Mesonic exchanges in hadronic vacuum polarization and light-by-light

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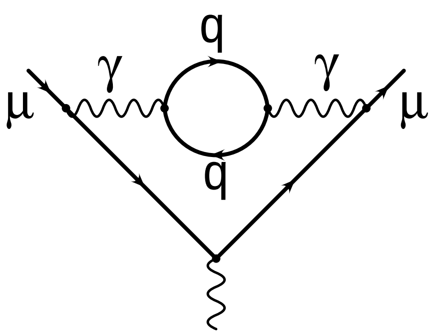
Outline

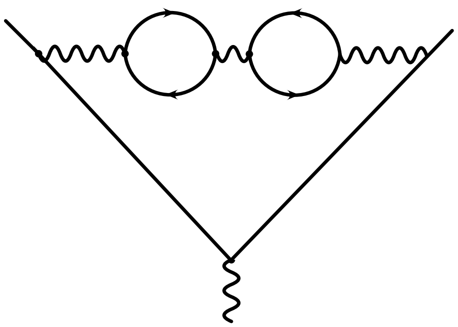
- Theoretical parameters: chiral limit and large number of colors
- Charged pion loop in polarization operator and in light-by-light
- Hadronic vacuum polarization in higher orders
- Models for hadronic light-by-light
- QCD constraints
- The model consistent with constraints
- Comparison with ENJL, thanks to the recent analysis by Bijnens and Prades
- Summary

Hadronic contributions

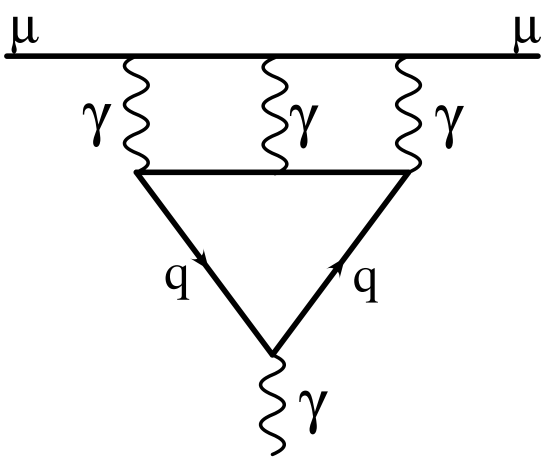
$$a_\mu^{\rm had} = a_\mu^{\rm had,LO} + a_\mu^{\rm had,HO} + a_\mu^{\rm LBL}$$



Lowest order hadronic contribution represented by a quark loop



An example of higher order hadronic contribution

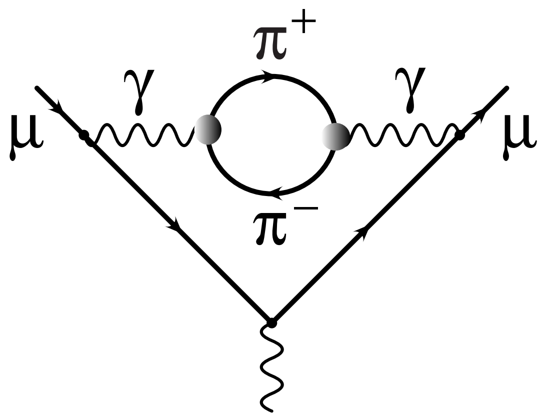


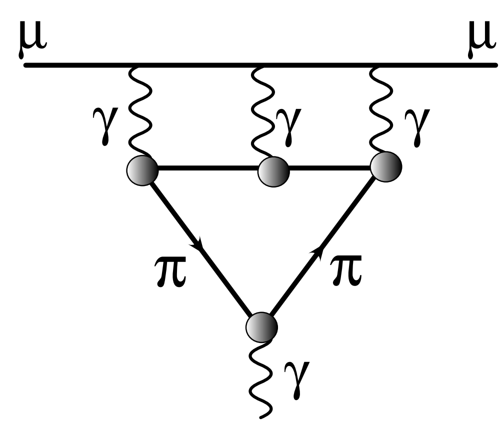
Light-by-light scattering contribution

In difference with $a_\mu^{\rm had,LO}$ there is no experimental input for the light-by-light contribution.What are possible theoretical parameters to exploit?

Smallness of chiral symmetry breaking, $m_\rho^2/m_\pi^2 \gg 1$

$$a_\mu^{(n)} \sim c_1 \left(\frac{\alpha}{\pi}\right)^n \frac{m_\mu^2}{m_\pi^2}\,, \qquad \mathrm{LO}: n=2\,, \quad \mathrm{LbL}: n=3$$

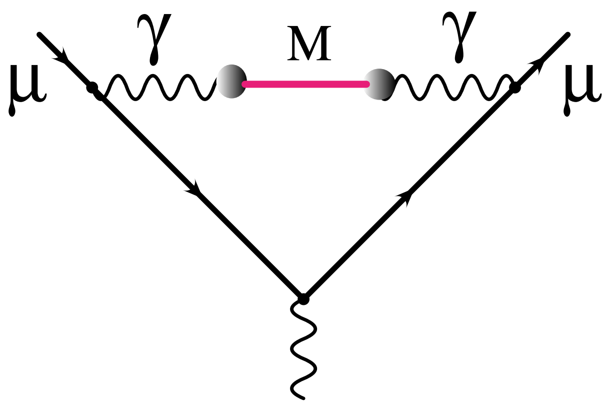


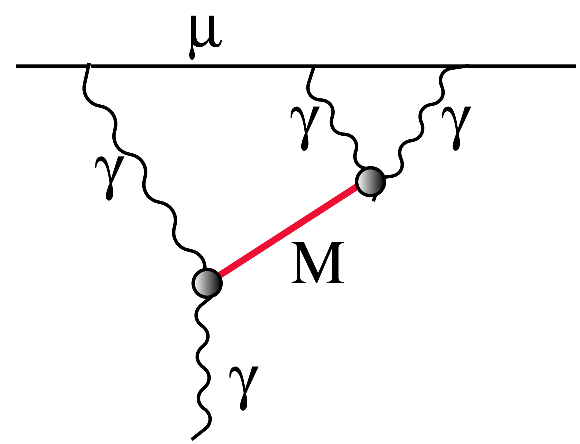


The Goldstone nature of pion implies $m_\pi^2 \propto m_q$ much less than typical $M_{\rm had}^2 \sim m_\rho^2$. Thus, the threshold range in pion loops produces the $1/m_\pi^2$ enhancement.

Large number of colors, N_c

Quark loops clearly give $a_\mu \propto N_c$. Dual not to pion loops but to meson exchanges.





No continuum in the large N_c limit.
$M = \rho^0, \omega, \phi, \rho', \ldots$ for the polarization operator
$M = \pi^0, \eta, \eta', a_0, a_1, \ldots$ (and any C-even meson) for the light-by-light

$$a_\mu^{(n)} \sim c_2 \left(\frac{\alpha}{\pi}\right)^n N_c \, \frac{m_\mu^2}{m_\rho^2}$$

We can check for $a_\mu^{\rm had,LO}$

$$a_\mu^{\rm had,LO} = \left(\frac{\alpha\, m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{{\rm d}s}{s^2}\, K(s) R(s)$$

$K(s)$ is the known function, $K(s) \to 1, \quad s \gg m_\mu^2$
$R(s)$ is the cross section of e^+e^- annihilation into hadrons in units of $\sigma(e^+e^- \to \mu^+\mu^-)$.

Two regions. The threshold region $s \sim 4m_\pi^2$ where

$$R(s) \approx \frac{1}{4}\left(1 - \frac{4m_\pi^2}{s}\right)^{3/2}$$

and the resonance region $s \sim m_\rho^2$ where by quark-hadron duality on average

$$R(s) \approx N_c \sum Q_q^2$$

The chirally enhanced threshold region gives numerically

$$a_\mu^{\rm had,LO}(4m_\pi^2 \le s \le m_\rho^2/2) \approx 400\times 10^{-11}$$

Compare with the N_c enhanced ρ peak,

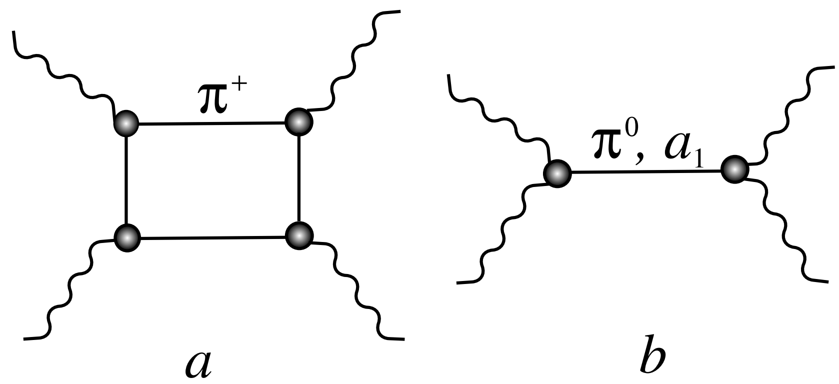
$$a_\mu^{\rm had,LO}(\rho) = \frac{m_\mu^2\,\Gamma(\rho\to e^+e^-)}{\pi\, m_\rho^3} \approx 5000\times 10^{-11}$$

This contribution is enhanced by N_c,

$$a_\mu(\rho) \sim c_2\left(\frac{\alpha}{\pi}\right)^2 N_c\,\frac{m_\mu^2}{m_\rho^2}$$

What is a lesson from this exercise? We see that the large N_c enhancement prevails over chiral one.

In light-by-light



The chirally enhanced pion box contribution does not result in large number, it is actually rather small,

$$a_\mu^{\mathrm{LbL}}(\text{pion box}) \approx -4\times 10^{-11}$$

Hayakawa, Kinoshita, Sanda; Melnikov

similarly to the hadronic polarization case above.

A larger value (-19) for the pion box was obtained by Bijnens, Pallante, Prades

Instability of the number is due to relatively large pion momenta in the loop, of order of $4m_\pi$ as we estimated. Then details of the model becomes important and theoretical control is lost. In HSL model few first terms of m_π^2/m_ρ^2 expansion are

$$a_\mu(\text{charged pion loop})\times 10^{11} = -46.37+35.46+10.98-4.7+\ldots = -4.9$$

If momenta were small compared with m_ρ the result would be close to the leading term – free pion loop.

In case of polarization operator the suppression of the leading term in the chiral expansion (larger momenta) can be related to the p-wave p^3 suppression. There is a suppression for s-wave in two-pion intermediate state near threshold in the case of LbL.

Hadronic Vacuum Polarization in Higher Orders

Based on discussions with Michel Davier

To see an impact of hadronic vacuum polarization (VP) in higher orders let us consider an extremely simplified model when there is only one narrow ρ^0 resonance in e^+e^- annihilation into hadrons. The polarization operator then has the form

$$\Pi(k^2) = \frac{4\pi\alpha}{g_0^2}\,\frac{k^2}{m_0^2 - k^2}\,,$$

where m_0 is the bare mass of ρ^0 and g_0 defines the bare coupling of ρ^0 and photon,

$$\langle\rho|j_\mu|0\rangle_0 = \frac{m_0^2}{g_0}\,\rho_\mu\,, \qquad j_\mu = \frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)\,.$$

Summation of higher orders results in the substitution of the photon propagator,

$$\frac{1}{k^2} \longrightarrow \frac{1}{k^2}\frac{1}{1+\Pi(k^2)} = \frac{1}{k^2} + \frac{4\pi\alpha}{g^2}\frac{1}{k^2-m^2},$$

where m and g represent physical mass and coupling related to the bare ones by

$$m^2 = \frac{m_0^2}{1-4\pi\alpha/g_0^2}, \qquad \frac{1}{g^2} = \frac{1}{g_0^2}\frac{1}{1-4\pi\alpha/g_0^2}.$$

The value of g is defined by the e^+e^- width,

$$\frac{\Gamma(\rho\to e^+e^-)}{m} = \frac{1}{12\pi}\left(\frac{4\pi\alpha}{g}\right)^2$$

The radiative effect shifts the ρ^0 mass up by 1.4 MeV. But it also shifts the coupling.

Let us now find the hadronic correction to a_μ in the model,

$$\Delta a_\mu = \frac{1}{12\pi^2}\Big(\frac{4\pi\alpha}{g}\Big)^2\frac{m_\mu^2}{m^2} = \frac{1}{12\pi^2}\Big(\frac{4\pi\alpha}{g_0}\Big)^2\frac{m_\mu^2}{m_0^2} = \frac{m_\mu^2\,\Gamma(\rho\to e^+e^-)}{\pi m^3}\,.$$

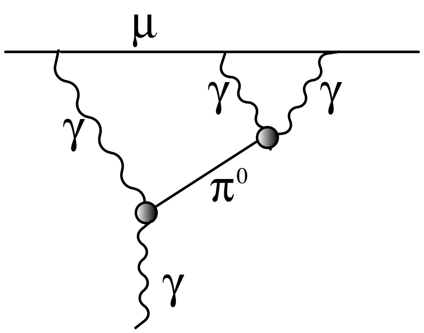
Note that the result has the same form in terms of bare and physical quantities.
It means that higher orders of VP insertion do not contribute.
Consistent with Krause '96 results.

Moreover, it also works for the sum over vector resonances provided that $m \gg m_\mu$ for all resonance masses.

The main higher order effect comes from the lepton VP which could accounted by running α,

$$\alpha \longrightarrow \alpha(m_\rho) \approx \alpha[1+(2\alpha/3\pi)\ln(m_\rho^2/m_e m_\mu)]\,.$$

Hadronic light-by-light



Hayakawa, Kinoshita, Sanda
Bijnens, Pallante, Prades
Barbieri, Remiddi
Pivovarov
Bartos, Dubničkova, Dubnička, Kuraev, Zemlyanaya
Knecht, Nyffeler
Knecht, Nyffeler, Perrotttet, de Rafael
Ramsey-Musolf, Wise
Blokland, Czarnecki, Melnikov
Melnikov, A.V.

Different models: constituent quark loop, extended Nambu–Jano-Lasinio model (ENJL), hidden local symmetry (HLS) model . . .

The π^0 pole part of LbL contains besides N_c the chiral enhancement in the logarithmic form, leading to the model-independent analytical expression

$$a_\mu^{\rm LbL}(\pi^0) = \left(\frac{\alpha}{\pi}\right)^3 N_c\,\frac{m_\mu^2\,N_c}{48\pi^2 F_\pi^2}\,\ln^2\frac{m_\rho}{m_\pi} + \ldots$$

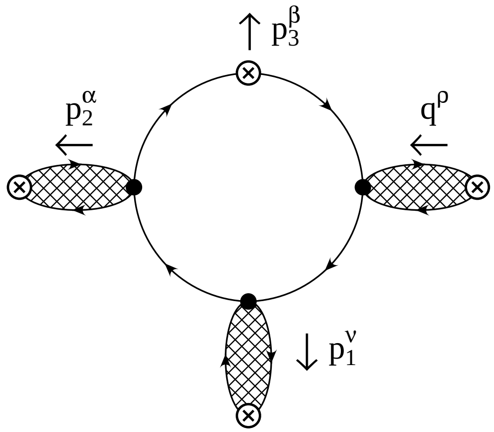
However next, model dependent, terms are comparable with the the leading one. Numerically

$$a_\mu^{\mathrm{LbL}}(\pi^0) = 58(10)\times 10^{-11}$$ Knecht, Nyffeler

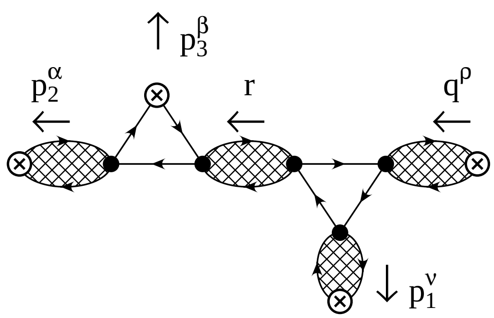
Models

HLS model is a modification the Vector Meson Dominance model.

ENJL model is represented by the following graphs



(a)



(b)

OPE constraints and hadronic model

$\epsilon_i^{\mu}(q_i)\,, \quad i=1,2,3,4, \quad \sum q_i = 0$

ϵ_4 represents the external magnetic field $f^{\gamma\delta} = q_4^{\gamma}\epsilon_4^{\delta} - q_4^{\delta}\epsilon_4^{\gamma}, \quad q_4 \to 0.$

The LbL amplitude

$$\begin{aligned}\mathcal{M} &= \alpha^2 N_c \,\mathrm{Tr}\,[\hat{Q}^4]\,\mathcal{A} = \alpha^2 N_c \,\mathrm{Tr}\,[\hat{Q}^4]\,\mathcal{A}_{\mu_1\mu_2\mu_3\gamma\delta}\epsilon_1^{\mu_1}\epsilon_2^{\mu_2}\epsilon_3^{\mu_3}f^{\gamma\delta} \\ &= -e^3\int \mathrm{d}^4x\,\mathrm{d}^4y\,\mathrm{e}^{-iq_1x-iq_2y}\,\epsilon_1^{\mu_1}\epsilon_2^{\mu_2}\epsilon_3^{\mu_3}\langle 0|T\,\{j_{\mu_1}(x)\,j_{\mu_2}(y)\,j_{\mu_3}(0)\}\,|\gamma\rangle\end{aligned}$$

The electromagnetic current $j_\mu = \bar{q}\,\hat{Q}\gamma_\mu q, \quad q=\{u,d,s\}$

Three Lorentz invariants: q_1^2, q_2^2, q_3^2

Consider the Euclidian range $q_1^2 \approx q_2^2 \gg q_3^2 \gg \Lambda^2_{\mathrm{QCD}}$

We can use OPE for the currents that carry large momenta q_1, q_2

$$i\int \mathrm{d}^4x\,\mathrm{d}^4y\,\mathrm{e}^{-iq_1x-iq_2y}\,T\left\{j_{\mu_1}(x), j_{\mu_2}(y)\right\} =$$

$$\int \mathrm{d}^4z\,\mathrm{e}^{-i(q_1+q_2)z}\,\frac{2i}{\hat{q}^2}\,\epsilon_{\mu_1\mu_2\delta\rho}\,\hat{q}^\delta j_5^\rho(z) + \cdots\,.$$

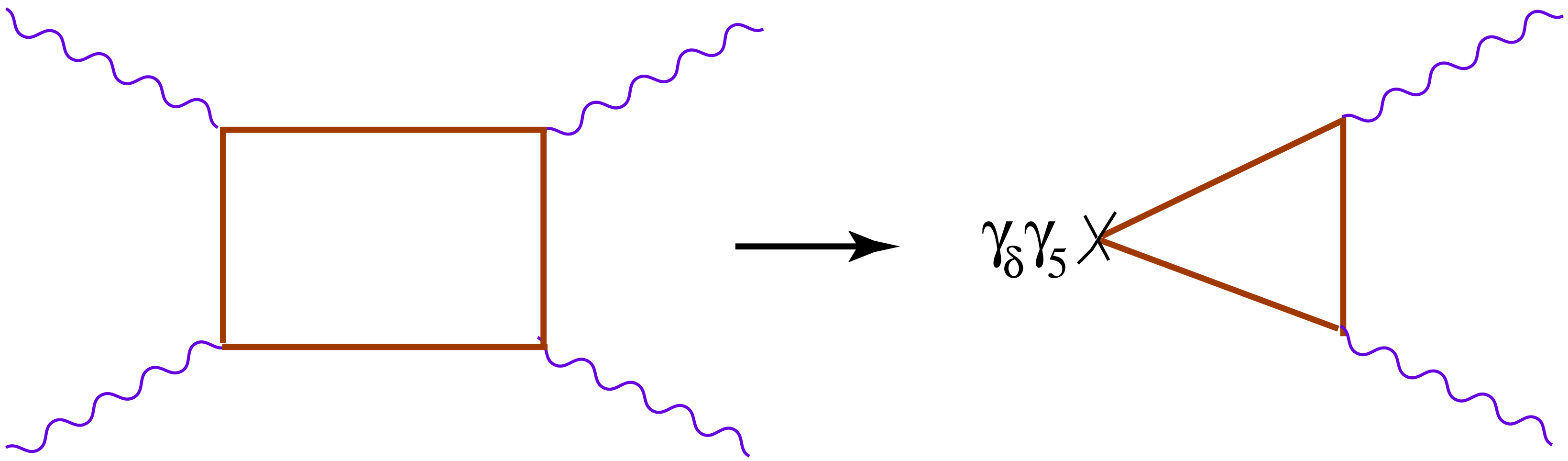
$\hat{q} = (q_1 - q_2)/2$, the axial current $j_5^\rho = \bar{q}\,\hat{Q}^2\gamma^\rho\gamma_5\,q$ is the linear combination of

$j_{5\rho}^{(3)} = \bar{q}\,\lambda_3\gamma^\rho\gamma_5\,q$ isovector

$j_{5\rho}^{(3)} = \bar{q}\,\lambda_8\gamma^\rho\gamma_5\,q$ hypercharge

$j_{5\rho}^{(3)} = \bar{q}\,\gamma^\rho\gamma_5\,q$ singlet

$$j_{5\rho} = \sum_{a=3,8,0}\frac{\mathrm{Tr}\,[\lambda_a\hat{Q}^2]}{\mathrm{Tr}\,[\lambda_a^2]}\,j_{5\rho}^{(a)}$$


γδγ5

The triangle amplitude

$$T^{(a)}_{\mu_3\rho} = i\,\langle 0|\int \mathrm{d}^4 z\, \mathrm{e}^{iq_3 z} T\{j^{(a)}_{5\rho}(z)\, j_{\mu_3}(0)\}|\gamma\rangle$$

kinematically is expressed via two scalar amplitudes

$$T^{(a)}_{\mu_3\rho} = -\frac{ie\,N_c \mathrm{Tr}\,[\lambda_a \hat{Q}^2]}{4\pi^2}\Big\{ w^{(a)}_L(q_3^2)\, q_{3\rho} q_3^\sigma \tilde{f}_{\sigma\mu_3} +$$
$$+ w^{(a)}_T(q_3^2)\Big(-q_3^2 \tilde{f}_{\mu_3\rho} + q_{3\mu_3} q_3^\sigma \tilde{f}_{\sigma\rho} - q_{3\rho} q_3^\sigma \tilde{f}_{\sigma\mu_3}\Big)\Big\}$$

Longitudinal w_L: pseudoscalar mesons exchange

Transversal w_T: pseudovector mesons exchange

In perturbation theory for massless quarks

$$w_L^{(a)}(q^2) = 2w_T^{(a)}(q^2) = -\frac{2}{q^2}$$

Nonvanishing w_L is the signature of the axial Adler–Bell–Jackiw anomaly.
Moreover, for nonsinglet $w_L^{(3,8)}$ it is the *exact* QCD result, no perturbative as well as nonperturbative corrections. So the pole behavior is preserved all way down to small q^2 where the pole is associated with Goldstone mesons $\pi^0,\ \eta$.
Comparing the pole residue we get the famous ABJ result

$$g_{\pi\gamma\gamma} = \frac{N_c \mathrm{Tr}\,[\lambda_3 \hat{Q}^2]}{16\pi^2\, F_\pi}$$

There exists the nonrenormalization theorem for w_T as well but only in respect to perturbative corrections. A.V. '02; Knecht, Peris, Perrottet, de Rafael '03
Higher terms in the OPE does not vanish in this case, they are responsible for shift of the pole $1/q^2 \to 1/(q^2 - m^2_{V,PV})$

Combining we get at $q_1^2 \approx q_2^2 \gg q_3^2$

$$\mathcal{A}_{\mu_1\mu_2\mu_3\gamma\delta}f^{\gamma\delta} = \frac{8}{\hat{q}^2}\epsilon_{\mu_1\mu_2\delta\rho}\hat{q}^{\delta}\sum_{a=3,8,0}W^{(a)}\Big\{w_L^{(a)}(q_3^2)\,q_3^{\rho}q_3^{\sigma}\,\tilde{f}_{\sigma\mu_3}$$
$$+\; w_T^{(a)}(q_3^2)\Big(-q_3^2\,\tilde{f}^{\rho}_{\mu_3}+q_{3\mu_3}q_3^{\sigma}\,\tilde{f}^{\rho}_{\sigma}-q_3^{\rho}q_3^{\sigma}\,\tilde{f}_{\sigma\mu_3}\Big)\Big\}+\cdots$$

where the weights $W^{(3)}=1/4,\ W^{(8)}=1/12,\ W^{(0)}=2/3$.

The model

$$\mathcal{A} = \mathcal{A}_{\rm PS} + \mathcal{A}_{\rm PV} \;+ \text{permutations},$$

$$\mathcal{A}_{\rm PS} = \sum_{a=3,8,0} W^{(a)} \phi_L^{(a)}(q_1^2, q_2^2)\, w_L^{(a)}(q_3^2)\, \{f_2 \tilde{f}_1\}\{\tilde{f} f_3\},$$

$$\mathcal{A}_{\rm PV} = \sum_{a=3,8,0} W^{(a)} \phi_T^{(a)}(q_1^2, q_2^2)\, w_T^{(a)}(q_3^2) \Big(\{q_2 f_2 \tilde{f}_1 \tilde{f} f_3 q_3\}$$

$$+\{q_1 f_1 \tilde{f}_2 \tilde{f} f_3 q_3\} + \frac{q_1^2 + q_2^2}{4}\{f_2 \tilde{f}_1\}\{\tilde{f} f_3\}\Big).$$

- For π^0

$$w_L^{(3)}(q^2) = \frac{2}{q^2 + m_\pi^2}\,,$$

$$\phi_L^3(q_1^2, q_2^2) = \frac{N_c}{4\pi^2 F_\pi^2} F_{\pi\gamma^*\gamma^*}(q_1^2, q_2^2)$$

$$= \frac{q_1^2 q_2^2 (q_1^2 + q_2^2) - h_2 q_1^2 q_2^2 + h_5 (q_1^2 + q_2^2) + (N_c M_1^4 M_2^4 / 4\pi^2 F_\pi^2)}{(q_1^2 + M_1^2)(q_1^2 + M_2^2)(q_2^2 + M_1^2)(q_2^2 + M_2^2)}$$

Following the form factor analysis by Knecht, Nyffeler

$M_1 = 769\ \mathrm{MeV}$, $M_2 = 1465\ \mathrm{MeV}$, $h_5 = 6.93\ \mathrm{GeV}^4$

They did not fix h_2 and put $h_2 = 0$ for the central value. Actually, it is fixed by the old QCD sum rule analysis Novikov et al ’84 $h_2 \approx -10\ \mathrm{GeV}^2$.

The model results in

$$a_\mu^{\pi^0} = 76.5 \times 10^{-11}\,, \qquad a_\mu^{\rm PS} = 114(10) \times 10^{-11}$$

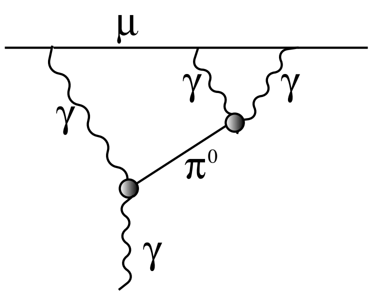
A similar analysis for pseudovector exchange gives

$$a_\mu^{\rm PV} = 22(5) \times 10^{-11}$$

and finally

$$a_\mu^{\rm LbL} = 136(25) \times 10^{-11}$$

Comparison with other models



The difference with meson exchange models, like Knecht, Nyffeler et al, is due to absence of the form factor in the vertex with the soft photon (magnetic field), 76.5×10^{-11} versus 58×10^{-11} for π^0 exchange.

ENJL model Bijnens, Pallante, Prades is conceptually not much different from our model. Indeed, we use meson exchange model which interpolates between the OPE at short distances and meson poles at large ones. It results in a less suppression at large momenta (no form factor in the vertex with magnetic field).

In the ENJL model high momenta asymptotics are provided by adding up the quark loops. Thus, our asymptotics are the same and difference is mostly in details of interpolations between high and low momenta.

Bijnens and Prades demonstrated nicely, in particular, that the asymmetric configuration of momenta $q_1 \approx q_2 \gg q_3$ plays a dominant role in both models.

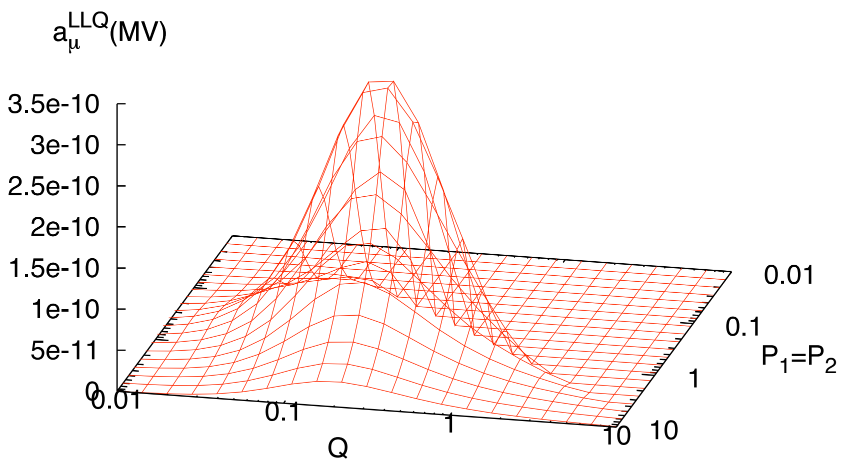


Fig. 8. The quantity a_μ^{LLQ} of Eq. 10) as a function of Q and $P_1 = P_2$ for the MV choice. a_μ is directly related to the volume under the surface as plotted.

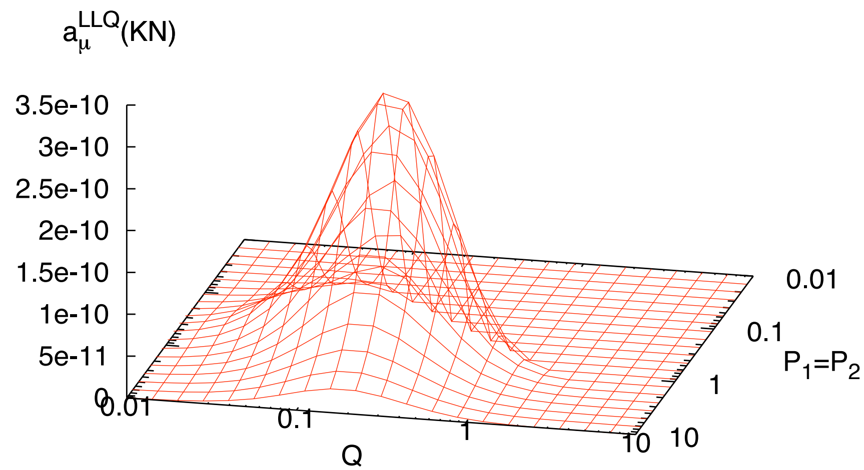


Fig. 9. The quantity a_μ^{LLQ} of Eq. 10) as a function of Q and $P_1 = P_2$ for the KN choice. a_μ is directly related to the volume under the surface as plotted.

Let us compare the sum of pseudoscalar exchanges.
We got it 114×10^{-11}, a 50% increase over the ENJL value 85×10^{-11}.
However, adding up the ENJL result for the quark loop, 22×10^{-11}, we get 109×10^{-11}. Of course, we imply here that the bulk of the quark loop refers to the pseudoscalar exchange.

The difference in results come also from few other sources:
(i) charge pion loop, zero versus $(-19) \times 10^{-11}$ in ENJL,
(ii) scalar exchange, zero versus $(-7) \times 10^{-11}$ in ENJL,
(iii) pseudovector exchange, 22×10^{-11} versus in 2.5×10^{-11} ENJL.

The first point was discussed above, we do not see this contribution as distinguishable from other unaccounted contributions suppressed by $1/N_c$.

The scalar exchange is not suppressed by $1/N_c$. We did not account it in our model because it does not show up at short distances. This means that the scalar exchange falls off at large momenta faster diminishing the integral. Indeed, numerically the scalar exchange is rather small contributions. Moreover, at this level other exchanges like spin two mesons are also relevant. It is not clear at all what would be a combined effect.

The pseudovector exchange occurs to be very sensitive to interpolation between low and high momenta and to the model of mixing in the flavor SU(3). We can some number of arguments in favor of our approach.

Summary

Our final result

$$a_\mu^{\mathrm{LbL}} = 136(25) \times 10^{-11}$$

looks significantly larger than the ENJL one, $83(32) \times 10^{-11}$. However, without the charged pion loop and scalar exchange contribution, the ENJL number is $109(32) \times 10^{-11}$.
Recently Bijnens and Prasad suggested $110(40) \times 10^{-11}$ as an educated guess.

We see that the difference in results refers to rather subtle issues where it is not easy to find solid arguments for resolution.

So my conclusion is rather pessimistic in regards to perspective of diminishing of theoretical error in the hadronic light-by-light contribution.