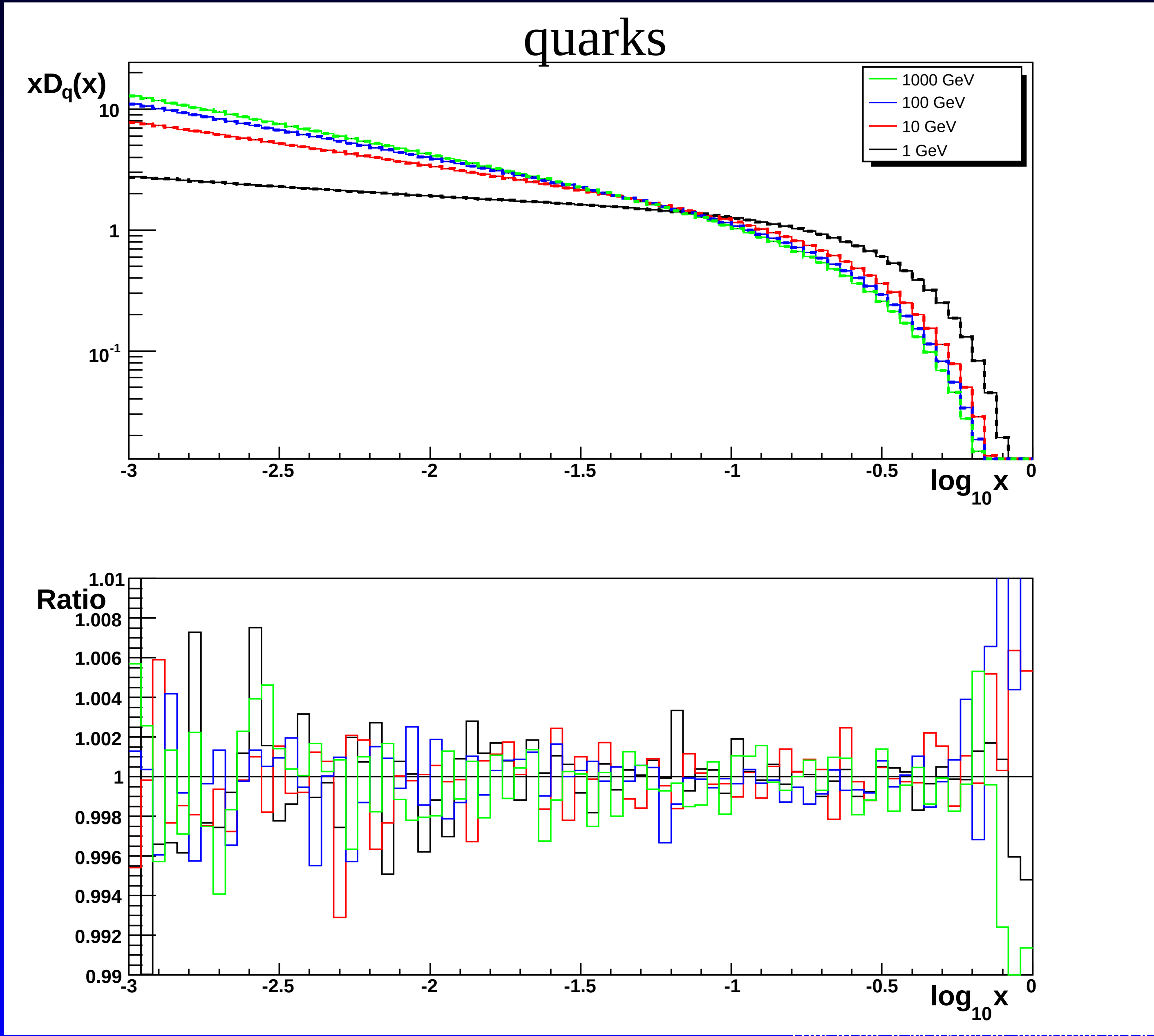


- external process - flavour changing emissions
- internal process - diagonal emissions

Results


gluon
xD_G(x)
1000 GeV
100 GeV
10 GeV
1 GeV
10
1
10^-1
-3
-2.5
-2
-1.5
-1
-0.5
0
log_10 x
Ratio
1.01
1.008
1.006
1.004
1.002
1
0.998
0.996
0.994
0.992
0.99

Results


quarks
xDq(x)
10
1
10-1
1000 GeV
100 GeV
10 GeV
1 GeV
-3
-2.5
-2
-1.5
-1
-0.5
0
log10x
Ratio
1.01
1.008
1.006
1.004
1.002
1
0.998
0.996
0.994
0.992
0.99
-3
-2.5
-2
-1.5
-1
-0.5
0
log10x

Bibliography

[1] K. Golec-Biernat, S. Jadach, W. Płaczek, M. Skrzypek, *Acta Phys. Pol. B* **37**, 1785 (2006)

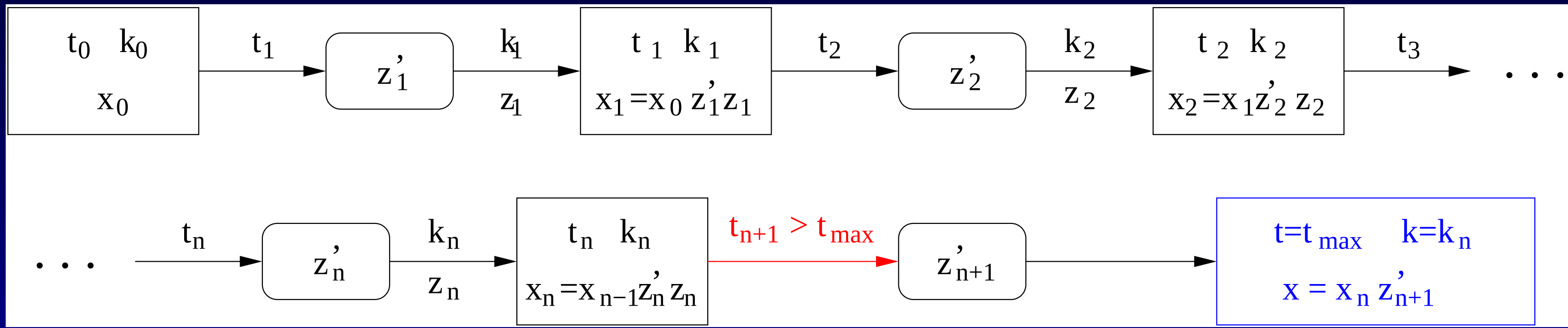
[2] S. Jadach, M. Skrzypek, *Acta Phys. Pol. B* **35**, 745 (2004) [hep-ph/0312355]

[3] S. Jadach, M. Skrzypek, Z. Wąs, Report IFJPAN-V-04-09, [hep-ph/0701174]

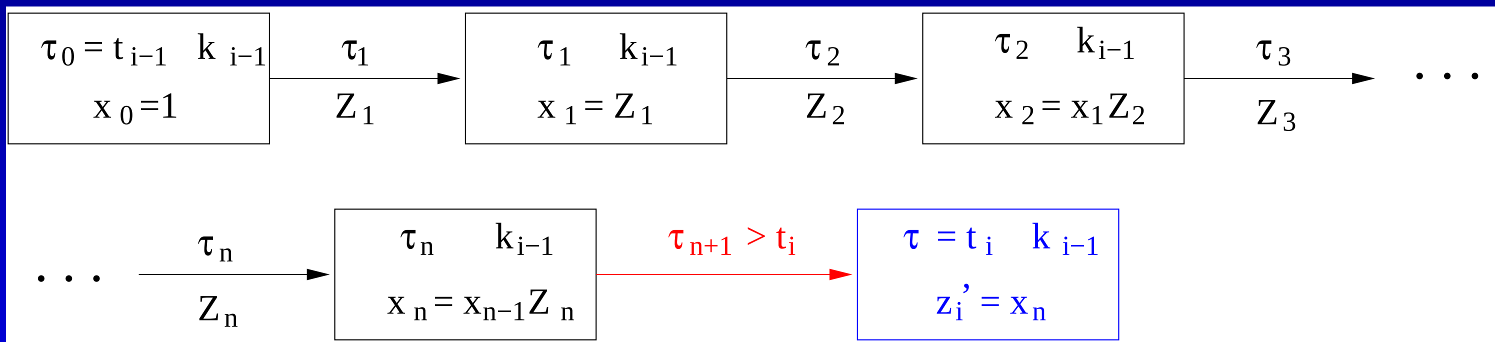
[4] R. Ellis, W. Stirling, B. Webber, *QCD and Collider Physics*, Cambridge University Press, 1996

Algorithm

Schematic visualisation of the algorithm



where the bremsstrahlung process is represented by:



Markovian iterative solution

$$D_k(t,x) = e^{-\Phi_k(t,t_0)} D_k(t_0,x) + \sum_{n=1}^{\infty} \sum_{k_{n-1}\dots k_0} \left(\prod_{i=1}^{n} \int_{t_0}^{t} dt_i \Theta(t_i - t_{i-1}) \int_0^1 dz_i \right) e^{-\Phi_k(t,t_n)}$$

$$\times \int_0^1 dx_0 \left(\prod_{i=1}^{n} P^{\Theta}_{k_i k_{i-1}}(t_i, z_i) e^{-\Phi_{k_{i-1}}(t_i,t_{i-1})} \right) D_{k_0}(t_0, x_0) \delta\Big(x - x_0 \prod_{i=1}^{n} z_i\Big)$$

$$\Phi_k(t,t_0) = \int_{t_0}^{t} dt' P^{\delta}_{kk}(t')$$

Flavour changing process

$$D_k(t,x) = \int_0^1 dz' dx_0 e^{-\Phi_k^B(t,t_0)} G_{kk}^A(t,t_0,z') D_k(t_0,x_0)\delta(x - z'x_0)$$

$$+ \sum_{n=1}^{\infty} \sum_{k_{n-1}\neq\ldots\neq k_1\neq k_0} \Big(\prod_{i=1}^{n} \int_{t_0}^{t} dt_i \Theta(t_i - t_{i-1}) \int_0^1 dz_i' \int_0^1 dz_i e^{-\Phi_{k_{i-1}}^B(t_i,t_{i-1})}$$

$$\times G_{k_{i-1}k_{i-1}}^A(t_i,t_{i-1},z_i') P_{k_i k_{i-1}}^{\Theta}(t_i,z_i)\Big) \int_0^1 dz_{n+1}' e^{-\Phi_k^B(t,t_n)} G_{kk}^A(t,t_n,z_{n+1}')$$

$$\int_0^1 dx_0 D_{k_0}(t_0,x_0)\delta\Big(x - x_0 \prod_{i=1}^{n} z_i \prod_{i=1}^{n+1} z_i'\Big)$$

Diagonal process

$$\partial_t G^A_{kk}(t, t_0, z') = P^A_{kk}(t, \cdot) \otimes G^A_{kk}(t, t_0, \cdot)(z')$$

$$G^A_{kk}(t, t_0, z') = e^{-\Phi^A_k(t,t_0)}\delta(1 - z') +$$
$$+ \sum_{n=1}^{\infty} \left[\prod_{i=1}^{n} \int_{t_0}^{t} dt_i \Theta(t_i - t_{i-1}) \int_0^1 dz'_i e^{-\Phi^A_k(t_i,t_{i-1})} P^{\Theta}_{kk}(t_i, z_i) \right] e^{-\Phi^A_k(t,t_n)} \delta(z - \prod_{i=1}^{n} z_i)$$