Evolution equations in QCD and QED

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Evolution equations

Generic form of evolution equation:

$$\boxed{\partial_t D(t,x) = \mathcal{K}(t,\cdot) \otimes D(t,\cdot)(x)}$$

- D structure function
- $\mathcal{K} = \left(\frac{1}{1-x}\right)_+$ evolution kernel
- $\otimes$ convolution rule

YFS-type evolution

$$\partial_t D_{YFS}(t,x) = \int_0^1 du \mathcal{K}(x-u) D_{YFS}(t,u)$$

DGLAP-type evolution

$$\partial_t D_{DGLAP}(t,x) = \int_x^1 \frac{dz}{z} \mathcal{K}\left(\frac{x}{z}\right) D_{DGLAP}(t,z)$$

Evolution kernels

$$\mathcal{K}(u) = \int_0^1 \frac{dy}{1-y} - \delta(1-u) \rightarrow \partial_t \int_0^1 D(x)dx < 0$$

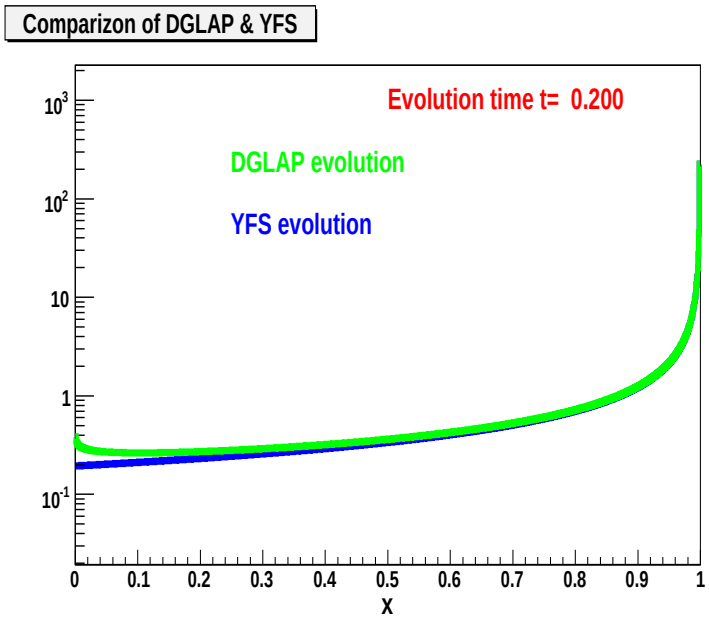
$$\partial_t \int_0^1 D(x)dx = 0 \rightarrow \int_0^1 \mathcal{K}(x)dx = 0$$

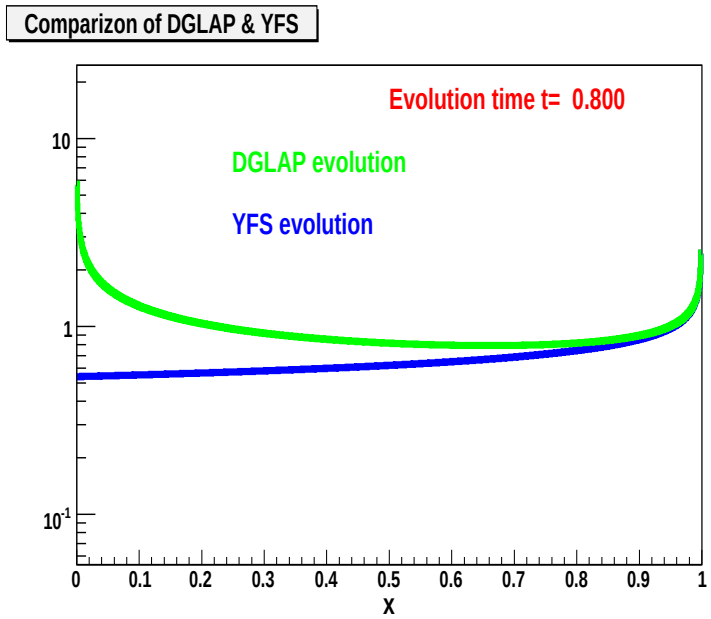
regularised and in a common representation:

$$\mathcal{K}_{YFS}(u,x) = \Theta(u-x-\epsilon)\Theta(u-1)\frac{1}{u-x} - \delta(u-x)\ln\frac{1}{\epsilon}$$

$$\mathcal{K}_{DGLAP}(x,u) = \Theta(u-x-\epsilon)\frac{1}{u-x} - \delta(u-x)\ln\frac{x}{\epsilon}$$

Comparison of solutions





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