Wave propagation in nonlinear gauge theory and corresponding phenomenology in deconfined strong interacting matter

LI Shi-Yuan

Shandong University, Jinan, P.R. China

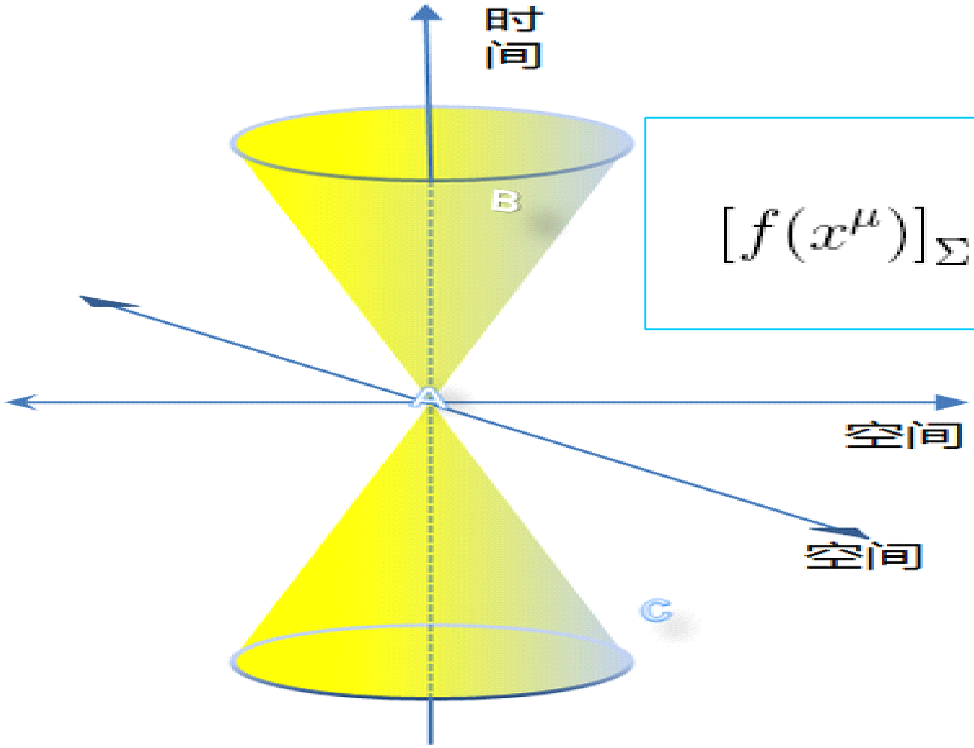
In collab. with VA De Lorenci, R. Klippert and JP Pereira (UNIFI, Itajuba, Brasil)

Wave Propagation:
dispertion relations and polarizations

- Abelian gauge theory-EM for non-linear material,non-linear Optics ⟶

  "non-linear Optics in non-Abelian gauge theories"
- framework to investigate dispertion relations and the polarizations of "shock wave" (Hadamard)
- the important results (velocity), eps. for a model
- extended to non-Abelian gauge dynamics...
- phenomenology for "QCD matter"

Hadamard method



$$[f(x^\mu)]_\Sigma \doteq \lim_{\{P^\pm\}\to P} [f(P^+) - f(P^-)]$$

Hadamard, Lecons sur la propagation des ondes et les equations de l'hydrodynamique (1903); Boillat, J. Math.Phys. 11, 941 (1970); Papapetrou, Lectures on General Relativity (1974)

$$L = L(F, G)$$

$$\overset{*}{F}_{\alpha\beta} = \frac{1}{2}\eta_{\alpha\beta}{}^{\sigma\tau} F_{\sigma\tau},$$

$$F = F^{\mu\nu} F_{\mu\nu}$$

$$G = F^{\mu\nu} \overset{*}{F}_{\mu\nu}.$$

$$2N^{\mu\nu\alpha\beta} F_{\alpha\beta,\nu} + L_F F^{\mu\nu}{}_{,\nu} = 0,$$

$$N^{\mu\nu\alpha\beta} \doteq L_{FF} F^{\mu\nu} F^{\alpha\beta} + L_{GG} \overset{*}{F}^{\mu\nu} \overset{*}{F}^{\alpha\beta} + L_{FG} \left(F^{\mu\nu} \overset{*}{F}^{\alpha\beta} + \overset{*}{F}^{\mu\nu} F^{\alpha\beta} \right)$$

$$\overset{*}{F}^{\mu\nu}{}_{,\nu} = 0.$$

$$Z^{\mu}{}_{\nu} \epsilon^{\nu} = 0,$$

$$Z^{\mu}{}_{\nu} \doteq \delta^{\mu}{}_{\nu} + \frac{4}{L_F k^2} N^{\mu\alpha}{}_{\nu\beta} k_{\alpha} k^{\beta}.$$

$$| \; Z_{\mu\nu} \; | = 0.$$

$$f^2 = (\vec{q}\cdot\vec{E})^2 - \omega^2 E^2 + (\hat{q}\cdot\vec{B})^2 - q^2 B^2 + 2\omega\vec{q}\cdot\vec{E}\times\vec{B}. \tag{17}$$

$$f^2 = F^{\alpha\mu} F_{\alpha}{}^{\nu} k_{\mu} k_{\nu}$$

The phase velocity $v \doteq \omega/|\vec{q}|$ of the electromagnetic waves can be obtained from Eq. (13). In fact, it is straighforward to show that this equation can be presented as a forth-degree equation for v as

$$a_4 v^4 + a_3 v^3 + a_2 v^2 + a_1 v + a_0 = 0, \tag{18}$$

where we have defined

$$a_4 \doteq \alpha - \beta E^2 + \gamma E^4 \tag{19}$$

$$a_3 \doteq 2(\beta - 2\gamma E^2)\hat{q}\cdot\vec{E}\times\vec{B} \tag{20}$$

$$\begin{aligned} a_2 \doteq& -2\alpha + \beta[E^2 - B^2 + (\hat{q}\cdot\vec{E})^2 + (\hat{q}\cdot\vec{B})^2] \\ &+2\gamma\{2(\hat{q}\cdot\vec{E}\times\vec{B})^2 - [(\hat{q}\cdot\vec{E})^2 + (\hat{q}\cdot\vec{B})^2 - B^2]E^2\} \end{aligned} \tag{21}$$

$$a_1 \doteq -2\{\beta - 2\gamma[(\hat{q}\cdot\vec{E})^2 + (\hat{q}\cdot\vec{B})^2 - B^2]\}\hat{q}\cdot\vec{E}\times\vec{B} \tag{22}$$

$$a_0 \doteq \alpha + \beta[B^2 - (\hat{q}\cdot\vec{E})^2 - (\hat{q}\cdot\vec{B})^2] + \gamma[B^2 - (\hat{q}\cdot\vec{E})^2 - (\hat{q}\cdot\vec{B})^2]^2. \tag{23}$$

A concrete model

- Savvidy(1977), Pagels and Tomboulis (1978), Nielsen and Ninomiya(1979).

- Effective Lagrangian for QCD
- but for QED/material, it has interesting magnetic 'vacuum' properties

$$L_{YM} \approx -\frac{1}{4} b_0 F \log \frac{F}{\lambda^2},$$

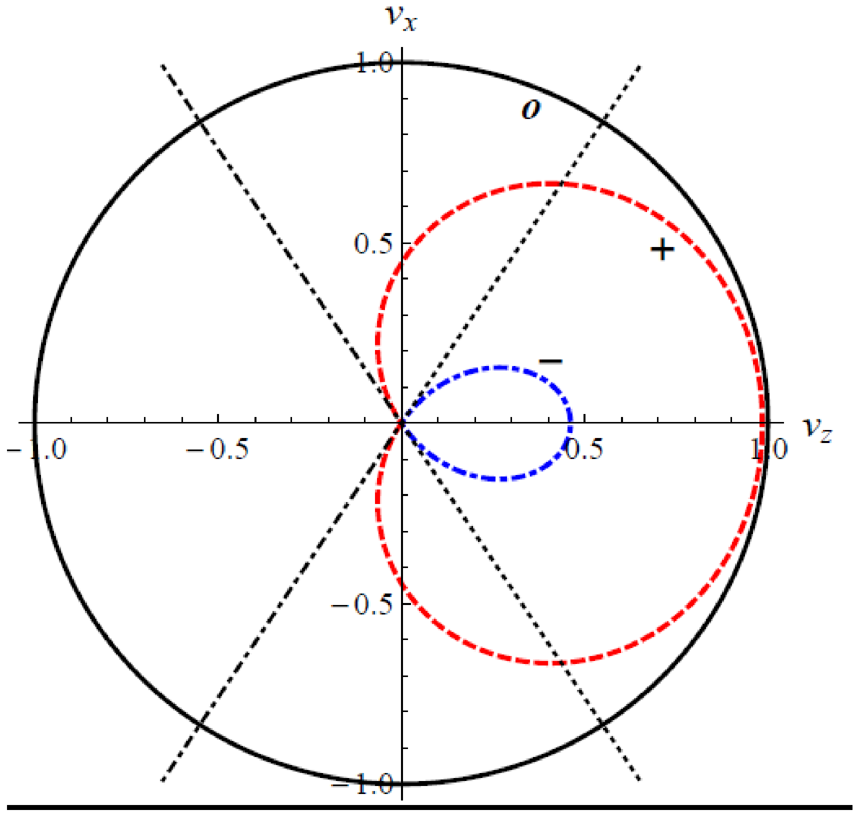
For the special external field configuaratiion, to look for the possible trirefregence

$$\vec{E} = E\hat{x} \quad \vec{B} = B\hat{y}$$

$$\hat{q} \cdot \hat{y} = 0, \ \hat{q} \cdot \hat{x} = \sin\theta$$

$$v_0 = 1,$$

$$v_\pm = \frac{2EB\cos\theta \pm \sqrt{\frac{F}{2}\left(3+\log\frac{F}{\lambda^2}\right)\left[\frac{F}{2}\left(1+\log\frac{F}{\lambda^2}\right) - E^2(1-\cos 2\theta)\right]}}{2E^2 - \frac{F}{2}\left(1+\log\frac{F}{\lambda^2}\right)}$$



Non-Abelian dynamics

- The discontinuity is on Field strength not A^{μ}
- So the result can be extended easily
- But for phenomenology, not as easily prepared as EM materials

- W-S early universe...
- The arena for QCD is deconfined strongly interacting matter (QGP?)

It is not easy to prepare external field configurations but…

- A symmetric average

$$\overline{E_i} = 0, \quad \overline{H_i} = 0, \quad \overline{E_i H_j - H_i E_j} = 0,$$

$$\overline{E_i E_j} = -\frac{1}{3} E^2 \eta_{ij},$$

$$\overline{H_i H_j} = -\frac{1}{3} H^2 \eta_{ij},$$

- v<1☺ ; v>1 not
- →confine/deconfine
- →polarizer

$$v_e^2 = 1 - \frac{8}{3} \frac{(Z^2+1)G(\bar{g})}{(Z^2-1)+4G(\bar{g})}.$$

$$Z^2 \doteq \frac{H^2}{E^2}$$

$$G(\bar{g}) \doteq \frac{\bar{g}\dot{\bar{g}} - 3\dot{\bar{g}}^2 + \bar{g}\ddot{\bar{g}}}{\bar{g}^2 - 2\bar{g}\dot{\bar{g}}}.$$

…*But also not possible to measure directly: Lambda polarization /correlation*

- the gluon propagate in the specified material have only certain velocity with coresponding polarization
- g to ssbar
- Lambda(uds)

$$\frac{dN}{dcos\theta} \sim 1 + \alpha\hat{p}\cdot\vec{P} = 1 + \alpha P cos\theta$$

$$P = 3\sqrt{<\hat{p}_1 \cdot \hat{p}_2>}/\alpha$$

Summary

- "Shock wave" propagation/Non-linear Optics; dynamics from Abelian to Non-Abelian
- Investigation of the framework to get all the possible dispertion relations and the corresponding polarizations
- A concrete model to demonstrate the above conclusion
- This model can serve for describing strongly interacting matter and measuring the propagation effect is possible for the case decofined
- Lambda local polarization observable is suggested.