Status of the proton radius puzzle

Richard J. Hill
Enrico Fermi Institute and Department of Physics
The University of Chicago, Chicago, Illinois, 60637, USA

This talk reviews the status of the proton radius puzzle, i.e., the discrepancy between inferred values for the proton charge radius. The focus is on a discussion of the uncertainties in extrapolations of electron scattering data, and of the uncertainties due to proton structure corrections in the muonic hydrogen bound state.

1 Introduction

In 2010 the CREMA collaboration reported a first measurement of the Lamb shift in muonic hydrogen [1]. Interpreted as a measurement of the proton charge radius, the result differs significantly from extractions based on electronic hydrogen, and extractions from electron-proton scattering, as summarized in Fig. 1. This $\sim 5\sigma$ anomaly is perplexing. It is an obstacle to precise determination of the Rydberg constant R_∞, [a] and it brings into question the reliability of electron scattering data.[b] It has also led to speculations on new forces acting in the muon-proton system [21], inadequate treatment of proton charge density correlations [22], and modifications of offshell photon vertices [23].

This talk begins by discussing effective field theory formalism for proton structure effects in atomic bound states. Dispersive analysis to constrain electron scattering determinations of coefficients in the effective theory is then described, and the status of the proton radius puzzle is summarized.

2 Proton structure in NRQED

Non-relativistic QED (NRQED) [24] is a field theory describing the interactions of photons and nonrelativistic matter. The NRQED lagrangian is constructed to yield predictions valid to any fixed order in small parameters α and $|\boldsymbol{q}|/M$, where $|\boldsymbol{q}|$ denotes a typical bound state momentum, and M is a mass scale for the nonrelativistic particle. NRQED provides a rigorous framework to study the effects of proton structure, avoiding problems of double counting in bound state energy computations [25]; eliminating difficulties of interpretation for the polarizability of a strongly interacting particle [26]; and providing trivial derivations of universal properties, such as the low

[a] "Data from muonic hydrogen are so inconsistent with the other data that they have not been included in the determination of r_p and thus do not have an influence on R_∞" [19].

[b] Until the difference between the e p and p values is understood, it does not make much sense to average all the values together. For the present, we stick with the less precise (and provisionally suspect) CODATA 2006 value. It is up to workers in this field to solve this puzzle [20].

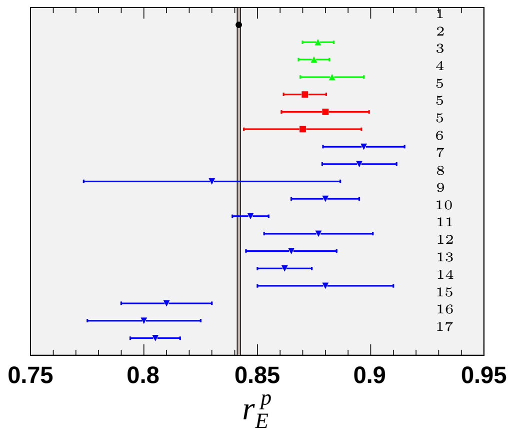

Figure 1: Values of the proton charge radius (in fm) extracted from muonic hydrogen (circle and vertical band); electronic hydrogen (green triangles); electron scattering employing the z expansion (red squares); and previous electron scattering extractions (blue downward triangles).

	Ref. [1]	Ref. [28]	
vertex correction	-0.0096 meV	-0.0108 meV	mismatch in r_E^p definition
two photon correction	0.051 meV	$\sim 0.05 \pm 0.05$ meV	model dependence
recoil finite size	0.013 meV	0	double counting
total	210.0011(45) - 5.2262 r^2 meV	209.987(50) - 5.2262 r^2 meV	
extracted radius	0.8421(6) fm	0.841(6) fm	

Table 1: Comparison between this and previous works for proton structure corrections to the $2P-2S$ Lamb shift in muonic hydrogen, in meV.

energy theorems of Compton scattering [27]. Neglecting the pure photon sector, the NRQED lagrangian has the expansion,

$$\begin{aligned}\mathcal{L}_e = \psi_e^\dagger \Big\{ & iD_t + \frac{\boldsymbol{D}^2}{2m_e} + \frac{\boldsymbol{D}^4}{8m_e^3} + c_F e \frac{\boldsymbol{\sigma}\cdot\boldsymbol{B}}{2m_e} + c_D e \frac{[\boldsymbol{\partial}\cdot\boldsymbol{E}]}{8m_e^2} + i c_S e \frac{\boldsymbol{\sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E}-\boldsymbol{E}\times\boldsymbol{D})}{8m_e^2} \\ & + c_{W1} e \frac{\{\boldsymbol{D}^2, \boldsymbol{\sigma}\cdot\boldsymbol{B}\}}{8m_e^3} - c_{W2} e \frac{D^i \boldsymbol{\sigma}\cdot\boldsymbol{B} D^i}{4m_e^3} + c_{p'p} e \frac{\boldsymbol{\sigma}\cdot\boldsymbol{D}\boldsymbol{B}\cdot\boldsymbol{D} + \boldsymbol{D}\cdot\boldsymbol{B}\boldsymbol{\sigma}\cdot\boldsymbol{D}}{8m_e^3} + i c_M e \frac{\{\boldsymbol{D}^i, [\boldsymbol{\partial}\times\boldsymbol{B}]^i\}}{8m_e^3} \\ & + c_{A1} e^2 \frac{\boldsymbol{B}^2 - \boldsymbol{E}^2}{8m_e^3} - c_{A2} e^2 \frac{\boldsymbol{E}^2}{16m_e^3} + \dots \Big\} \psi_e + d_1 \frac{\psi_p^\dagger \boldsymbol{\sigma}\psi_p \cdot \psi_e^\dagger \boldsymbol{\sigma}\psi_e}{m_e m_p} + d_2 \frac{\psi_p^\dagger \psi_p \psi_e^\dagger \psi_e}{m_e m_p} + \dots \,. \end{aligned} \quad (1)$$

Here ψ_e is a two-component spinor representing the nonrelativistic electron field, $\boldsymbol{\sigma}$ is the Pauli spin matrix, D_t and $\boldsymbol{D}$ are covariant derivatives and $\boldsymbol{E}$, $\boldsymbol{B}$ are the electric and magnetic fields. The proton charge radius is defined by the matching condition for c_D, while structure-dependent contributions to two-photon exchange enter the contact interactions d_1,d_2.

Once a regularization scheme for the field theory is specified, and coefficients $c_D, d_2, \dots$ are determined, spectroscopic intervals can be computed. The bound state computation is not essentially different for a point particle nucleus (e.g. a muon-electron bound state) versus a composite nucleus (e.g. a proton-electron bound state). Some discrepancies between the tabulation of Ref. [1] are noted in Table 1. For details, see Ref. [28].

3 Electron scattering

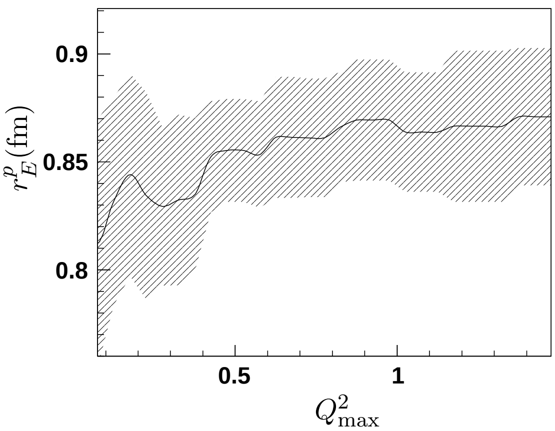

Figure 2: Variation of the fitted proton charge radius as a function of maximum Q^2. Fits of the proton data were performed with $k_{\rm max} = 10$, $\phi = 1$, $t_0 = 0$, $|a_k| \le 10$. Data from Ref. [33].

Electron scattering is a promising method to determine structure-dependent constants in the NRQED lagrangian. Let us consider the electromagnetic form factors satisfying $F_1(0) = 1$, $F_2(0) = a_p$, and

$$F_1'(0) = \frac{1}{6}(r_E^p)^2 - \frac{a_p}{4m_p^2} + \frac{Z^2\alpha}{3\pi m_p^2}\log\frac{m_p}{\lambda}\,, \quad F_2'(0) = \frac{1}{6}\left[(1+a_p)(r_M^p)^2 - (r_E^p)^2\right] + \frac{a_p}{4m_p^2}\,, \qquad (2)$$

where λ is a photon mass introduced for convenience. When extracting the form factor slope we must account for the unknown form factor shape while retaining predictive power. This problem can be addressed using constraints from analyticity [32], both to constrain the functional form of $F_i(q^2)$, and to allow systematic inclusion of electron-neutron scattering data and $\pi\pi \to N\bar{N}$ data [5]. Figure 2 illustrates the radius extraction from a representative dataset [33], as a function of the maximum momentum transfer included in the fit. The corresponding radius, for $Q^2_{\rm max} = 0.5\,{\rm GeV}^2$, is displayed in Fig. 1, together with more precise determinations including neutron and $\pi\pi$ data. For details see Ref. [5].

4 Outlook

			Ref. [1]	Ref. [28]
H	CODATA06	0.876(8)	4.2σ	3.5σ
ep	Sick 2005	0.895(18)	2.9σ	2.8σ
	JLab 2011	0.875(10)	3.3σ	2.9σ
	Mainz 2011	0.879(8)	4.6σ	3.8σ
H, ep	CODATA10	0.8775(51)	6.9σ	4.6σ
ep	this work	0.870(26)	1.1σ	1.1σ
ep, en	this work	0.880(20)	1.9σ	1.9σ
ep, en, $\pi\pi \to N\bar{N}$	this work	0.871(10)	2.9σ	2.6σ

Table 2: Discrepancy between the proton charge radius from muonic hydrogen and from other determinations.

The proton radius remains a puzzle. Table 2 displays the discrepancy between the proton charge radius from muonic hydrogen, and other determinations. The final two columns of the table correspond to the reference values in the final row of Table 1. Future work should provide a more robust estimation of uncertainties due to radiative corrections in the electron scattering determination. New measurements in electronic hydrogen are being undertaken to assess the possibility of systematic effects in the Rydberg and proton radius determinations. Proposed muon-proton scattering measurements could provide an independent determination of the proton radius, and directly measure the poorly constrained contact interaction parameterized by d_2.

Acknowledgments

The work reported here summarizes Refs. [5,28], done in collaboration with Gil Paz. Research supported by NSF Grant 0855039.

References

1. R. Pohl *et al.*, Nature **466**, 213 (2010).
2. P. J. Mohr, B. N. Taylor and D. B. Newell, Rev. Mod. Phys. **80**, 633 (2008).
3. P. J. Mohr and B. N. Taylor, Rev. Mod. Phys. **77**, 1 (2005).
4. K. Melnikov and T. van Ritbergen, Phys. Rev. Lett. **84**, 1673 (2000).
5. R. J. Hill and G. Paz, Phys. Rev. D **82**, 113005 (2010).
6. P. G. Blunden and I. Sick, Phys. Rev. C **72**, 057601 (2005).
7. I. Sick, Phys. Lett. B **576**, 62 (2003).
8. I. M. Gough Eschrich *et al.* [SELEX Collaboration], Phys. Lett. B **522**, 233 (2001).
9. R. Rosenfelder, Phys. Lett. B **479**, 381 (2000).
10. P. Mergell, U. G. Meissner and D. Drechsel, Nucl. Phys. A **596**, 367 (1996).
11. C. W. Wong, Int. J. Mod. Phys. E **3**, 821 (1994).
12. M. McCord *et al.*, Nucl. Instrum. Meth. B **56/57**, 496 (1991).
13. G. G. Simon, C. Schmitt, F. Borkowski and V. H. Walther, Nucl. Phys. A **333**, 381 (1980).
14. F. Borkowski, P. Peuser, G. G. Simon, V. H. Walther and R. D. Wendling, Nucl. Phys. A **222**, 269 (1974).
15. Yu. K. Akimov *et al.*, Sov. Phys. **JETP35**, 651 (1972).
16. D. Frerejacque, D. Benaksas and D. J. Drickey, Phys. Rev. **141**, 1308 (1966).
17. L. N. Hand, Rev. Mod. Phys. **35**, 335 (1963).
18. V. Lyubushkin *et al.* [NOMAD Collaboration], Eur. Phys. J. C **63**, 355 (2009).
19. P. J. Mohr, B. N. Taylor and D. B. Newell, arXiv:1203.5425 [physics.atom-ph]].
20. K. Nakamura *et al.* [Particle Data Group Collaboration], J. Phys. G G **37**, 075021 (2010).
21. V. Barger, C. -W. Chiang, W. -Y. Keung and D. Marfatia, Phys. Rev. Lett. **106**, 153001 (2011). D. Tucker-Smith and I. Yavin, Phys. Rev. D **83**, 101702 (2011). B. Batell, D. Mc-Keen and M. Pospelov, Phys. Rev. Lett. **107**, 011803 (2011).
22. A. De Rujula, Phys. Lett. B **693**, 555 (2010). A. De Rujula, Phys. Lett. B **697**, 26 (2011). I. C. Cloet and G. A. Miller, Phys. Rev. C **83**, 012201 (2011).
23. G. A. Miller, A. W. Thomas, J. D. Carroll and J. Rafelski, Phys.Rev. A**84**, 020101 (2011).
24. W. E. Caswell and G. P. Lepage, Phys. Lett. B **167**, 437 (1986).
25. U. D. Jentschura, Eur. Phys. J. D **61**, 7 (2011).
26. A. I. Lvov, Int. J. Mod. Phys. A **8**, 5267 (1993).
27. S. Ragusa, Phys. Rev. D **47**, 3757 (1993).
28. R. J. Hill and G. Paz, Phys. Rev. Lett. **107**, 160402 (2011).
29. K. Pachucki, Phys. Rev. A **60**, 3593 (1999).
30. E. Borie, Phys. Rev. A **71**, 032508 (2005).
31. C. E. Carlson and M. Vanderhaeghen, Phys. Rev. A **84**, 020102 (2011).
32. For a review and references see: R. J. Hill, [arXiv:hep-ph/0606023].
33. J. Arrington, W. Melnitchouk and J. A. Tjon, Phys. Rev. C **76**, 035205 (2007).