FERMILAB-CONF-10-118-T

CALCULATION OF QCD LOOPS USING TREE-LEVEL MATRIX ELEMENTS

JAN WINTER [a]
Theoretical Physics Department,
Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510, USA

The possibility of treating colour in one-loop amplitude calculations alike the other quantum numbers is briefly discussed for semi-numerical algorithms based on generalized unitarity and parametric integration techniques. Numerical results are presented for the calculation of virtual corrections in multi-gluon scattering.

1 An algorithmic solution based on generalized unitarity for the evaluation of colour-dressed one-loop amplitudes

The evaluation of QCD higher-order corrections in particular for multi-particle processes is indispensable for a good understanding of total and differential cross sections at hadron colliders such as the Tevatron or the LHC. The automation of these calculations at the next-to-leading order (NLO) in the strong coupling constant has been a very appealing goal ever since the Monte Carlo tools dealing with the computation of tree-level cross sections have matured and been greatly optimized. Over the past few years a number of subtraction codes has been developed handling the cancellations of singularities occurring in NLO calculations in a general way. Accordingly a standard interface has lately been agreed on between Monte Carlo tools and one-loop matrix-element programs [1,2]. Recently these programs have also shown a remarkable advancement. New techniques based on combining generalized unitarity and parametric integration methods have become available and made the computation of multi-leg one-loop corrections feasible. Prominent examples are given by the vector boson plus 3 jet NLO predictions provided by the BlackHat and Rocket groups [3,4,5,6] and the $t\bar{t}$ plus 1 jet NLO result [7]. These calculations separate the treatment of the colour quantum numbers from the other degrees of freedom, which makes it harder to fully automatize these approaches. In a recent publication [8] it has been shown that the Ellis–Giele–Kunszt–Melnikov algorithm [9] can be extended to treat colour along the same line with the other quantum numbers. Given the implementation for ordered gluon one-loop amplitudes [10], the important changes are: the sums over ordered cuts present in the decomposition of the one-loop integrands and amplitudes are replaced by sums over unordered partitions including their non-cyclic and non-reflective permutations. Gluon bubble contributions now come with a symmetry factor of 1/2!. The integrand's residues are calculated through products of colour-dressed tree-level amplitudes obtained from dressed recursion relations where internal colour degrees of freedom are summed over. Finally, to extract the lower-point coefficients, the subtraction terms due to higher-cut contributions are identified by a de-pinching procedure that may involve loop-momentum shifts by external momenta.

[a] Talk presented at the 45th Rencontres de Moriond, Workshop on QCD and High Energy Interactions, March 13-20, 2010 at La Thuile, Italy.

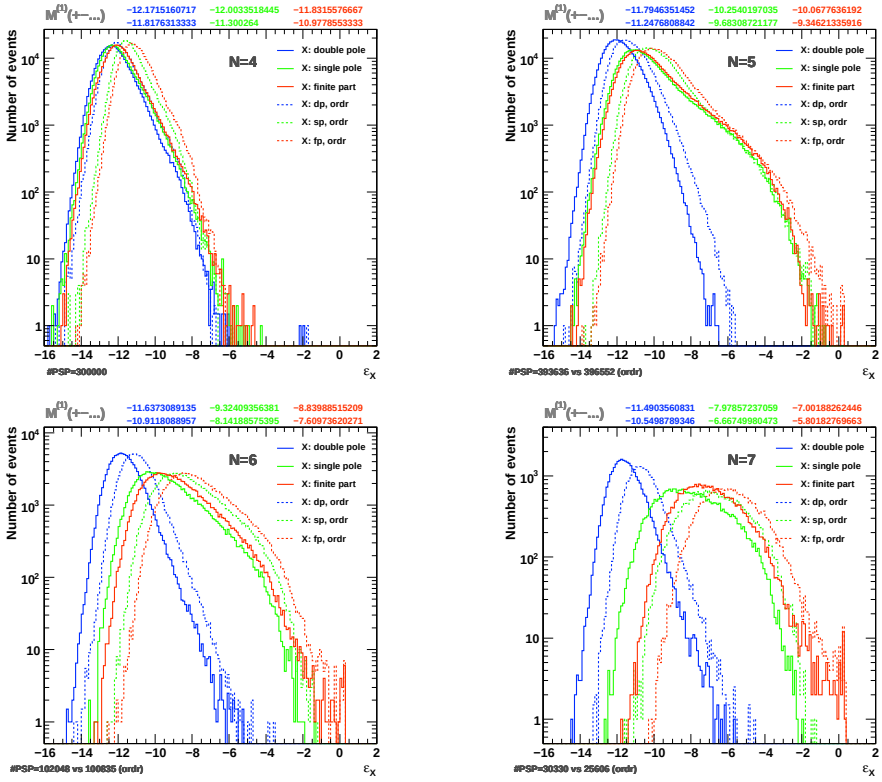

Figure 1: Double-, single-pole and finite-part accuracy distributions obtained from double-precision computations of one-loop amplitudes for $N = 4, 5, 6, 7$ gluons with polarizations $\lambda_k = + - + - \ldots$ and randomly chosen non-vanishing colour configurations. Results from the colour-dressed algorithm are compared to those of the colour-ordered method (labeled "ordr") indicated by dashed curves. The mean accuracies and the number of randomly picked phase-space points (subject to the cuts detailed in the text) are displayed at the top and bottom left of the plots, respectively. Phase-space points required to be calculated at higher precision were vetoed.

2 Numerical calculation of multi-gluon one-loop corrections

Taking the example of multi-gluon scattering, the performance and major results of the algorithm briefly introduced in Section 1 are discussed below. A more comprehensive description of the outcomes of these numerical calculations can be found in the original paper [8].

The convenient way to cross-check the results of the colour-dressed algorithm for the full one-loop amplitudes for N gluons is to make use of the colour decomposition approach. Given the polarization states $\lambda_k \in \{+, -\}$ of the $k = 1, \ldots, N$ gluons and their colours $(ij)_k$ using the colour-flow notation $i_k, j_k \in \{1, 2, 3\}$, one sums up all relevant ordered amplitudes after having multiplied them by their corresponding colour factors. The colour-ordered N-gluon one-loop amplitudes are computed with the algorithm [10] based on the original Ellis–Giele–Kunszt–Melnikov method. The quality of the one-loop amplitude determination can be analyzed by means of the logarithmic relative deviations of the double (dp) and single poles (sp) and the finite part (fp). They are defined as follows:

$$\varepsilon_{\rm dp} \;=\; \log_{10} \frac{|\mathcal{M}^{(1)[1]}_{\rm dp,num} - \mathcal{M}^{(1)}_{\rm dp,th}|}{|\mathcal{M}^{(1)}_{\rm dp,th}|}\,, \qquad \varepsilon_{\rm s/fp} \;=\; \log_{10} \frac{2\,|\mathcal{M}^{(1)[1]}_{\rm s/fp,num} - \mathcal{M}^{(1)[2]}_{\rm s/fp,num}|}{|\mathcal{M}^{(1)[1]}_{\rm s/fp,num}| + |\mathcal{M}^{(1)[2]}_{\rm s/fp,num}|}\,, \tag{1}$$

Two independent solutions denoted by [1] and [2] are used to test the accuracies of the single poles and finite parts. The analytic structure of the double poles is particularly simple and proportional to the Born amplitudes, $\mathcal{M}^{(1)}_{\rm dp,th} = -(c_\Gamma/\epsilon^2)\, N\, N_{\rm C}\, \mathcal{M}^{(0)}$ where $N_{\rm C}$ denotes the number of colours. This allows for a direct comparison with the semi-numerical results. Figure 1 shows

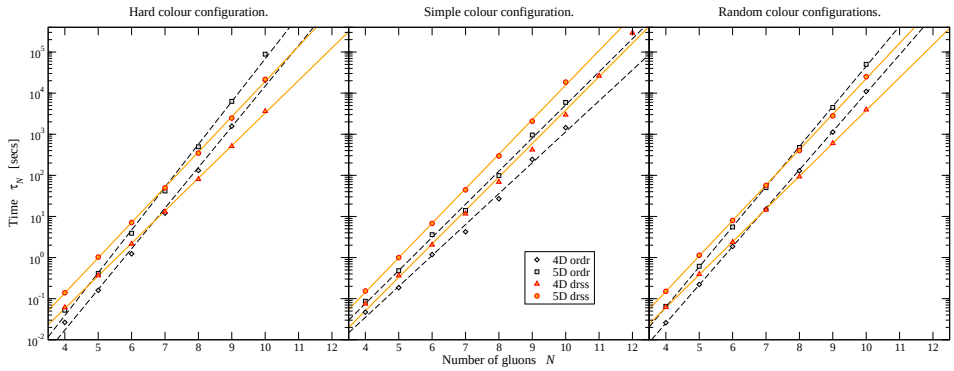

Figure 2: Times τ_N to compute two N-gluon one-loop amplitudes for a hard, a simple and random colour configurations using 3.0 and 2.66 GHz Intel Core2 Duo processors. The solid and dashed curves represent fit results for the colour-dressed (drss) and colour-ordered (ordr) approach, respectively. The times to compute the cut-constructible part of the amplitudes using the 4-dimensional (4D) versions of the algorithms are also shown.

the ε-distributions in absolute normalization obtained by using double-precision computations for various numbers of external gluons with alternating polarization states. The phase-space points are accepted if the generated momenta satisfy the cuts: $|\eta_n| < 2\,,\ p_{\perp,n} > 0.1\,|E_1 + E_2|$ and $\Delta R_{kn} > 0.7$ where η_n and $p_{\perp,n}$ respectively denote the pseudo-rapidity and transverse momentum of the n-th outgoing gluon ($n = 3, \ldots, N$); the ΔR_{kn} describe the pairwise geometric separations in η and azimuthal-angle space of gluons k and n. For $N \geq 5$ gluons, a small fraction of events requires higher than double precision calculations to reliably determine the master-integral coefficients.[b] These phase-space points can be identified by a simple procedure testing the stability of the solutions for the bubble coefficients that contribute to the cut-constructible part. As seen in the plots of Figure 1 the gluon one-loop amplitudes can be determined quite accurately for the bulk of the events. The tails are sufficiently under control and fall off more steeply for larger ε-values owing to the veto on points that yield unstable solutions in double-precision calculations. The distributions and peak positions of the double poles are rather stable while those of the single poles and finite parts noticeably shift to larger ε with an increasing number N of gluons. In all cases the dressed approach is seen to provide more accurate results than the method relying on the colour decomposition.

The complexity of the colour-dressed tree-level recursion relations and the asymptotic behaviour of the Stirling numbers of the second kind that govern the growth of the number of the unitarity cuts both follow an exponential scaling law when more legs are added. The colour-dressed algorithm is hence expected to scale similarly with N, i.e. the computing time τ_N is proportional to x^N where x is an attribute of the implemented algorithm. Relying on the colour decomposition, the ordered one-loop amplitude calculations exhibit polynomial complexity while the number of orderings that need be evaluated for one colour configuration grows factorially. The effective growth however will be determined by the increase in the number of non-vanishing orderings. Figure 2 depicts how the N-gluon one-loop amplitude computation times scale with increasing N for different examples of assigning the gluons' colour states. Amplitudes with a small number of gluons, $N \leq 6$, can always be calculated faster using the colour decomposition. This remains true as long as there is a sufficient number of vanishing orderings, i.e. one deals with simple colour configurations only. In all other cases – for random colour assignments most importantly – the colour-dressed method takes over for $N \geq 7$ owing to its milder exponential growth. The curve fitting for the general case indicates base-values of $x = 7.3(1)$ for the dressed algorithm versus $x = 9.5(1)$ when using the colour decomposition. One notices that the factorial growth caused by the sum over orderings has been tamed to become effectively exponential.

[b] For $N = 6$ and 7 gluons, about 3% and 10% of the events need be treated in higher precision, respectively.

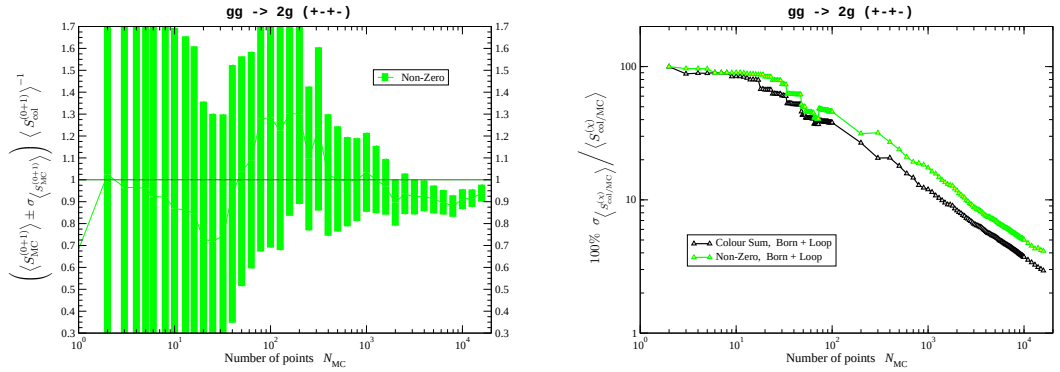

Figure 3: Consistency check for and relative errors of the phase-space integrations of colour-summed and colour-sampled amplitudes at the Born plus virtual-correction level. The distributions are given as functions of the number of generated uniform phase-space points; cuts are specified in the text. After 15900 events one obtains $\langle S_{\rm MC}^{(0+1)}\rangle/\langle S_{\rm col}^{(0+1)}\rangle = 0.939 \pm 0.039({\rm MC}) \pm 0.028({\rm col})$ with the left plot showing errors due to colour sampling only.

Utilizing the ability of calculating the full one-loop amplitudes for multiple gluons, one can study the effect of the Monte Carlo sampling over non-vanishing ("Non-Zero") colour configurations on the performance of phase-space integrations for the Born plus virtual contributions. Figure 3 shows the simplest example of $2 \to 2$ gluon scattering. The colour-sampled and colour-summed integrals are defined as $S_{\rm MC}^{(0+1)} = W_{\rm col} \times \mathcal{K}$ and $S_{\rm col}^{(0+1)} = \sum_{\rm col} \mathcal{K}$, respectively, where the colour weight $W_{\rm col}$ depends on the actual colour configuration ("col") and the kernel is given by

$$\mathcal{K} \;=\; \left|\mathcal{M}^{(0)}\right|^2 + \frac{\widehat{\alpha}_s}{2\,\pi}\,\Re\left(\mathcal{M}_{\rm fp}^{(1)}\,\mathcal{M}^{(0)\,\dagger}\right) \qquad \text{with} \quad \widehat{\alpha}_s \equiv 0.12\ . \tag{2}$$

In the left panel of Figure 3 the sampled and summed integrations are shown to converge after about 10^3 Monte Carlo steps, $N_{\rm MC}$, while in the right panel their relative errors are displayed as a function of $N_{\rm MC}$. One finds that the colour sampling only introduces an additional integration uncertainty, which can be reduced more easily than the full colour sum can be carried out.

Acknowledgments

I thank the organizers for creating a fantastic atmosphere during the course of the workshop. I also would like to acknowledge my collaborators Walter Giele and Zoltan Kunszt for many exciting and important discussions while working on this subject. Fermilab is operated by the Fermi Research Alliance under CN DE-AC02-07CH11359 with the U.S. Department of Energy.

References

1. T. Binoth *et al.*, arXiv:1001.1307 [hep-ph].
2. J. R. Andersen *et al.* [SM and NLO Multileg Working Group], arXiv:1003.1241 [hep-ph].
3. C. F. Berger *et al.*, Phys. Rev. D **80** (2009) 074036 [arXiv:0907.1984 [hep-ph]].
4. C. F. Berger *et al.*, arXiv:1004.1659 [hep-ph].
5. R. K. Ellis, K. Melnikov, G. Zanderighi, Phys. Rev. D **80** (2009) 094002 [arXiv:0906.1445].
6. K. Melnikov and G. Zanderighi, Phys. Rev. D **81** (2010) 074025 [arXiv:0910.3671 [hep-ph]].
7. K. Melnikov and M. Schulze, arXiv:1004.3284 [hep-ph].
8. W. T. Giele, Z. Kunszt and J. Winter, arXiv:0911.1962 [hep-ph].
9. R. K. Ellis, W. T. Giele, Z. Kunszt, JHEP **0803** (2008) 003 [arXiv:0708.2398 [hep-ph]]; W. T. Giele, Z. Kunszt, K. Melnikov, JHEP **0804** (2008) 049 [arXiv:0801.2237 [hep-ph]].
10. J. Winter and W. T. Giele, arXiv:0902.0094 [hep-ph].