GEOMETRICAL SCALING IN HIGH ENERGY HADRONIC COLLISIONS

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After introducing the concept of geometrical scaling (GS) on the example of deep inelastic ep scattering, we show that GS is also present in the $p_{\rm T}$ spectra measured by the LHC. We discuss simple phenomenological signatures of GS and its applications.

It is known that gluonic parton density rapidly increases at low Bjorken x [1]. Such growth has to be tamed at some point. The scale at which this happens is called saturation scale $Q_{\rm s}(x)$ and it depends on the Bjorken x. The explicit form of the saturation scale follows from the fact that $Q_{\rm s}^2(x)$ is related to the gluon distribution in the proton at low x [2]:

$$Q_{\rm s}^2(x) = Q_0^2\,(x/x_0)^{-\lambda} \tag{1}$$

where $Q_0 \sim 1$ GeV and $x_0 \sim 10^{-3}$ are free parameters whose precise values can be extracted from fits to the HERA data. Power λ is known to be of the order $\lambda \sim 0.2 \div 0.3$ and Bjorken x is defined as

$$x = Q^2/(Q^2 + W^2 - M_p^2) \tag{2}$$

where M_p stands for the proton mass.

Geometrical scaling [3] consists in the fact that for sufficiently low x the reduced $\gamma^* p$ cross section $\sigma_{\gamma^* p}(W, Q^2) \sim F_2(x, Q^2)/Q^2$ depends in fact only on the scaling variable τ:

$$\sigma_{\gamma^* p} = \text{function}(\tau), \text{ where } \tau = Q^2/Q_{\rm s}^2(x). \tag{3}$$

This is depicted in Fig.1. where the combined HERA data [1] for different scattering energies W are plotted in terms of Q^2(left) and in terms of τ (right). Quantitative analysis of the combined HERA data and the details of the W binning will be presented elsewhere [4].

In pp collisions particles of low and moderate $p_{\rm T}$ (and given rapidity y) are produced mainly from scattering of gluons carrying longitudinal momentum fractions $x_{1,2}$:

$$x_{1,2} = e^{\pm y}\, p_{\rm T}/W \quad \text{with} \quad W = \sqrt{s}. \tag{4}$$

If gluonic densities in pp collisions are characterized by the saturation scale (1), then also $dN/d\eta d^2 p_{\rm T}$ should scale. Therefore geometrical scaling for the multiplicity distribution in pp collisions [5,6] states that particle spectra depend on the scaling variable

$$\tau = p_{\rm T}^2/Q_{\rm s}^2(p_{\rm T}, W) \tag{5}$$

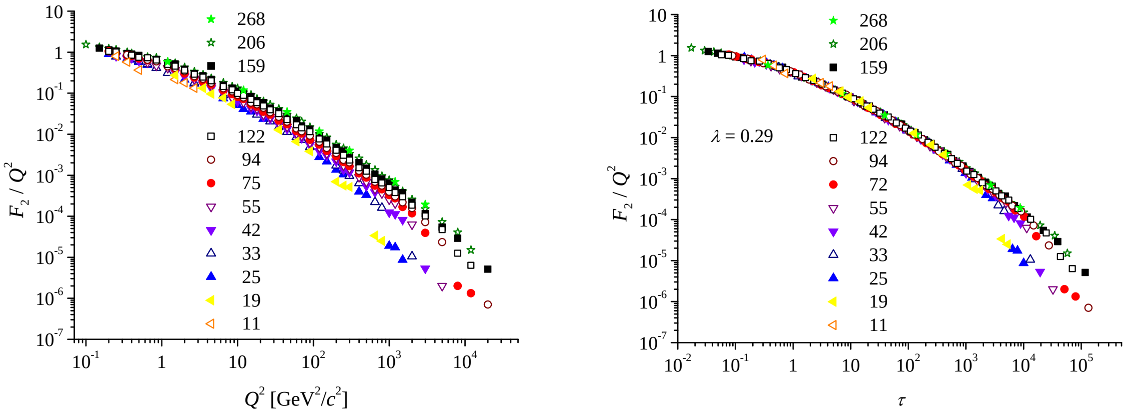

Figure 1: Geometrical scaling in DIS.

where $Q_{\rm s}^2(p_{\rm T}, W)$ is the saturation scale (1) at $x_1 \sim x_2$ (4):

$$Q_{\rm s}^2(p_{\rm T}, W) = Q_0^2 \left(p_{\rm T}/(W \times 10^{-3})\right)^{-\lambda} \tag{6}$$

where we have neglected rapidity dependence of $x_{1,2}$. Factor 10^{-3} corresponds to the choice of the energy scale (arbitrary at this moment x_0 in Eq.(1)). Hence

$$N_{\rm ch}(W, p_{\rm T}) = \left.\frac{dN_{\rm ch}}{d\eta d^2 p_{\rm T}}\right|_W = \frac{1}{Q_0^2} F(\tau) \tag{7}$$

with $Q_0 \sim 1$ GeV. Here $F(\tau)$ is a universal function of τ. This is depicted in Fig. 2 where the $p_{\rm T}$ spectra measured by CMS [7] ale plotted in terms of $p_{\rm T}^2$ (left) and τ (right).

In order to examine the quality of geometrical scaling in pp collisions in Ref.[8] we have considered ratios R_{W_1/W_2}

$$R_{W_1/W_2}(p_{\rm T}) = \frac{N_{\rm ch}(W_1, p_{\rm T})}{N_{\rm ch}(W_2, p_{\rm T})}. \tag{8}$$

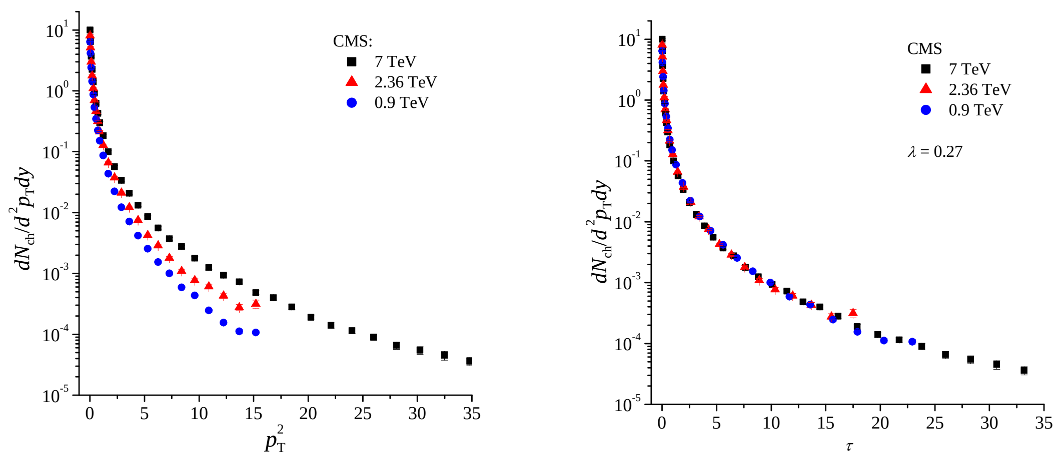

Figure 2: Geometrical scaling in pp.

Here, following Ref.[9] we shall discuss another way of establishing geometrical scaling, at least qualitatively. Note that if at two different energies W_1 and W_2 multiplicity distributions are equal

$$N_{\rm ch}(W_1, p_{\rm T}^{(1)}) = N_{\rm ch}(W_2, p_{\rm T}^{(2)}) \tag{9}$$

then this means that they correspond to the same value of variable τ (5). As a consequence

$$p_{\rm T}^{(1)\,2} \left(p_{\rm T}^{(1)}/W_1\right)^{\lambda} = p_{\rm T}^{(2)\,2} \left(p_{\rm T}^{(2)}/W_2\right)^{\lambda} \tag{10}$$

for constant λ. Equation (10) implies

$$S_{W_1/W_2}^{p_{\rm T}} = p_{\rm T}^{(1)}/p_{\rm T}^{(2)} = (W_1/W_2)^{\frac{\lambda}{2+\lambda}}\,. \tag{11}$$

Ratios $S_{W_1/W_2}^{p_{\rm T}}$ for pp non-single diffractive spectra measured by the CMS [7] collaboration at the LHC are plotted in Fig. 3 together with the straight horizontal lines corresponding to the r.h.s. of Eq.(11) for $\lambda = 0.27$. We see approximate constancy of $S_{W_1/W_2}^{p_{\rm T}}$ over the wide range of $N_{\rm ch}$. A small rise of $S_{W_1/W_2}^{p_{\rm T}}$ with decreasing $N_{\rm ch}$ corresponds to the residual $p_{\rm T}$-dependence [6] of the exponent λ.

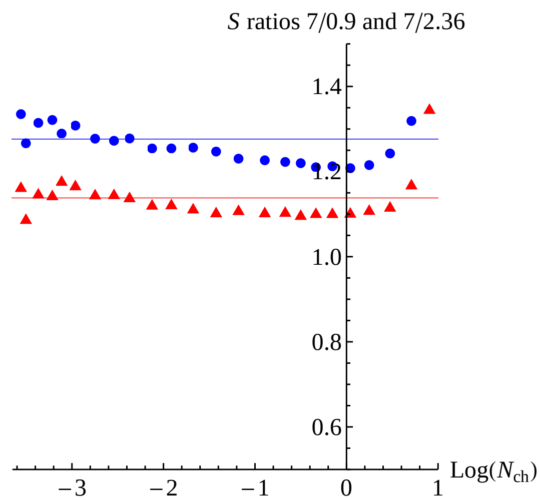

Figure 3: $S_{W_1/W_2}^{p_{\rm T}}$ ratios for CMS pp spectra

This is the simplest way of looking for GS in the $p_{\rm T}$ spectra. An obvious advantage is that it is very easy to do. An obvious disadvantage consists in the fact that it is difficult to attribute sensible error to the ratios $S_{W_1/W_2}^{p_{\rm T}}$, so for quantitative purposes it is better to consider ratios R_{W_1/W_2}.

One of the immediate applications of GS is its ability to *predict* $p_{\rm T}$ spectra at yet unmeasured energies. This was a crucial problem in calculating the so called nuclear modification factor R_{AA} for Pb-Pb collisions at the LHC. R_{AA} is essentially a ratio of nuclear to pp spectra at the same scattering energy normalized by the number of binary collisions. In the first heavy ion LHC run the c.m.s. energy per nucleon was 2.76 GeV and there was no data for pp collisions at this energy until the late run in 2011. In Fig. 4 we plot R_{AA} as published by ALICE [10]. Black (upper) stars correspond the 2010 data where proton spectrum has been interpolated from the measurements at other energies (two solid grey lines correspond to the estimated uncertainty). Pink (lower) stars in turn correspond to the preliminary data where pp spectrum has been measured in a dedicated 2.76 GeV run [11]. Triangles and circles correspond to our rough estimate of R_{AA} where ALICE measured Pb-Pb spectrum [10] has been divided by the theoretical pp spectrum obtained from the hypothesis of geometrical scaling for two different values of λ.

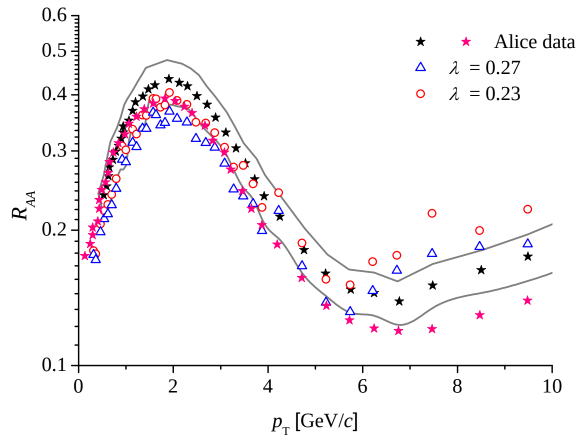

Figure 4: R_{AA} measured by ALICE (stars) and GS prediction (triangles and circles, see text) .

In this talk we have argued that geometrical scaling is a universal phenomenon observed in DIS and in pp scattering at the LHC (for theoretical background see lectures by L. McLerran [12]). After illustrating how GS works in these two processes we have proposed a simple procedure to look for geometrical scaling in the p_{T} spectra, namely to construct ratios of transverse momenta corresponding to the same multiplicity. We have used GS to predict the p_{T} spectra at yet unmeasured energies.

Many aspects of GS require further studies. Firstly, new data at higher energies (to come) have to be examined. Secondly, more detailed analysis including identified particles and rapidity dependence has to be performed. On theoretical side the universal shape $F(\tau)$ has to be found and its connection to the unintegrated gluon distribution has to be studied. That will finally lead to perhaps the most difficult part, namely to the breaking of GS in pp.

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References

1. F. D. Aaron *et al.* [H1 and ZEUS Collaboration], JHEP **1001** (2010) 109.
2. K.J. Golec-Biernat, M. Wüsthoff, Phys. Rev. D **59** (1998) 014017, D **60** (1999) 114023.
3. A.M. Stasto, K.J. Golec-Biernat, J. Kwiecinski, Phys. Rev. Lett. **86** (2001) 596.
4. M. Praszalowicz, T. Stebel, in preperation.
5. L. McLerran, M. Praszalowicz, Acta Phys. Pol. B **41** (2010) 1917 and **42** (2011) 99.
6. M. Praszalowicz, Phys. Rev. Lett. 106 (2011) 142002.
7. V. Khachatryan *et al.* [CMS Collaboration], JHEP **1002** (2010) 041, Phys. Rev. Lett. **105** (2010) 022002, JHEP **1101** (2011) 079.
8. M. Praszalowicz, Acta Phys. Pol. **B42** (2011) 1557.
9. M. Praszalowicz, arXiv:1112.0997 [hep-ph].
10. K. Aamodt *et al.* [ALICE Collaboration], Phys. Lett. B **696** (2011) 30.
11. For pp spectra at 2.76 GeV see talk of M.L. Knichel, this proceedings.
12. L. McLerran, Acta Phys. Pol. B **41** (2010) 2799.