



Light scalar mesons: 2-quark or four-quark states

- a model independent way to distinguish

Cai-Dian Lü (呂才典)

IHEP, Beijing

Wei Wang, CD Lu, [arXiv:0910.0613](https://arxiv.org/abs/0910.0613) [hep-ph]



Outline

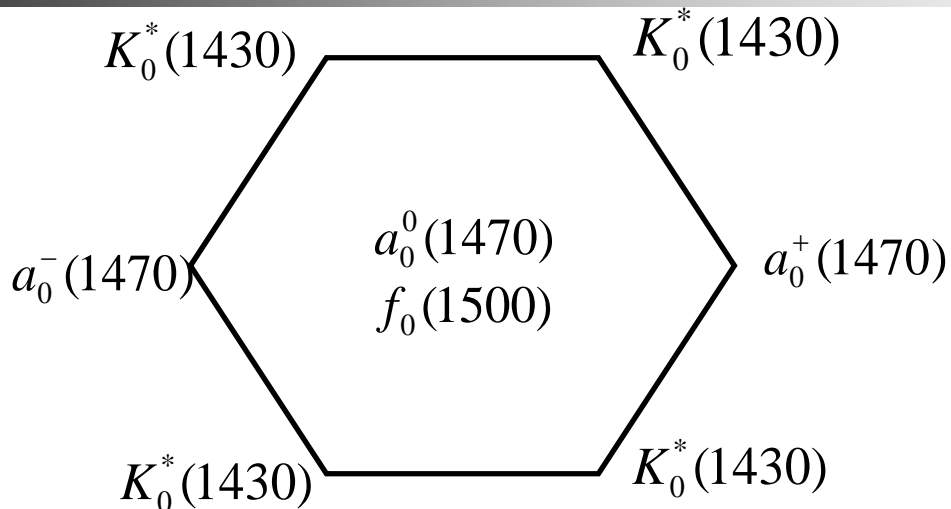
- **Scalar meson study status**
- **Four quark states or two-quark states:
semileptonic decays**
- **Non-leptonic decays**

- **Summary**



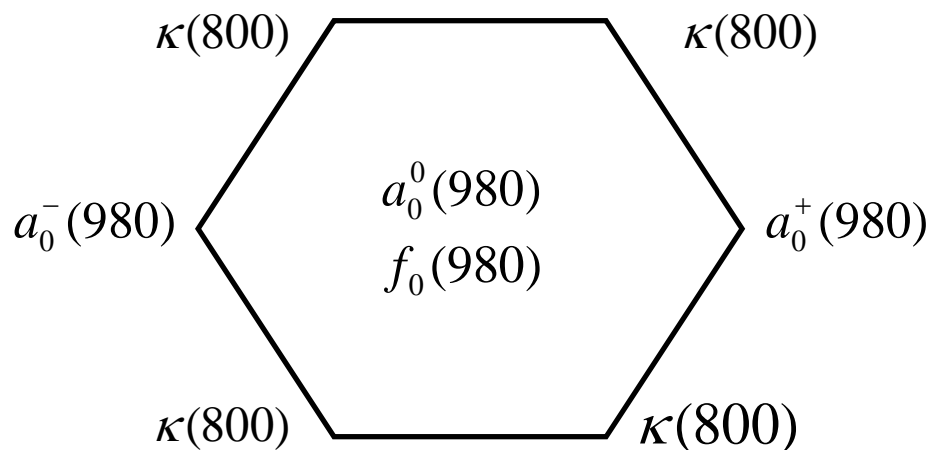
Scalar Mesons ($J^P=0^+$)

$q\bar{q}$



$f_0(1370)$
 $f_0(1710)$

$q^2\bar{q}^2$
or $q\bar{q}$



$\sigma(600)$

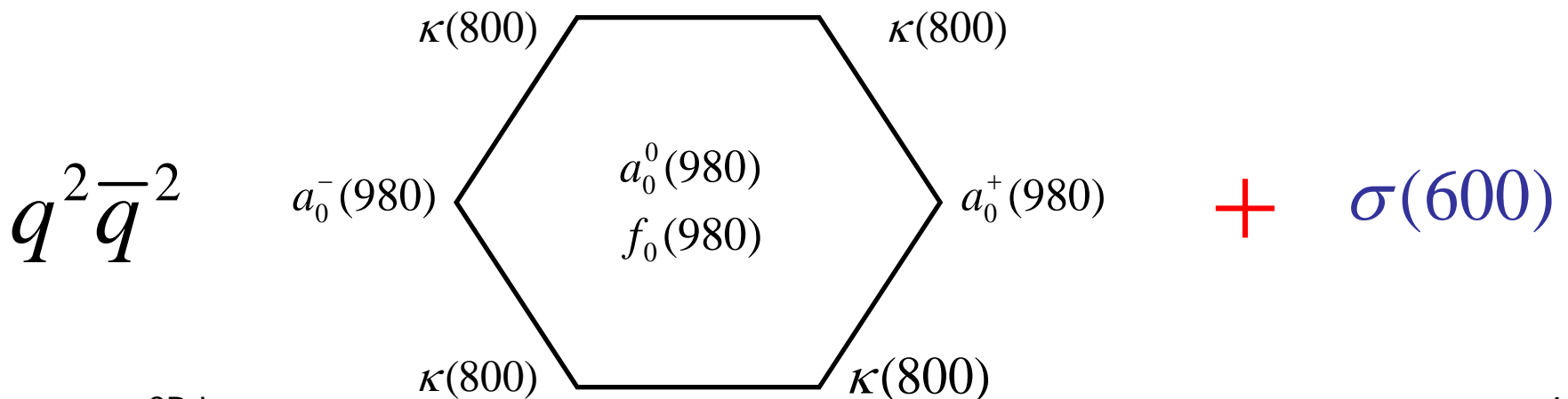


Scalar Mesons around 1GeV ($J^P=0^+$)

σ exist or not for many years

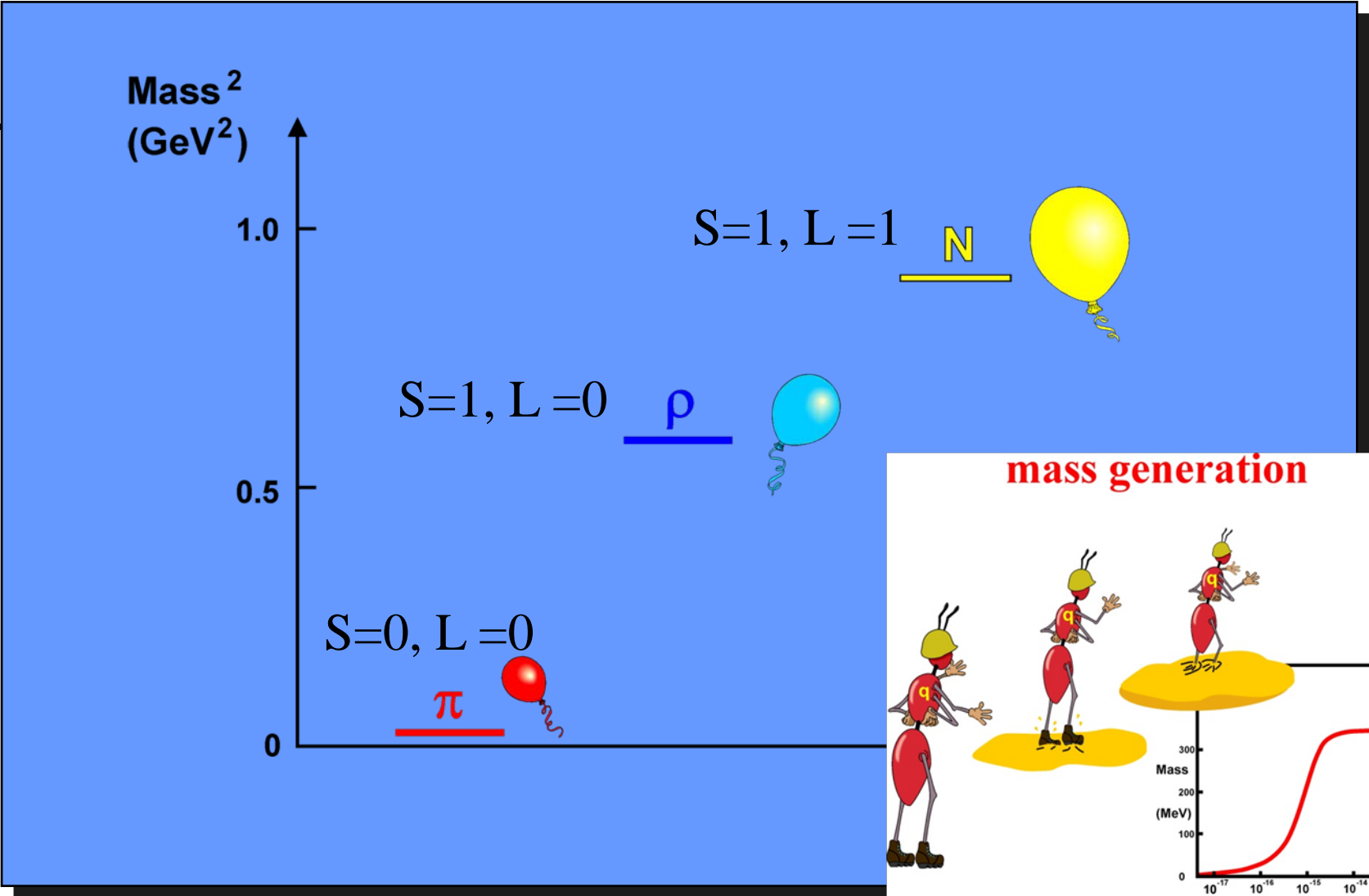
Mass too light to be $q\bar{q}$

Because 0^{++} states are $S=1, L=1$, should be heavier than 1^- (ρ, ϕ, K^*)





Hadron masses ²





Comparison with $\bar{c}c$ family

0^{++}

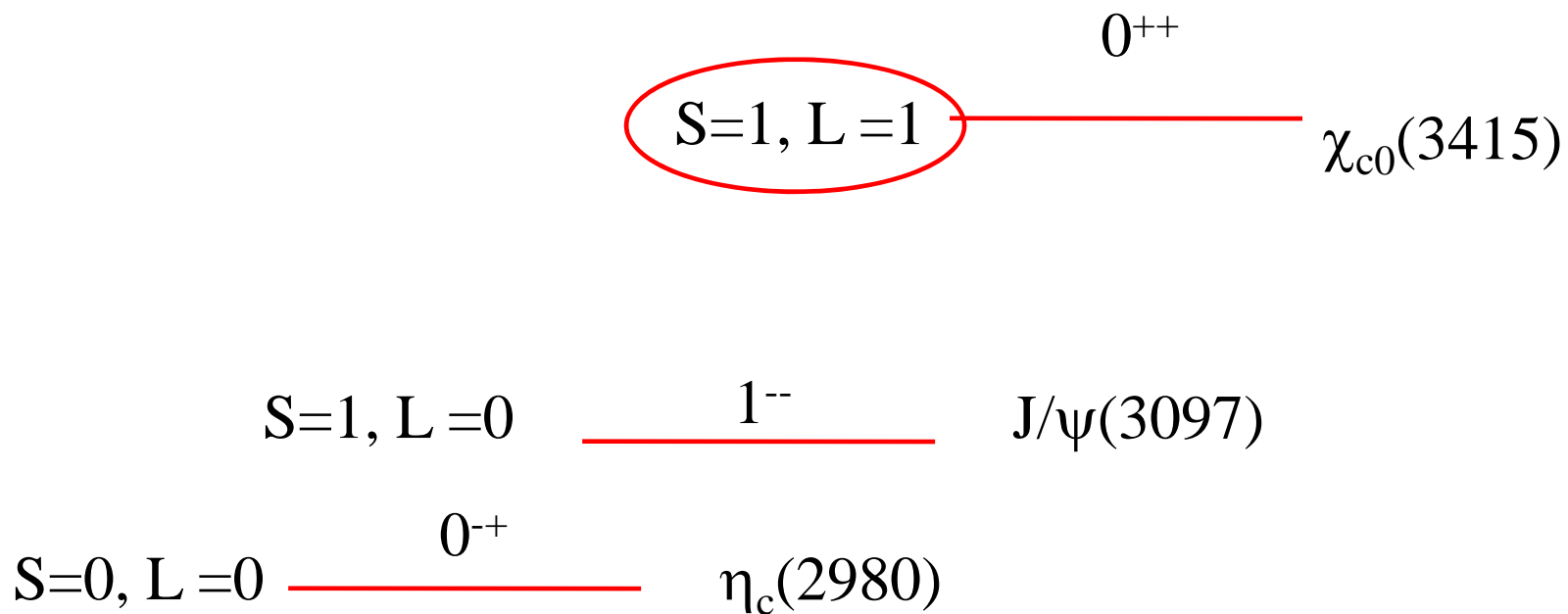
$S=1, L=1$ ————— $\chi_{c0}(3415)$

$S=1, L=0$ ————— 1^{--} $J/\psi(3097)$

$S=0, L=0$ ————— 0^{-+} $\eta_c(2980)$



Comparison with $\bar{c}c$ family



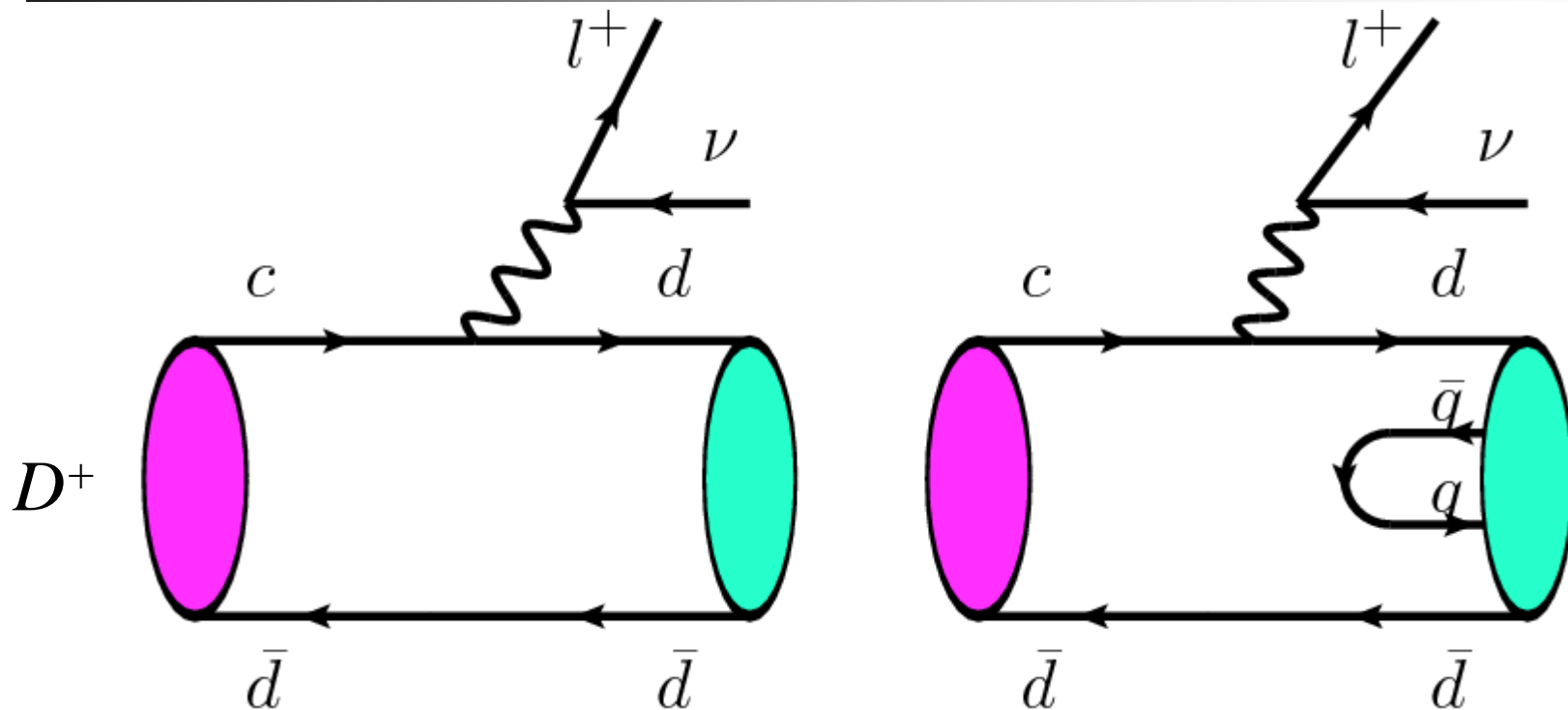


the neutral scalar mesons 0^{++} has the same quantum number with vacuum

-
- They can mix with vacuum, **glueball**, even molecular states
 - So a lot of explanations on the market—**tetra quark states**
 - Most study focus on the **decay property** of the scalar mesons
 - The **production** of scalar mesons from **heavy quark decays** are more interesting



Feynman diagrams of semi-leptonic decays of D to Scalar Mesons



$D^+ \rightarrow f_0 \pi^+$, $D^+ \rightarrow \sigma \pi^+$ have been measured



2-quark picture of ordinary light Scalar Mesons

$$|\sigma\rangle = \frac{1}{\sqrt{2}}(|\bar{u}u\rangle + |\bar{d}d\rangle) \equiv |\bar{n}n\rangle, \quad |f_0\rangle = |\bar{s}s\rangle, \quad (1)$$

$$|a_0^0\rangle = \frac{1}{\sqrt{2}}(|\bar{u}u\rangle - |\bar{d}d\rangle), \quad |a_0^-\rangle = |\bar{u}d\rangle, \quad |a_0^+\rangle = |\bar{d}u\rangle$$

σ - f_0 mixing:

$$\begin{pmatrix} |f_0\rangle \\ |\sigma\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} |\bar{s}s\rangle \\ |\bar{n}n\rangle \end{pmatrix}$$

with

$$25^\circ < \theta < 40^\circ, \quad 140^\circ < \theta < 165^\circ$$



2-quark picture of ordinary light Scalar Mesons

$$|\sigma\rangle = \frac{1}{\sqrt{2}}(|\bar{u}u\rangle + |\bar{d}d\rangle) \equiv |\bar{n}n\rangle, \quad |f_0\rangle = |\bar{s}s\rangle, \quad (1)$$

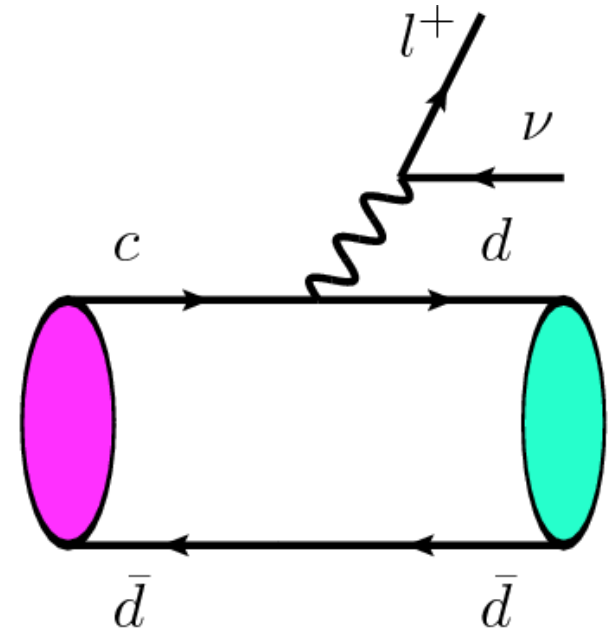
$$|a_0^0\rangle = \frac{1}{\sqrt{2}}(|\bar{u}u\rangle - |\bar{d}d\rangle), \quad |a_0^- \rangle =$$

σ - f_0 mixing:

$$\begin{pmatrix} |f_0\rangle \\ |\sigma\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} |\bar{s}s\rangle \\ |\bar{n}n\rangle \end{pmatrix}$$

with

$$25^\circ < \theta < 40^\circ, \quad 140^\circ < \theta < 165^\circ$$





2-quark picture of Scalar Mesons

only $d\bar{d}$ component contribute, in isospin symmetry, we have

$$\begin{aligned}A(D^+ \rightarrow f_0 l^+ \nu) &= -\sin \theta \hat{A}, \\A(D^+ \rightarrow \sigma l^+ \nu) &= -\cos \theta \hat{A}\end{aligned}$$

where

$$\hat{A} \equiv A(D^+ \rightarrow a_0^0 l^+ \nu)$$

$\text{Br} \sim |\mathcal{A}|^2$, **We can get sum rule as**

$$\mathcal{B}(D^+ \rightarrow a_0^0 l^+ \nu) = \mathcal{B}(D^+ \rightarrow f_0 l^+ \nu) + \mathcal{B}(D^+ \rightarrow \sigma l^+ \nu)$$



Hadronic picture

- Isospin 0 and isospin 1 contribution should be 1:1, derived from the Clebsch-Gordan coefficients
- Isospin conserved by strong interaction, no matter perturbative or non-perturbative
--model independent

similarly, $B^+ / D^+ \rightarrow \rho e^+ \nu_e = B^+ / D^+ \rightarrow \omega e^+ \nu_e$
already verified by exp.



4-quark picture of ordinary light Scalar Mesons

Group theory
assignment

$$|\sigma\rangle = \bar{u}u\bar{d}d, \quad |f_0\rangle = |\bar{n}n\bar{s}s\rangle, \quad (5)$$

$$|a_0^0\rangle = \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d)\bar{s}s, \quad |a_0^+\rangle = |\bar{d}u\bar{s}s\rangle, \quad |a_0^-\rangle = |\bar{u}d\bar{s}s\rangle$$

σ - f_0 mixing:

$$\begin{pmatrix} |f_0\rangle \\ |\sigma\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} |\bar{n}n\bar{s}s\rangle \\ |\bar{u}u\bar{d}d\rangle \end{pmatrix}$$

with

$$\phi = (174.6^{+3.4}_{-3.2})^\circ$$

$$n\bar{n} = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}$$



4-quark picture of ordinary light Scalar Mesons

Group theory assignment

$$|\sigma\rangle = \bar{u}u\bar{d}d, \quad |f_0\rangle = |\bar{n}n\bar{s}s\rangle,$$

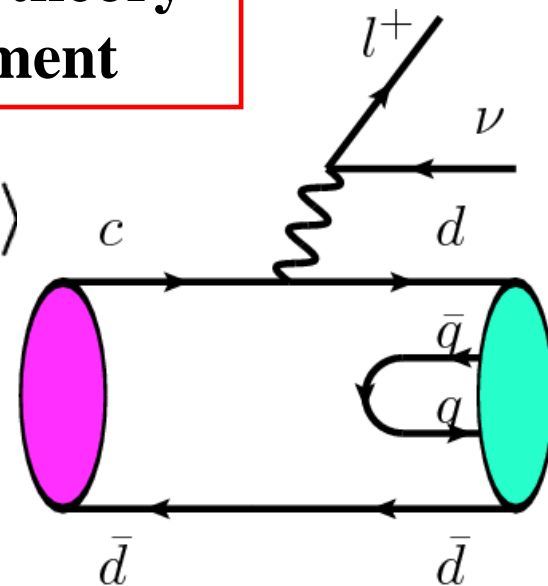
$$|a_0^0\rangle = \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d)\bar{s}s, \quad |a_0^+\rangle = |\bar{d}u\bar{s}s\rangle$$

σ - f_0 mixing:

$$\begin{pmatrix} |f_0\rangle \\ |\sigma\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} |\bar{n}n\bar{s}s\rangle \\ |\bar{u}u\bar{d}d\rangle \end{pmatrix}$$

with

$$\phi = (174.6^{+3.4}_{-3.2})^\circ$$



$$n\bar{n} = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}$$



4-quark picture of Scalar Mesons

$$\mathcal{A}(D^+ \rightarrow f_0 l^+ \nu) = -(\cos \phi + \sqrt{2} \sin \phi) \hat{\mathcal{A}}$$

$$\mathcal{A}(D^+ \rightarrow \sigma l^+ \nu) = (\sin \phi - \sqrt{2} \cos \phi) \hat{\mathcal{A}},$$

where

$$\hat{\mathcal{A}} \equiv \mathcal{A}(D^+ \rightarrow a_0^0 l^+ \nu)$$

We can get sum rule as

$$\mathcal{B}(D^+ \rightarrow a_0^0 l^+ \nu) = \frac{1}{3} [\mathcal{B}(D^+ \rightarrow f_0 l^+ \nu) + \mathcal{B}(D^+ \rightarrow \sigma l^+ \nu)]$$



Define a ratio R

$$R = \frac{\mathcal{B}(D^+ \rightarrow f_0 l^+ \nu) + \mathcal{B}(D^+ \rightarrow \sigma l^+ \nu)}{\mathcal{B}(D^+ \rightarrow a_0^0 l^+ \nu)}$$

It is one for 2-quark picture, while 3 for 4-quark picture

Similarly, for B meson decays, we have

$$R = \frac{\mathcal{B}(B^+ \rightarrow f_0 l^+ \nu) + \mathcal{B}(B^+ \rightarrow \sigma l^+ \nu)}{\mathcal{B}(B^+ \rightarrow a_0^0 l^+ \nu)}$$
$$= \begin{cases} 1 & \text{two quark} \\ 3 & \text{tetra-quark} \end{cases}$$



These channels have large enough BRs to be measurable

If the mixing angle is modest, all three $D^+ \rightarrow Sl^+\nu$ have similar branching ratios. The branching ratio of the semileptonic $D_s \rightarrow f_0$ decay is measured [9] as

$$\begin{aligned} & \mathcal{B}(D_s \rightarrow f_0 l \bar{\nu}) \times \mathcal{B}(f_0 \rightarrow \pi^+ \pi^-) \\ & = (2.0 \pm 0.3 \pm 0.1) \times 10^{-3}. \end{aligned} \quad (16)$$

Thus as an estimation, branching ratios for the cascade $D^+ \rightarrow Sl^+\nu$ decays are expected to have the order

$$\frac{V_{cd}^2}{V_{cs}^2} \times 2 \times 10^{-3} \sim 1 \times 10^{-4}. \quad (17)$$



As for the B decays, the branching ratio of $B \rightarrow Sl\bar{\nu}$ can be estimated utilizing the $B \rightarrow \rho l\bar{\nu}$ and $D_s^+ \rightarrow \phi l^+ \nu$ decays. If the mixing angle is moderate, the branching ratio can be estimated as

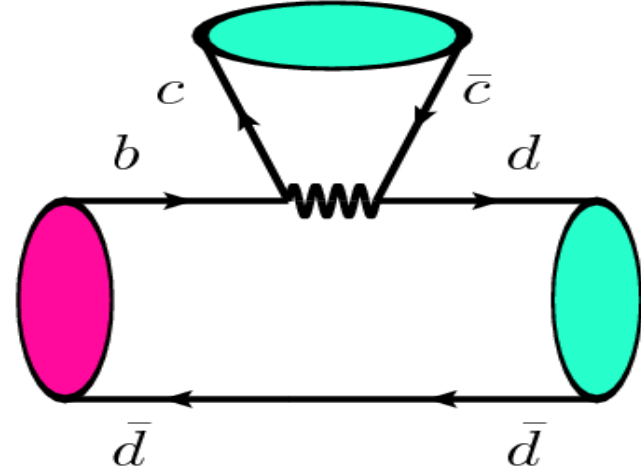
$$\begin{aligned} \mathcal{B}(B \rightarrow f_0 l \bar{\nu}) &\sim \mathcal{B}(B \rightarrow \rho l \bar{\nu}) \frac{\mathcal{B}(D_s \rightarrow f_0 l \bar{\nu})}{\mathcal{B}(D_s \rightarrow \phi l \bar{\nu})} \\ &\sim 10^{-4} \times \frac{10^{-3}}{10^{-2}} = 10^{-5}. \end{aligned} \quad (18)$$

Compared with the recently measured semileptonic $B \rightarrow \eta$ decay [11]

$$\mathcal{B}(B^- \rightarrow \eta l^- \bar{\nu}) = (3.1 \pm 0.6 \pm 0.8) \times 10^{-5}, \quad (19)$$



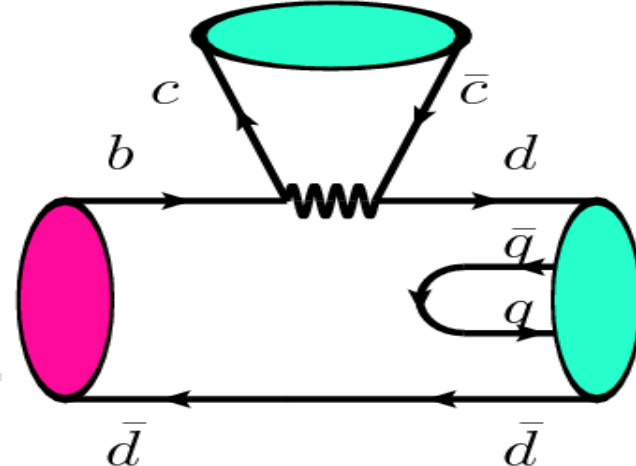
$$B \rightarrow J/\psi (\eta_c) f_0$$



- Semileptonic B decays $B^+ \rightarrow f_0 l^+ \nu_l$ are clean, but the neutrino is identified as missing energy, thus the **efficiency is limited**
- The lepton pair can also be replaced by a **charmonium state** such as J/ψ , since J/ψ does not carry any light flavor either.
- $B \rightarrow J/\psi f_0$ decays may provide another ideal probe to detect the internal structure of the scalar mesons.



$$B \rightarrow J/\psi (\eta_c) f_0$$



- Semileptonic B decays $B^+ \rightarrow f_0 l^+ \nu_l$ are clean, but the neutrino is identified as missing energy, thus the **efficiency is limited**
- The lepton pair can also be replaced by a **charmonium state** such as J/ψ , since J/ψ does not carry any light flavor either.
- $B \rightarrow J/\psi f_0$ decays may provide another ideal probe to detect the internal structure of the scalar mesons.



$B \rightarrow J/\psi(\eta_c) S$

$$R = \frac{\mathcal{B}(\bar{B}^0 \rightarrow f_0 J/\psi) + \mathcal{B}(\bar{B}^0 \rightarrow \sigma J/\psi)}{\mathcal{B}(\bar{B}^0 \rightarrow a_0^0 J/\psi)} = 1 \quad (20)$$

in the $\bar{q}q$ picture, and

$$R = \frac{\mathcal{B}(\bar{B}^0 \rightarrow f_0 J/\psi) + \mathcal{B}(\bar{B}^0 \rightarrow \sigma J/\psi)}{\mathcal{B}(\bar{B}^0 \rightarrow a_0^0 J/\psi)} = 3 \quad (21)$$

in the $\bar{q}q\bar{q}q$ picture. Although these are hadronic decays



The branching fraction is expected to have the order

$$\begin{aligned}\mathcal{B}(B \rightarrow f_0 J/\psi) &\sim \mathcal{B}(\bar{B}^0 \rightarrow \rho^0 J/\psi) \frac{\mathcal{B}(D_s \rightarrow f_0 l \bar{\nu})}{\mathcal{B}(D_s \rightarrow \phi l \bar{\nu})} \\ &\sim 10^{-5} \times \frac{10^{-3}}{10^{-2}} = 10^{-6}.\end{aligned}\quad (22)$$

On experimental side, the J/ψ is easily detected through a lepton pair l^+l^- and thus this mode may be more useful. If the J/ψ meson is replaced by η_c in eq.(20,21), one can get the similar sum rules.



Uncertainties mainly from SU(3) breaking

- **Form factor difference** – no problem, since it makes the R for 4-quark picture even larger than 3
- **Mostly by mass difference**, only problem:
 $m_\sigma = (0.4 \sim 1.2) \text{ GeV}$, but any way phase space is easy to calculate
- D decays not very good. But **in B decays**, there is **no problem**, since m_σ negligible
- Governed by **heavy quark effective theory**, the SU(3) breaking is suppressed by $1/m_Q$



2-quark picture of ordinary light Scalar Mesons

$$|\sigma\rangle = \frac{1}{\sqrt{2}}(|\bar{u}u\rangle + |\bar{d}d\rangle) \equiv |\bar{n}n\rangle, \quad |f_0\rangle = |\bar{s}s\rangle, \quad (1)$$

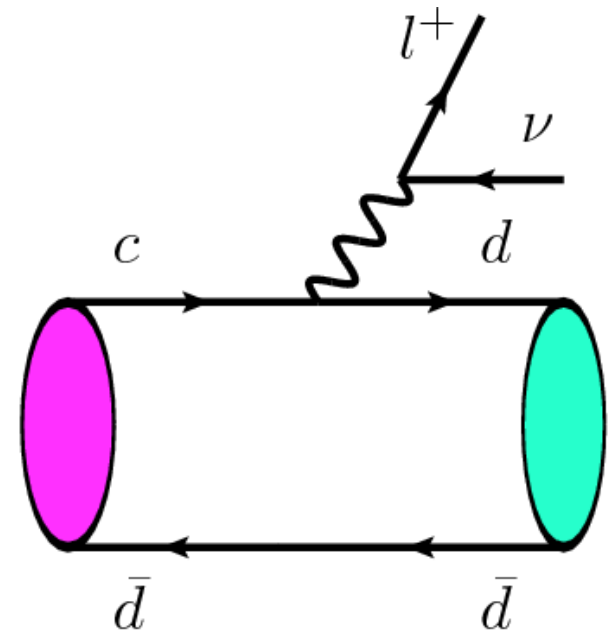
$$|a_0^0\rangle = \frac{1}{\sqrt{2}}(|\bar{u}u\rangle - |\bar{d}d\rangle), \quad |a_0^- \rangle =$$

σ - f_0 mixing:

$$\begin{pmatrix} |f_0\rangle \\ |\sigma\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} |\bar{s}s\rangle \\ |\bar{n}n\rangle \end{pmatrix}$$

with

$$25^\circ < \theta < 40^\circ, \quad 140^\circ < \theta < 165^\circ$$





2-quark picture of Scalar Mesons

only dd component contribute, in SU(3) symmetry, we have

$$\begin{aligned} \mathcal{A}(D^+ \rightarrow f_0 l^+ \nu) &= -\sin \theta \hat{\mathcal{A}}, \\ \mathcal{A}(D^+ \rightarrow \sigma l^+ \nu) &= -\cos \theta \hat{\mathcal{A}} \end{aligned}$$

where

$$\hat{\mathcal{A}} \equiv \mathcal{A}(D^+ \rightarrow a_0^0 l^+ \nu)$$

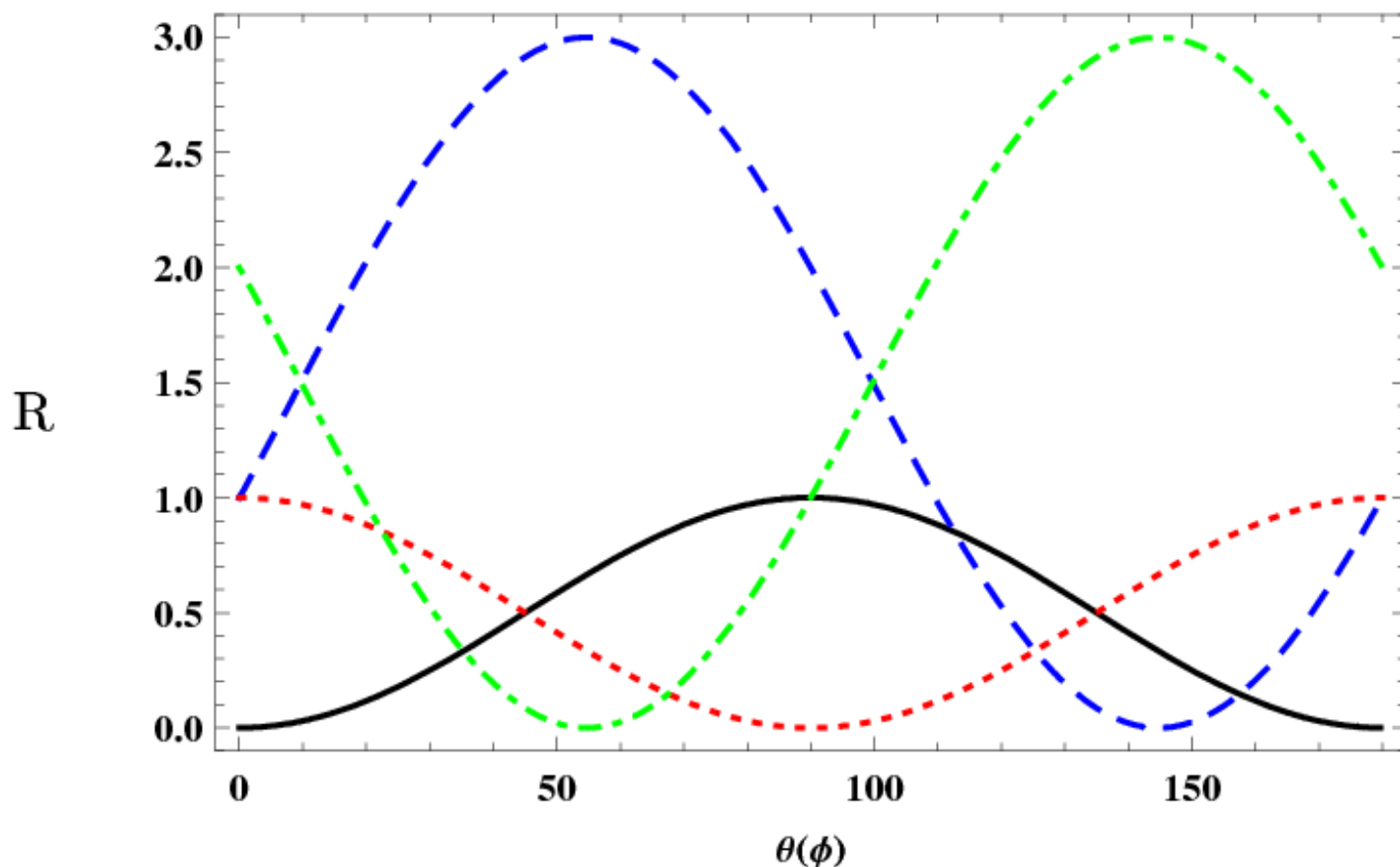
Br $\sim |\mathcal{A}|^2$, **We can get s sum rule as**

$$\mathcal{B}(D^+ \rightarrow a_0^0 l^+ \nu) = \mathcal{B}(D^+ \rightarrow f_0 l^+ \nu) + \mathcal{B}(D^+ \rightarrow \sigma l^+ \nu)$$



$$R_{f_0} = \frac{\mathcal{B}(D/B \rightarrow f_0 l \nu)}{\mathcal{B}(D/B \rightarrow a_0^0 l \nu)} \quad \left(R_{f_0} = \frac{\mathcal{B}(B \rightarrow f_0 J/\psi)}{\mathcal{B}(B \rightarrow a_0^0 J/\psi)} \right)$$

The measurement of ratio R can determine the mixing angle of f_0 and σ





Summary

- The **semi-leptonic decays** of **D** and/or **B** mesons to scalar mesons can provide a **theoretically clean** way to distinguish the **2-quark** and **4-quark picture** of light scalar mesons
- So as the **non-leptonic decays** of **$B \rightarrow J/\psi(\eta_c) S$**
- They can also measure the mixing angle of f_0 and σ



Thank you!