NNLL threshold resummation for the total top-pair production cross section

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(Based on M.Beneke, P.Falgari, S. Klein, CS, arXiv:1109.1536 [hep-ph])

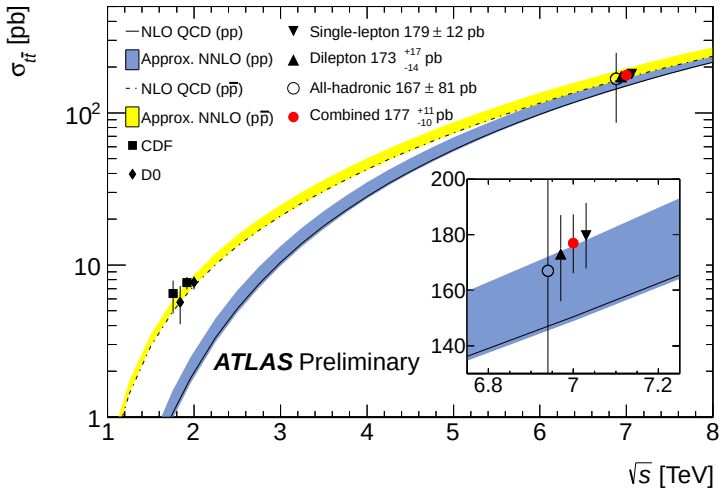
Total cross section measurements

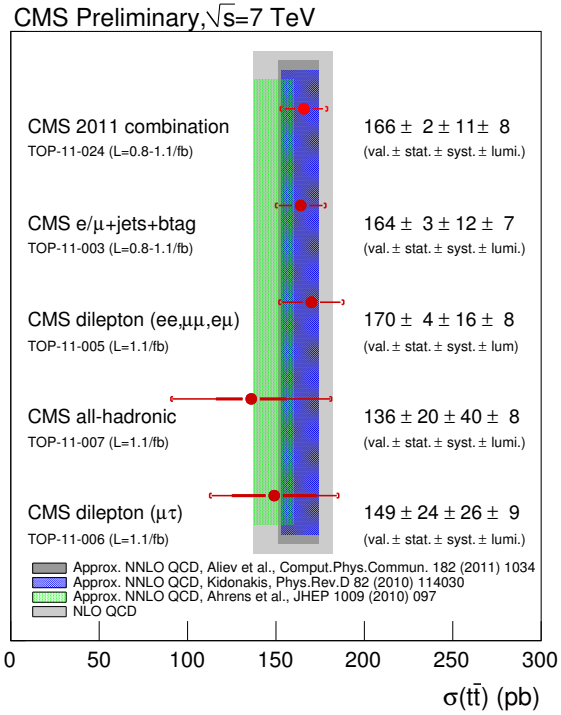
at Tevatron and LHC

$$\sigma_{t\bar{t}}^{\text{Tevatron}} = \begin{cases} 7.56^{+0.63}_{-0.56} & \text{(D0)} \\ 7.50^{+0.48}_{-0.48} & \text{(CDF)} \end{cases}$$

$$\sigma_{t\bar{t}}^{\text{LHC @7 TeV}} = \begin{cases} 166^{+13}_{-13} & \text{(CMS)} \\ 177^{+11}_{-10} & \text{(ATLAS)} \end{cases}$$

$$\Rightarrow \Delta\sigma_{t\bar{t}}/\sigma_{tt} \sim 7\%$$





NLO+NLL QCD:

(Nason, Dawson Ellis 88, Bonciani et al. 98)

$$\sigma_{t\bar{t}}^{\text{Tevatron}} = 6.93^{+0.28+0.51}_{-0.50-0.45}$$

$$\sigma_{t\bar{t}}^{\text{LHC @7 TeV}} = 167^{+14+14}_{-15-13}$$

$\Rightarrow$ Need higher-order predictions!

Introduction

NNLO: in progress, several ingredients known ($\Rightarrow$ Alex Mitov's talk)

(Czakon et al.; Bonciani et al. 08-11; Körner et al. 05-09, Anastasiou/Mert-Aybert 08;. . .)

Progress for soft gluon resummation:

1-loop hard functions for singlet/octet (Czakon/Mitov 08)

massive 2-loop soft anomalous dimension (Becher/Neubert; Kidonakis; Mitov/Sterman/Sung; Beneke/Falgari/CS; Czakon Mitov/Sterman; Ferroglia et al 09)

NNLO$_{\text{app.}}$ (Moch/Uwer(Langenfeld) 08/09, Beneke et al.; Ahrens et al. 09, Kidonakis 10)

NNLL resummation (Ahrens et al. 10/11; Beneke et al. 11, Cacciari et al. 11)

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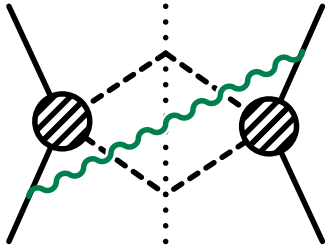
Further issues:

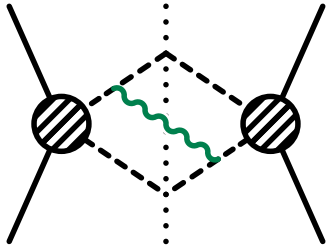
- EW corrections $\sim -2\%$ (Bernreuther et al.; Kühn et al. 05/06)
- non-resonant corrections (Denner et al. Bevilacqua et al. 10)
- PDF uncertainties

Enhanced QCD corrections in threshold limit $\beta = \sqrt{1 - 4m_t^2/\hat{s}} \to 0$

Soft corrections: (Resummation in Mellin space: Sterman 87; Catani, Trentadue 89, Kidonakis,Sterman 97, Bonciani et al. 98, ...)

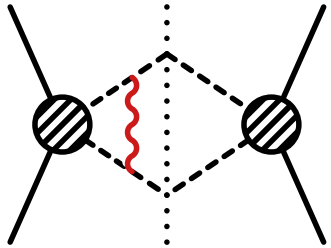


$\Rightarrow \alpha_s \ \log^2(8\beta^2)$



$\Rightarrow \alpha_s \ \log(8\beta^2)$

Coulomb gluon corrections (Fadin, Khoze 87; Peskin, Strassler 90, NRQCD,...)



$\Rightarrow \alpha_s \ \frac{1}{\beta}$

Resummed cross section

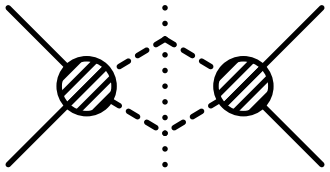
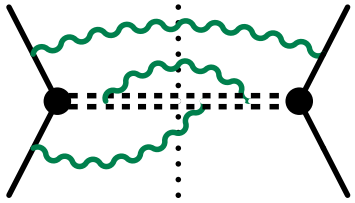
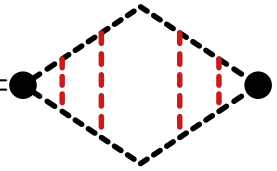
$$\hat{\sigma}_{pp'} \propto \sigma^{(0)} \exp\left[\ln\beta\, g_{\mathsf{LL}}(\alpha_s \ln\beta) + g_{\mathsf{NLL}}(\alpha_s \ln\beta) + \alpha_s g_{\mathsf{NNLL}}(\alpha_s \ln\beta) + \ldots\right]$$
$$\times \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \times \{1\,(\mathsf{LL,NLL}); \alpha_s, \beta\,(\mathsf{NNLL}); \ldots\} :$$

- Factorization into soft, hard and Coulomb functions

(Beneke, Falgari, CS 10)

$$\hat{\sigma}_{pp'\to HH'}\big|_{\hat{s}\to 4M^2} = \sum_{R,i} H_i W_i^R \otimes J^R$$

Hard, soft and Coulomb functions:

$H_i =$  , $W_i{}^R =$  , $J^R =$ 

Soft radiation "sees" only total colour charge R of heavy particles

(Singlet, octet,...)

- Factorization into soft, hard and Coulomb functions

(Beneke, Falgari, CS 10)

$$\hat{\sigma}_{pp'\to HH'}|_{\hat{s}\to 4M^2} = \sum_{R,i} H_i W_i^R \otimes J^R$$

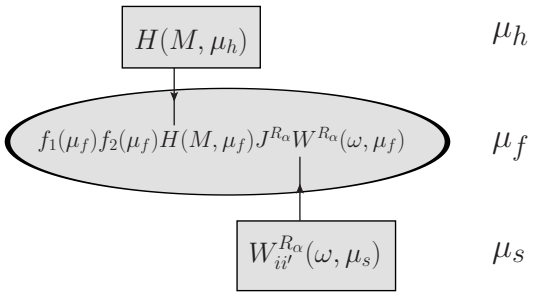
- can include Coulomb resummation to all orders

((N)LO Coulomb-Green function: Fadin/Khoze 87; Beneke/Signer/Smirnov 99,...)

- RGEs for hard and soft function

(Becher, Neubert; Ferroglia et al.;
Beneke/Falgari/CS; Czakon/Mitov/Sterman 09)

Momentum space solution
(Becher/Neubert 06)



- $\mu_h \sim 2m_t$, Choice of μ_s:
  - RGE approach: fixed μ_s (Becher, Neubert, Xu 07)
  - Running scale $\mu_s \sim m_t\beta^2$ (Beneke, Falgari, Klein, CS 11)

(frozen to $\mu_s \sim m_t\beta_{\text{cut}}^2$ for $\beta < \beta_{\text{cut}} = 0.35$ (Tevatron), 0.54 (LHC))

Total top-pair production cross-section

$\sigma_{t\bar{t}}$[pb]	Tevatron	LHC (7 TeV)	LHC (8 TeV)	LHC (14 TeV)
NLO	$6.68\,^{+0.36+0.51}_{-0.75-0.45}$	$158.1\,^{+19.5+13.9}_{-21.2-13.1}$	$226.2\,^{+27.8+19.1}_{-29.7-17.8}$	$884\,^{+107+65}_{-106-58}$
NLL	$7.31\,^{+0.40+0.57}_{-0.54-0.54}$	$172.8\,^{+20.3+15.9}_{-15.5-14.6}$	$246.5\,^{+28.8+21.7}_{-21.5-19.8}$	$954\,^{+111+74}_{-76-66}$
NNLO_{app}	$7.06\,^{+0.27+0.69}_{-0.34-0.53}$	$161.1\,^{+12.3+15.2}_{-11.9-14.5}$	$230.0\,^{+16.7+20.5}_{-15.7-19.8}$	$891\,^{+76+64}_{-69-63}$
NNLL	$7.22\,^{+0.31+0.71}_{-0.47-0.55}$	$162.6\,^{+7.4+15.4}_{-7.6-14.7}$	$231.9\,^{+10.5+20.8}_{-10.3-20.1}$	$896\,^{+40+65}_{-37-64}$

($m_t = 173.3$ GeV, (N)NLO MSTW08PDFs, 90% CL PDF+α_s uncertainty)

Different uncertainties for NNLL (added in quadrature above):

- Scale variation (μ_f, μ_h, μ_C):

$$\Delta_\mu \sigma_{\text{NNLL}}(\text{TeV}) = {}^{+0.21}_{-0.41}\,, \quad \Delta_\mu \sigma_{\text{NNLL}}(\text{LHC7}) = {}^{+4.2}_{-1.9}$$

- Uncertainty in resummation procedure:
  (vary β_{cut} by 20%, envelope of NNLO_{app}, N3LO_{app}, NNLL, ambiguity $\sqrt{\hat{s}} - 2m_t \leftrightarrow m_t\beta^2$)

$$\Delta_{\text{Res}} \sigma_{\text{NNLL}}(\text{TeV}) = {}^{+0.20}_{-0.21}\,, \quad \Delta_{\text{Res}} \sigma_{\text{NNLL}}(\text{LHC7}) = {}^{+3.9}_{-5.6}$$

- Estimate of missing constant at $\mathcal{O}(\alpha_s^2)$

$$\Delta_{\text{Const}} \sigma_{\text{NNLL}}(\text{TeV}) = \pm 0.10\,, \quad \Delta_{\text{Const}} \sigma_{\text{NNLL}}(\text{LHC7}) = \pm 4.7$$

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($m_t = 173.3$ GeV, (N)NLO MSTW08PDFs, 90% CL PDF+α_s uncertainty)

NNLL results include

- Coulomb effects beyond NNLO
- bound-state corrections below threshold

  (take $\Gamma_t \neq 0$ approximately into account)

$\Rightarrow$ small effect:

$$\Delta\sigma_{\text{BS}} = 0.014\text{pb (Tevatron)}, \quad 0.67\text{pb (LHC7)}, \quad 3.1\text{pb (LHC14)}$$

$$\Delta\sigma_{\text{C}} = -0.052\text{pb (Tevatron)}, \quad 0.13\text{pb (LHC7)}, \quad -0.3\text{pb (LHC14)}$$

Comparison to other NNLO$_{\text{app}}$/NNLL predictions

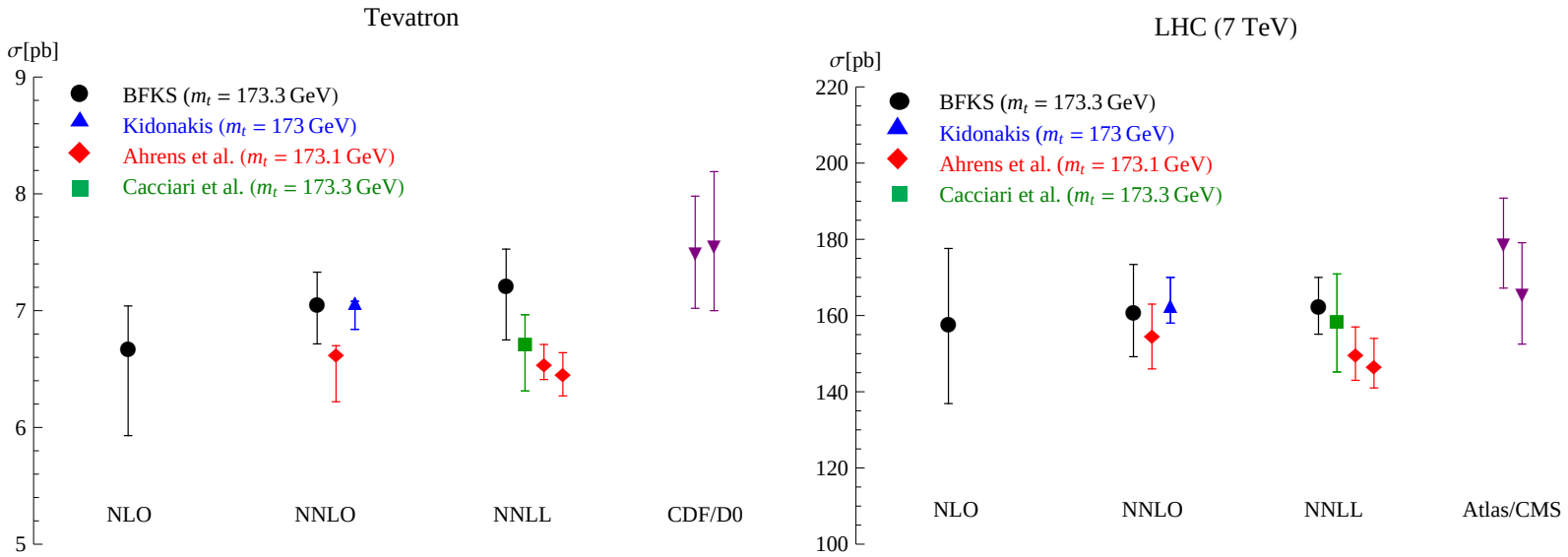
NNLL soft resummation (Cacciari/Czakon/Mangano/Mitov/Nason 11)

(β-expansion, Mellin space, no Coulomb resummation, different α_s^2 constant)

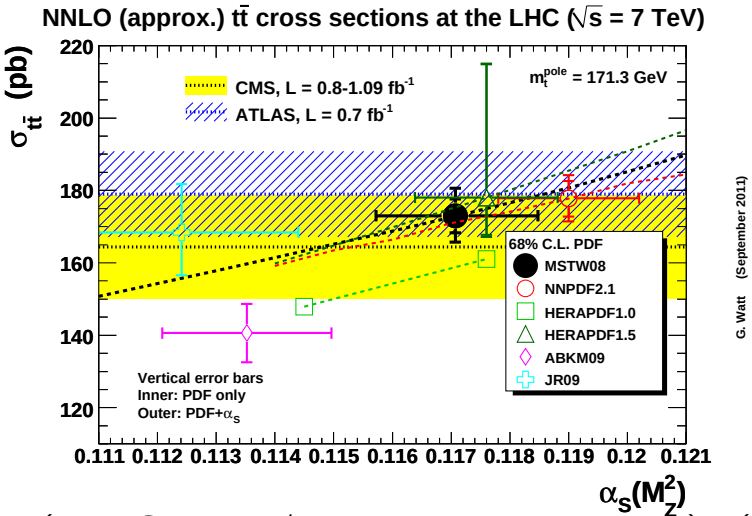
Resummed differential cross sections

- Pair-invariant mass : $\frac{d\hat{\sigma}(t\bar{t})}{dM_{t\bar{t}}}$, (Ahrens/Ferroglia/Neubert/Pecjak/Yang 10)
- One-particle inclusive: $\frac{d\hat{\sigma}(t+X)}{dp_T}$ (Kidonakis 11; Ahrens et al. 11)

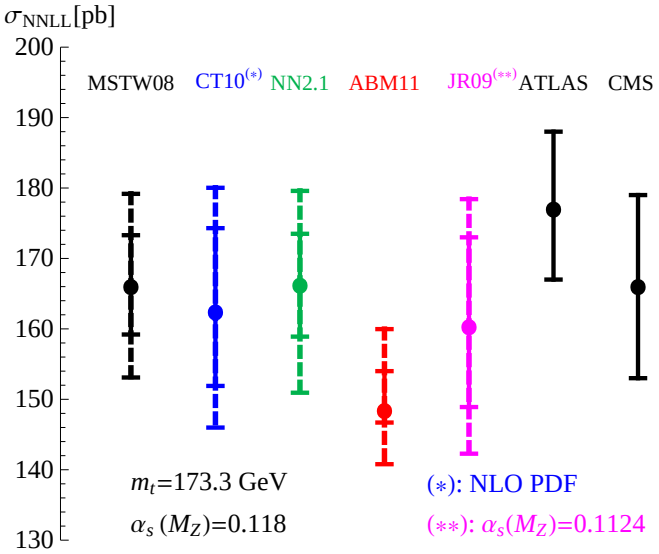
(include some higher-order terms in β)



Dependence on PDF and α_s (Watt 12 using HATHOR (Aliev et al. 10))



Fix $\alpha_s(M_Z) = 0.118$ (68% CL PDF/α_s error + theory error) (Numbers from J. Piclum)



Fit m_t-dependence of NNLL-cross-section:

$$\sigma^{\text{th}}_{t\bar{t}}(m_t^{\text{pole}}) = \left(\frac{172.5}{m_t^{\text{pole}}}\right)^4 \left(c_0 + c_1(m_t^{\text{pole}} - 172.5) + c_2(m_t^{\text{pole}} - 172.5)^2 + c_3(m_t^{\text{pole}} - 172.5)^3\right) \text{pb}\,,$$

$$c_0 = 166.5,\ c_1 = -1.15,\ c_2 = 5.1 \times 10^{-3},\ c_3 = 8.5 \times 10^{-5}$$

Use ATLAS-CONF-2011-121:

$$\sigma_{t\bar{t}} = 179.0 \pm 11.8\,\text{pb}$$

Dependence on m_t^{MC}:

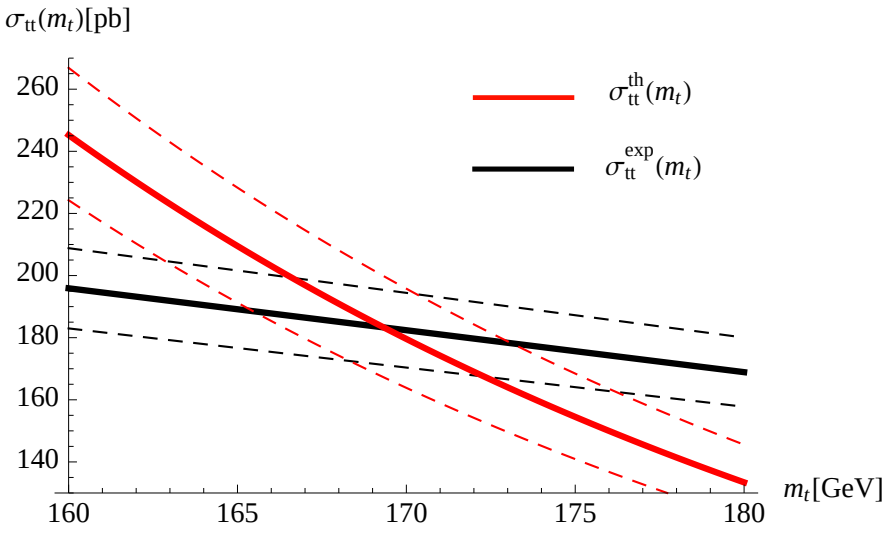
$$\sigma^{\text{exp}}_{t\bar{t}}(m_t^{\text{MC}}) = \left(411.9 - 1.35\, m_t^{\text{MC}}\right) \text{pb}$$

maximize joined likelihood

$$f(m_t) = \int f_{\text{th}}(\sigma|m_t) \cdot f_{\text{exp}}(\sigma|m_t) d\sigma\,,$$

with normalized Gaussians $f_{\text{th/exp}}$

$$\Rightarrow\ m_t^{\text{pole}} = (169.8\,^{+5.2}_{-5.0}\ \underbrace{^{+0.3}_{-0.3}}_{m_t^{\text{MC}} \neq m_t^{\text{pole}}})\ \text{GeV}$$



CMS result from $\sigma_{t\bar{t}} = 169.9 \pm 18.4$ pb:

Approx. NNLO × MSTW08NNLO	m_t^{pole} / GeV	$m_t^{\overline{\text{MS}}}$ / GeV
Langenfeld et al. [7]	$170.3^{+7.3}_{-6.7}$	$163.1^{+6.8}_{-6.1}$
Kidonakis [8]	$170.0^{+7.6}_{-7.1}$	–
Ahrens et al. [9]	$167.6^{+7.6}_{-7.1}$	$159.8^{+7.3}_{-6.8}$

Summary

Threshold corrections $\sim \log^n \beta$, $\frac{1}{\beta^n}$

- Factorization of soft and Coulomb corrections
- $\log \beta$ resummation from momentum space solution to RGEs
- combined Soft and Coulomb resummation possible

NNLL resummation for $t\bar{t}$

- small effects beyond NNLO (2 % Tevatron, 1 % LHC)
- theory uncertainty $+4.2/-6.5\%$ Tevatron, $\pm 4.5\%$ LHC including resummation ambiguities (PDF$+\alpha_s$ uncertainty larger)
- Mass determination from $\sigma_{t\bar{t}}$:

$$\Delta m_t \approx \pm 5 \text{ GeV appears viable}$$

Outlook

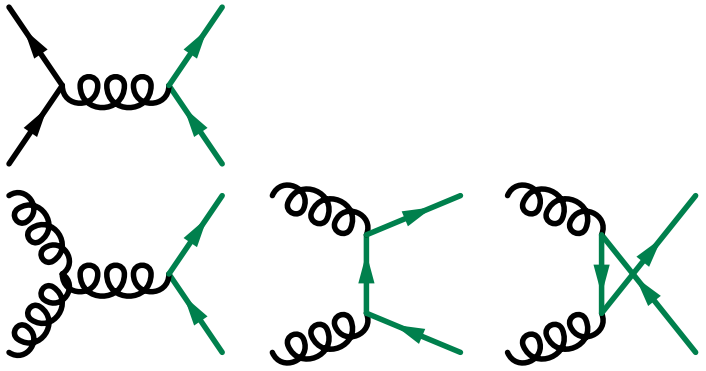
- public program (in preparation w. J. Piclum)
- resummation for squarks/gluinos (NLL: Falgari/CS/Wever 12)

Bonus slides

Top-pair production: two LO subprocesses:

$$q\bar{q} \to t\bar{t}: \begin{cases} 90\% & \text{Tevatron} \\ 20-10\% & \text{LHC7}-14 \end{cases}$$

$$go \to t\bar{t}: \begin{cases} 10\% & \text{Tevatron} \\ 80-90\% & \text{LHC7}-14 \end{cases}$$



Behaviour at production threshold $\hat{s} \sim 4m_t^2$: $\left(\beta = \sqrt{1-\frac{4m_t^2}{\hat{s}}}\right)$

- $q\bar{q}$ channel: colour octet, spin triplet

$$\hat{\sigma}_{q\bar{q}}^{(8)} = \frac{\pi\beta}{9m_t^2}\left[1+\frac{\alpha_s}{4\pi}\left(\frac{-2\pi^2}{2N_c}\frac{1}{\beta}+8C_F\log^2 8\beta^2-(32C_F+4N_C)\log 8\beta^2\right)+\ldots\right]$$

- gg channel: colour singlet/octet, spin singlet

$$\hat{\sigma}_{gg}^{(1)} = \frac{5\pi\beta}{192m_t^2}\left[1+\frac{\alpha_s}{4\pi}\left(2C_F\pi^2\frac{1}{\beta}+8N_C\log^2 8\beta^2-32N_C\log 8\beta^2\right)+\ldots\right]$$

$$\hat{\sigma}_{gg}^{(8)} = \frac{\pi\beta}{96m_t^2}\left[1+\frac{\alpha_s}{4\pi}\left(\frac{-2\pi^2}{2N_C}\frac{1}{\beta}+8N_C\log^2 8\beta^2-(32N_C+4N_C)\log 8\beta^2\right)+\ldots\right]$$

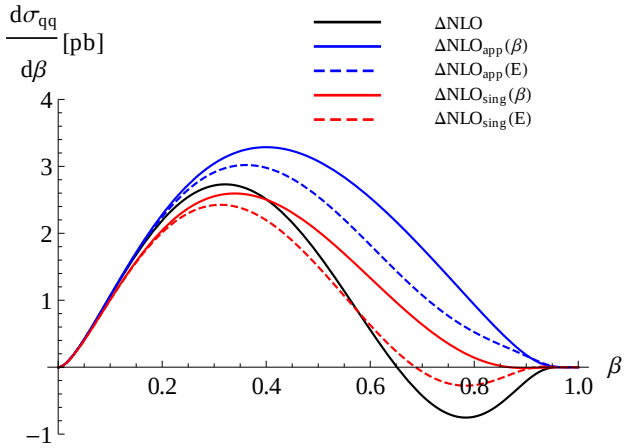
$\Rightarrow$ Universal behaviour depending on initial/final colour states

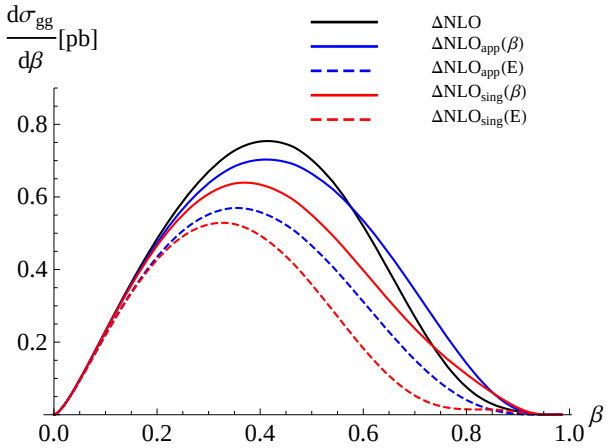
Accuracy of threshold approximation

NLO$_{\text{sing}}$: only $\log\beta$, $1/\beta$-terms

NLO$_{\text{app}}$: also constant terms (hard/soft function)

Ambiguity: $E = \sqrt{\hat{s}} - 2m_t \approx m_t\beta^2$





$\frac{d\sigma}{d\beta} = \frac{8\beta m_{\tilde{q}}^2}{s(1-\beta^2)^2} L(\beta,\mu_f)\hat{\sigma}$

(Tevatron, L: parton luminosity (MSTW08))

$\Rightarrow$ large ambiguities, but exact result covered: ($\mu_f = \mu_r = m_t = 173.1$ GeV)

$$\sigma_{\text{NLO,app}}(\text{Tev}) = 6.42 - 7.45\text{pb} \qquad \sigma_{\text{NLO}}(\text{Tev}) = 6.50\text{pb}$$

$$\sigma_{\text{NLO,app}}(\text{LHC7}) = 130 - 158\ \text{pb} \qquad \sigma_{\text{NLO}}(\text{LHC7}) = 150\ \text{pb}$$

Coulomb resummation

in Coulomb Green function: (Fadin, Khoze 87; Peskin, Strassler 90)

$$\hat{\sigma}_{t\bar{t}}(\hat{s}) \sim \sum_{R=1,8} \sigma_{t\bar{t}}^{0,R}(\hat{s}) \mathrm{Im}\, G_{\mathrm{C}}^{R}(0,0;E+i\Gamma)$$

Singlet channel, neglecting decay width:

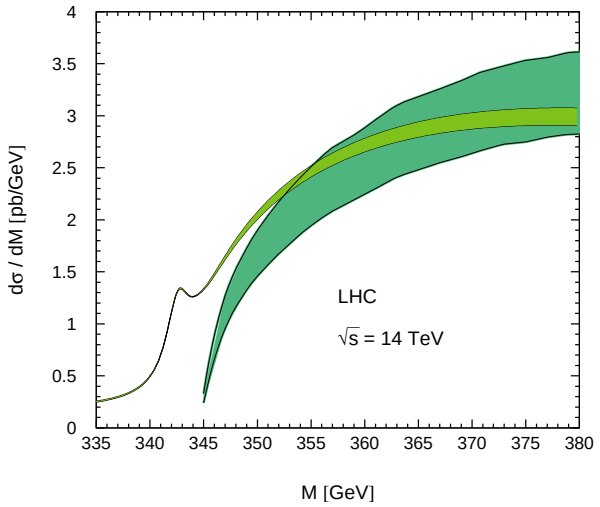
$$\mathrm{Im}\, G_{\mathrm{C}}^{1}(0,0;E) = \begin{cases} \frac{-m_t^2 \pi C_F \alpha_s}{4\pi} \left(e^{-\pi C_F \alpha_s \sqrt{\frac{m_t}{E}}} - 1 \right)^{-1} & E > 0 \\ \sum_{n=1}^{\infty} \delta(E - E_n) R_n & E < 0 \end{cases}$$

$$E = \sqrt{\hat{s} - 2m_t} \approx m_t \beta^2 .$$

Bound-state poles at

$$E_n = -\frac{\alpha_s^2 C_F^2 m_t}{4n^2}$$

smeared out by $\Gamma_t \neq 0$.



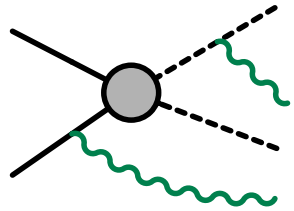
(Hagiwara et al. 08, Kiyo et al. 08)

All threshold enhanced $\mathcal{O}(\alpha_s^2)$ terms (Beneke, Czakon, Falgari, Mitov, CS 09

Implemented in HATHOR, Aliev et.al. 10)

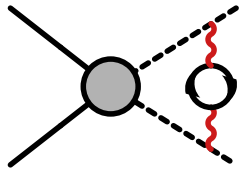
Pure soft corrections: (also Moch/Uwer+Langenfeld (08/09))

$$\Delta\sigma_{\mathsf{s}}^{(2)} \sim \alpha_s^2(c_{\mathsf{LL}}^{(2)} \ln^4\beta + c_{\mathsf{NLL}}^{(2)} \ln^3\beta + c_{\mathsf{NNLL,2}}^{(2)} \ln^2\beta + \underbrace{c_{\mathsf{NNLL,1}}^{(2)} \ln\beta}_{\text{2-loop } \gamma_{H,s}})$$



Potential corrections: 2nd Coulomb, NLO potentials

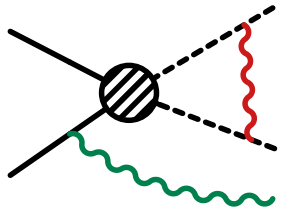
$$\Delta\sigma_{\mathsf{p}}^{(2)} \sim \alpha_s^2 \left(\frac{c_{C^2}}{\beta^2} + \frac{1}{\beta}(c_{\mathsf{C},0}^{(2)} + c_{\mathsf{C},1}^{(2)} \log\beta) + \underbrace{c_{\mathsf{n\text{-}C}}^{(2)} \ln\beta}_{\text{spin-dependent}}\right)$$



(using Beneke, Signer, Smirnov 99, Czarnecki/Melnikov 97/01)

mixed Coulomb/soft, hard corrections:

$$\Delta\sigma_{\mathsf{p}\otimes\ \mathsf{sh}}^{(2)} \sim\sim \frac{\alpha_s}{\beta}\alpha_s(c_{\mathsf{LL}}^{(1)} \ln\beta^2 + c_{\mathsf{NLL}}^{(1)} \ln\beta + c + \underbrace{H^{(1)}}_{\text{process dependent}})$$



Comparison to other NNLO$_{\text{app}}$/NNLL predictions

$\sigma_{t\bar{t}}$[pb]	Tevatron	LHC (7 TeV)	
NNLO$_{\text{app}}$	$7.06\,^{+0.27+0.69}_{-0.34-0.53}$	$161.1\,^{+12.3+15.2}_{-11.9-14.5}$	
NNLO$_{\text{app}}$+NNLL	$7.22\,^{+0.31+0.71}_{-0.47-0.55}$	$162.6\,^{+7.4+15.4}_{-7.6-14.7}$	(Beneke, Klein, Falgari, CS 11)
NLO+NNLL$_N$	$6.72\,^{+0.24+0.16}_{-0.41-0.12}$	$158.7\,^{+12.2+4.3}_{-13.5-4.4}$	(Cacciari et al. 11)

Ambiguities

- in momentum space results: fixed soft scale:

$$\sigma_{\text{NNLL}} = 7.08 \text{ pb (TeV)}; \quad 157.4 \text{ pb (LHC 7TeV)}$$

- in N-space results (Cacciari et al. 11)
  - set $\mathcal{O}(\alpha_s^2)$ constant in β expansion to zero:

$$\sigma^N_{\text{NNLL}} = 6.97 \text{ pb (TeV)}; \quad 167.9 \text{ pb (LHC 7TeV)}$$

  - don't include $1/N$ suppressed terms:

$$\sigma^N_{\text{NNLL}} = 6.84 \text{ pb (TeV)}; \quad 164.7 \text{ pb (LHC 7TeV)}$$

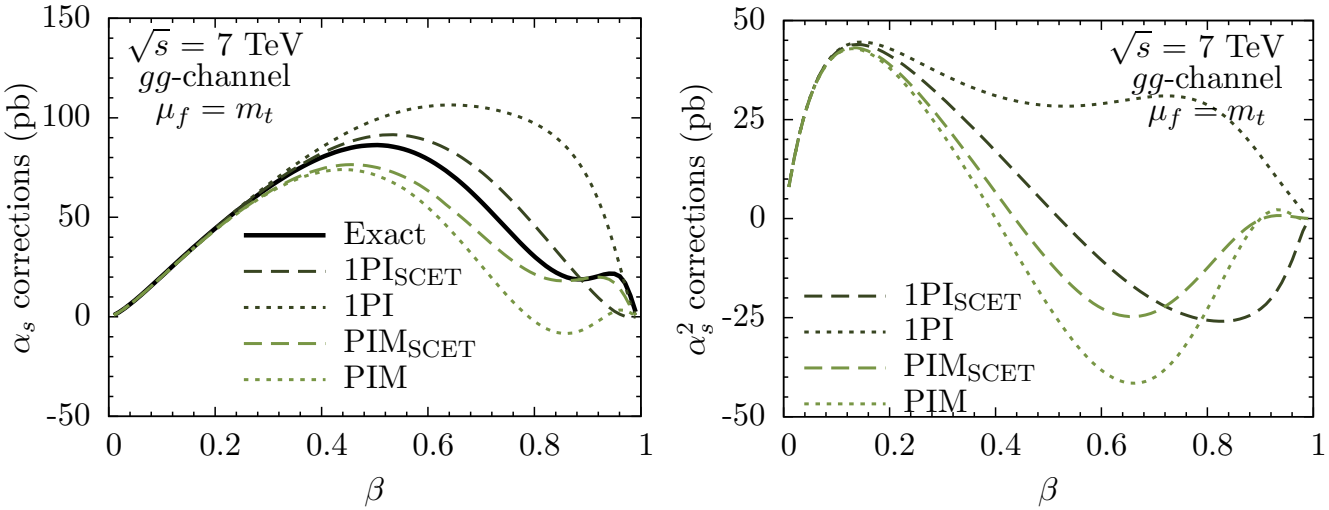
Comparison to other NNLO$_{\text{app}}$/NNLL predictions

Pair invariant mass cross sections (Kidonakis,Sterman 97, Ahrens et al. 10)

$$\frac{d\hat{\sigma}(t\bar{t})}{dM_{t\bar{t}}} \Rightarrow \left[\frac{\log^n(1-z)}{1-z}\right]_+ , \; z = \frac{M^2_{t\bar{t}}}{\hat{s}} , \qquad \text{PIM}_{\text{SCET}} : \log\left(\frac{1-z}{\sqrt{z}}\right)$$

One particle inclusive cross sections: (Laenen et al. 98, Ahrens et al. 11)

$$\frac{d\hat{\sigma}(t+X)}{ds_4} \Rightarrow \left[\frac{\log^n\left(s_4/m^2\right)}{s_4}\right]_+ ; \; s_4 = p_X^2 - m_t^2 , \qquad \text{1PI}_{\text{SCET}} : \log\left(s_4/\sqrt{m^2+s_4}\right)$$



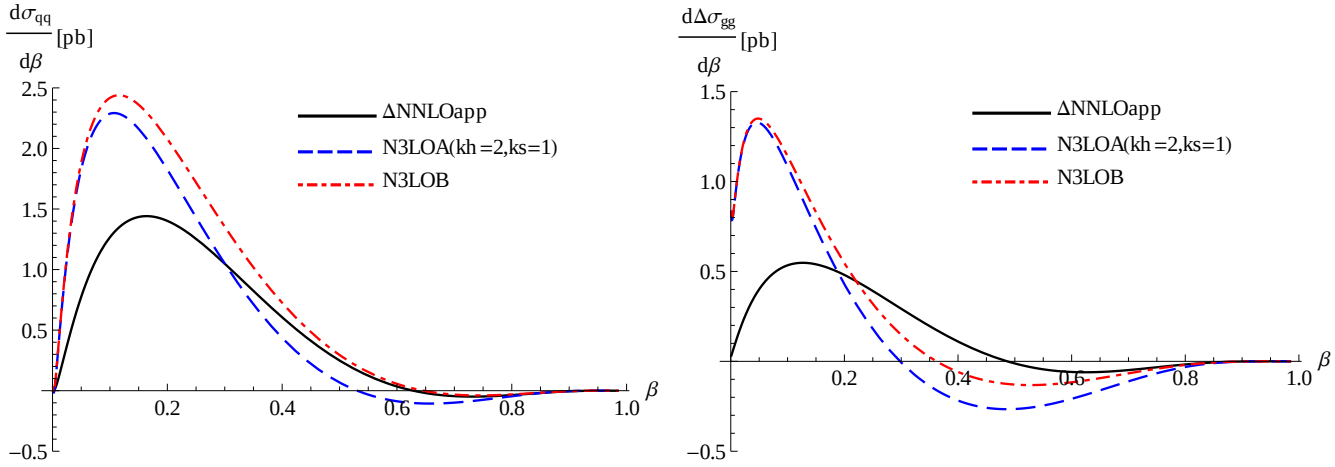
Approximate N^{3}LO results

Expand NNLL to $\mathcal{O}(\alpha_s^3)$, e.g.

$$\Delta\sigma^{(3)}_{qq,\text{NNLL}} = 12945.4\log^6\beta - 37369.1\log^5\beta + 27721.4\log^4\beta + 41839.4\log^3\beta$$
$$+ \frac{1}{\beta}\left(-6278.5\log\beta + 3862.5\log^2\beta + 2804.7\log^3\beta - 2994.5\log^4\beta\right)$$
$$+ \frac{153.9\log^2\beta + 122.9\log\beta - 145}{\beta^2} + \underbrace{\left\{\log\beta^{1,2}, 1/\beta, C^{(3)}\right\}}_{\text{not known exactly}} + \text{scale dep.}$$

N^{3}LO$_A$: keep all terms, including k-dependence and constants

N^{3}LO$_B$: only keep terms known exactly



Soft scale choice in momentum-space resummation

- RGE approach: fixed μ_s, vary by $0.5 \ldots 2$ (Becher, Neubert, Xu 07)
- Running scale with cutoff (Beneke, Falgari, Klein, CS 11)

$$\mu_s \sim \begin{cases} m_t \beta_{\text{cut}}^2, & \text{for} \beta < \beta_{\text{cut}} \\ m_t \beta^2, & \text{for} \beta > \beta_{\text{cut}} \end{cases}$$

$\mathcal{O}(\alpha_s^2)$ **constant** in threshold expansion

$$\Delta\sigma_{tt}^{(2)}(7\text{TeV}) = \left[1.70 \left(\frac{C_{qq}^{(2)}}{1000}\right) + 4.31 \left(\frac{C_{gg,\mathbf{8}}^{(2)}}{1000}\right) + 1.31 \left(\frac{C_{gg,\mathbf{1}}^{(2)}}{1000}\right)\right] \text{pb}$$

Estimate

$$C_2 \approx C_1^2\text{:} \qquad \Delta\sigma_{tt}^{(2)} \sim \pm 5 \text{ pb}$$

Kinematic ambiguity

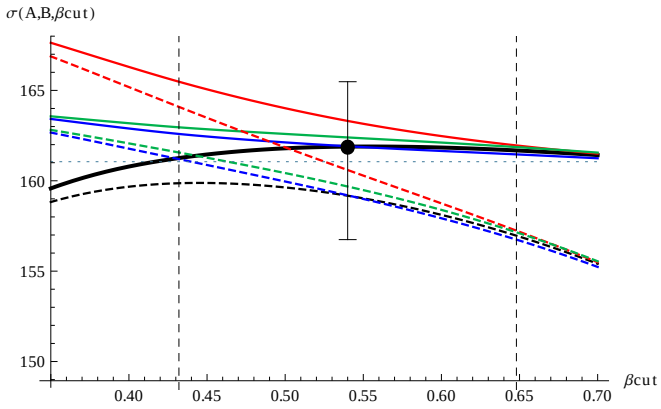
$$\log\beta \Leftrightarrow \log((\sqrt{\hat{s}} - 2m_t)/m_t): \qquad \Delta\sigma_{tt} \sim 1 \text{ pb}$$

Running scale: Introduce β_{cut}

- allow for different implementations

  $\beta < \beta_{\text{cut}}$: NNLL ($\mu_s = k_s m_t \beta^2_{\text{cut}}$) with/without constant at $\mathcal{O}(\alpha_s^2)$

  $\beta > \beta_{\text{cut}}$: NNLL ($\mu_s = k_s m_t \beta^2$); $\text{NNLO}_{\text{approx}}$; NNNL_3(A/B)



- Choose β_{cut} so that not too sensitive to
  - ambiguities for $\beta \to 1$
  - breakdown of perturbation theory for $\beta \to 0$

(E.g. LHC7: $\mu_s = 2 m_t \beta^2$, $\beta_{\text{cut}} = 0.54 \Rightarrow \mu_s > 100$ GeV)