NNLL threshold resummation for the total top-pair production cross section

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We present predictions for the total top-quark pair production cross section at the Tevatron and the LHC with 7, 8 and 14 TeV centre-of-mass energy, including the resummation of threshold logarithms and Coulomb corrections through next-to-next-to-leading logarithmic order, and $t\bar{t}$ bound-state contributions. The remaining theoretical and PDF uncertainties and prospects for the measurement of the top mass from the total cross section are discussed.

1 Introduction

After the discovery of the top quark at the Tevatron and the initial determination of its properties like mass, decay width, and the coupling to other particles, the LHC is currently opening the door to precision studies with hundreds of thousands of top-antitop pairs produced per year. One of the key observables is the top-quark pair production cross-section that has now been measured with an accuracy of seven percent both at Tevatron and LHC. At the LHC, the combinations of measurements in different channels yield the results [1]

$$\sigma_{t\bar{t}} = \begin{cases} 177 \;\;\; \pm 3 \;\; (\text{stat.}) \pm 8 \;\;\;\;\; (\text{syst.}) \pm 7 \;\;\; (\text{lumi.}) \;\text{pb}, & \text{ATLAS} \\ 165.8 \pm 2.2 \; (\text{stat.}) \pm 10.6 \; (\text{syst.}) \pm 7.8 \; (\text{lumi.}) \;\text{pb}, & \text{CMS} \end{cases} \tag{1}$$

Since the uncertainty of theoretical predictions based on next-to-leading order (NLO) calculations [2] in QCD and next-to-leading-logarithmic (NLL) higher-order effects [3] is larger than 10%, higher-order corrections have to be included in order to match the experimental precision. In the absence of the complete NNLO corrections that are currently being computed,[a] the theoretical precision can be improved by including higher-order QCD corrections that are enhanced in the partonic threshold limit, $\beta = \sqrt{1 - 4m_t^2/\hat{s}} \to 0$, where $\hat{s}$ is the partonic centre-of-mass energy. These contributions take the form of logarithmic corrections proportional to $(\alpha_s \log^{2,1} \beta)^n$ due to the emission of soft gluons, and of Coulomb corrections $(\alpha_s/\beta)^n$ due to the virtual exchange of gluons between the slowly moving top quarks. Both corrections can be resummed to all

[a] For a status report see. [4] The result for the quark-antiquark initial state was obtained very recently. [5]

$\sigma_{t\bar{t}}$[pb]	Tevatron	LHC ($\sqrt{s}$ =7 TeV)	LHC ($\sqrt{s}$ =8 TeV)	LHC ($\sqrt{s}$ =14 TeV)
NLO	$6.68^{+0.36+0.51}_{-0.75-0.45}$	$158.1^{+18.5+13.9}_{-21.2-13.1}$	$226.2^{+27.8+19.1}_{-29.7-17.8}$	$884^{+107+65}_{-106-58}$
NNLO_{app}	$7.06^{+0.27+0.69}_{-0.34-0.53}$	$161.1^{+12.3+15.2}_{-11.9-14.5}$	$230.0^{+16.7+20.5}_{-15.7-19.8}$	891^{+76+64}_{-69-63}
NNLL	$7.22^{+0.31+0.71}_{-0.47-0.55}$	$162.6^{+7.4+15.4}_{-7.5-14.7}$	$231.9^{+10.5+20.8}_{-10.3-20.1}$	896^{+40+65}_{-37-64}

Table 1: $t\bar{t}$ cross section at Tevatron and LHC in various approximations, for $m_t = 173.3\,\text{GeV}$ using the MSTW08 PDFs. The first error denotes the total theoretical uncertainty, the second the 90% c.l. PDF+α_s uncertainty.

orders in perturbation theory, leading to a representation of the partonic cross sections for the subprocesses $pp' \to t\bar{t}X$ (with $p, p' \in \{g, q, \bar{q}\}$) of the form

$$\hat{\sigma}_{pp'} = \hat{\sigma}^{(0)}_{pp'} \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{(NNLL)}} + \ldots\Big]$$
$$\times \left\{1\,(\text{LL,NLL}); \alpha_s\,(\text{NNLL}); \alpha_s^2, \beta^2\,(\text{N}^3\text{LL}); \ldots\right\}\,. \qquad (2)$$

Several recent developments have made it possible to perform resummation at NNLL accuracy: the function g_2 has been computed [6] using the infrared structure of massive QCD amplitudes, [7] while the $\mathcal{O}(\alpha)$ coefficient functions [8] and the NNLO Coulomb effects [9] became available as well. The combined resummation of soft and Coulomb corrections has been established in. [10]

2 Results from NNLL resummation

In [11] we have performed the combined NNLL resummation of soft and Coulomb effects using the momentum-space approach to soft-gluon resummation [12] and results for the higher-order Coulomb corrections, [13] including would-be bound-state contributions to the cross section. We extend these results in table 1 by providing predictions for the LHC at a centre-of-mass energy of 8 TeV in addition to the results for Tevatron and the LHC at 7 and 14 TeV. Results for different values of m_t at 8 TeV (updated to include the NNLO $q\bar{q}$ partonic cross section [5]), as well as for hypothetical heavy quarks will be presented elsewhere. [14] For comparison, the table also includes the NLO cross section [2] and the approximate NNLO results [9] obtained by expanding the resummed corrections to $\mathcal{O}(\alpha_s^2)$. The theoretical uncertainty of the approximate NNLO results includes an estimate of the unknown constant NNLO contribution to the cross section; the NNLL uncertainties include in addition an estimate of higher-order ambiguities based on comparing different NNLL implementations and expansions to N^{3}LO accuracy as discussed in detail in. [11] Compared to the NLO results, the NNLL corrections increase the cross section by 8% at the Tevatron and $1-3\%$ at the LHC. The main effect of the NNLL corrections is included in the NNLO_{app} result, with further higher-order corrections of about 2% at the Tevatron, and $\lesssim 1\%$ at the LHC. The NNLO_{app} and NNLL results include two-loop Coulomb and soft/Coulomb interference effects of the order of $1-2\%$, while Coulomb corrections beyond NNLO and bound-state contributions of the order of 0.5% [11] are included in addition at NNLL.

In the left panel of figure 1 we compare our results (denoted by black circles) for the LHC at $\sqrt{s} = 7$ TeV to predictions by other groups and experimental measurements. The NNLL resummation of soft-gluon corrections using the traditional Mellin-space approach [15] (denoted by a green square) differs from our results in the treatment of constant NNLO and power-suppressed terms, in addition to the different resummation formalism. Further results have been obtained by integrating NNLL or approximate NNLO predictions for invariant-mass or p_T-distributions. [16,17] These calculations (denoted by a blue triangle/red diamonds) include some power suppressed contributions in β, but not the NNLO potential terms. [9] All the approximations [15,16,17] differ

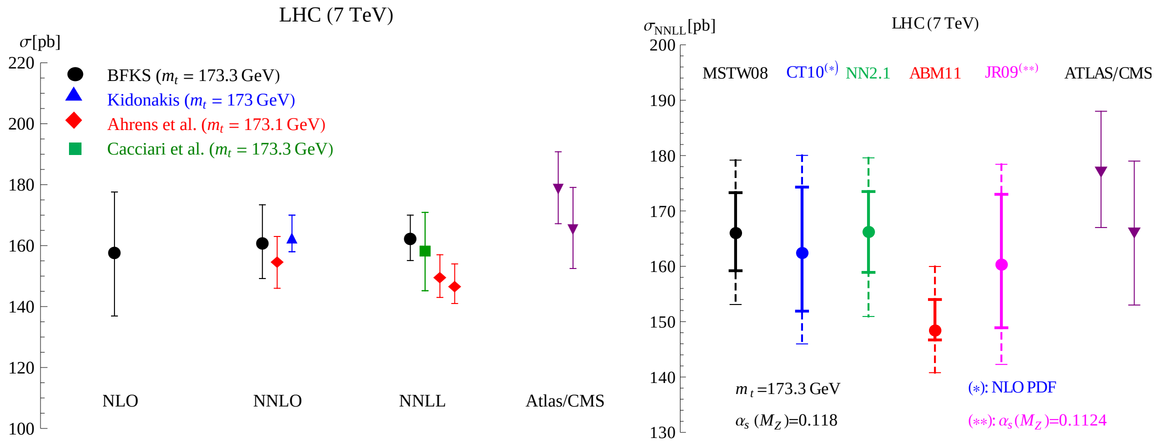

Figure 1: Left: Comparison of different NNLO and NNLL predictions, see the text for explanation and references. The error bands include theoretical uncertainties, but no PDF+α_s errors. Right: NNLL predictions for different PDF sets with fixed $\alpha_s(M_Z) = 0.118$. The inner (solid) error bar denotes the 68% confidence level PDF+α_s error, the outer (dashed) error bar includes in addition the uncertainty of the NNLL cross-section calculation. Both figures include also the most recent experimental measurements that assume $m_t = 172.5$ GeV.

from ours by neglecting the higher-order Coulomb effects. While the predictions agree within the quoted uncertainties at the LHC,[b] the different central values indicate the ambiguities inherent in threshold approximations and illustrate the possible impact of a full NNLO calculation.

In addition to the top-quark mass and the strong coupling, the top-pair cross section depends also on the parton distribution functions (PDFs). Due to the dominant gluon fusion channel, uncertainties in the determination of the gluon PDF have a large impact on the cross section at the LHC and constitute the main theoretical uncertainty. This is illustrated in the right panel of figure 1 where the results of the MSTW08NNLO PDF [18] used as our default are compared to the NNLO PDF sets by NNPDF2.1, [19] ABM11 [20] and JR09VR. [21] As illustration, results from the NLO PDF CT10 [22] are also included. In this comparison, a common central value $\alpha_s(M_Z) = 0.118$ of the strong coupling [c] is employed instead of the best-fit values used as default by the PDF sets (note that the MSTW08NNLO default $\alpha_s(M_Z) = 0.1171$ is used in the left panel of Figure 1). For the common α_s-value, the predictions of most PDFs agree within the 68% confidence level of the PDF+α_s uncertainty (denoted by the inner, solid error bar), but it is also seen that the spread is larger than estimated by the uncertainty of a single PDF set.

As an application of a precise theoretical prediction of the total top-pair cross section, the top-quark mass can be extracted in a theoretically well defined mass definition from the measured cross section, assuming the latter is free of new-physics contributions. [23] Using our NNLL predictions discussed above, we have estimated that the top-quark pole mass could be extracted with an accuracy of ±5 GeV from the currently available ATLAS data on the total cross section, and our result [11] $m_t = 169.8^{+4.9}_{-4.7}$ GeV is compatible with the direct mass determination $m_t = 173.2 \pm 0.8$ GeV at the Tevatron.[24] A CMS analysis[25] of a cross section measurement with a smaller central value but larger uncertainty, $\sigma_{t\bar{t}} = 169.90 \pm 3.9$ (stat.) ± 16.3 (syst.)± 7.6 (lumi.) pb, obtained the comparable result $m_t = 170.3^{+7.3}_{-6.7}$ GeV using the calculation of.[23]

3 Summary and outlook

We have reviewed the results of the NNLL resummation of soft and Coulomb-gluon corrections performed in [11] and extended them by including predictions for the LHC at $\sqrt{s} = 8$ TeV and

[b] Somewhat larger discrepancies are found at the Tevatron, see for instance. [11]
[c] With the exception of the JR09 PDF where $\alpha_s(M_Z) = 0.1124$ is used.

for different PDF sets. Our results are in good agreement with experimental measurements. We plan to make our calculation available in form of a public program in the near future.[14] We have discussed the impact of the uncertainty of current PDF sets on the cross-section predictions. The prospects to constrain the gluon PDF by the top-pair cross-section measurement will be investigated in.[14] It has been estimated that the top-quark pole mass could be measured with a precision of ±5 GeV from current LHC cross-section measurements.

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