# Hadronic Cross Section Measurements with ISR and the Implications on $g_\mu - 2$

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on behalf of the B-Factories

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Moriond QCD Conference,
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# Outline

1. Introduction
   - Optical theorem
   - Experimental setup at *BABAR*
2. Recent Results
   - $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$
   - $e^+e^- \to K^+K^-$
3. Summary

# The contributions to $a_\mu$ and its uncertainty

from Phys.Rev.Lett., 109:111808, 2012 and Eur.Phys.J., C71:1515, 2011 [1, 3]

$$\vec{\mu} = g\frac{e}{2m}\vec{s}$$

$$(g_\mu - 2)/2 =: a_\mu = 0 \qquad \text{(Dirac)}$$

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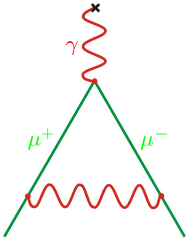

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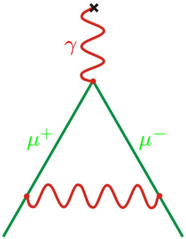


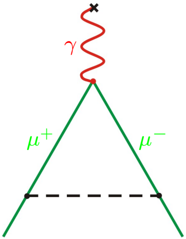

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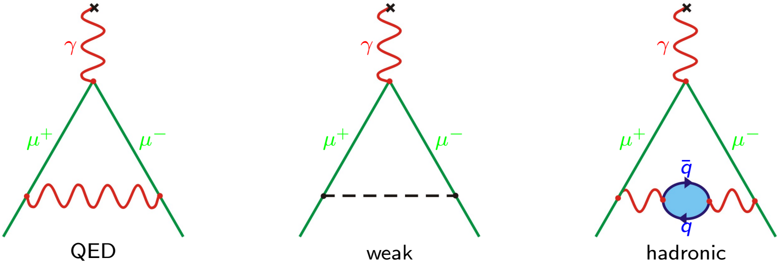

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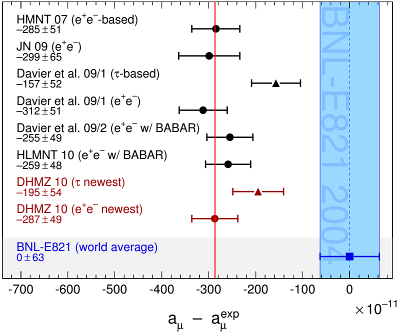
| Interaction | Contribution $[\cdot 10^{-11}]$ | Uncertainty $[\cdot 10^{-11}]$ |
|---|---|---|
| QED | 116 584 718.951 | 0.080 |
| weak | 154 | 2 |
| hadronic | 6930 | 49 |
| BNL E821 | 116 592 089 | 63 |
| Deviation | 287 | 80 |
| | $\rightarrow 3.6\sigma$ | |

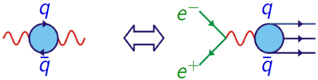
# Discrepancy between SM prediction and direct measurement from Eur.Phys.J., C71:1515, 2011 [3].




## Just a fluctuation?

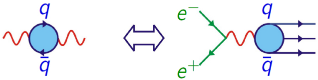
3.6$\sigma$ effect, thus reduction of uncertainties necessary!

# The optical theorem




$$a_\mu^{had} = \frac{1}{4\pi^3}\int_0^\infty K(s)\sigma_{\rm had}(s)ds \ , \qquad K(s) \propto {}^1/_s$$

# The optical theorem

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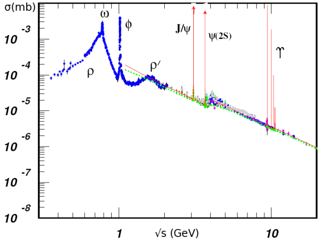
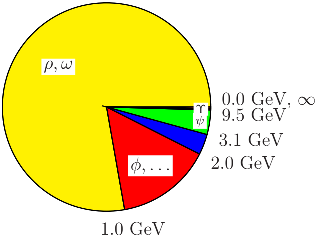
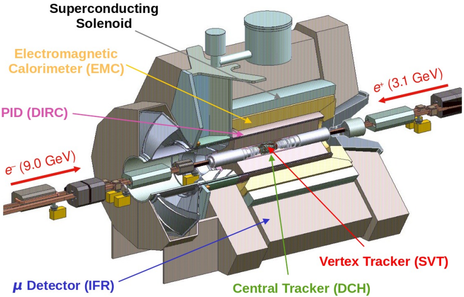



$\sigma_{\rm had}$ (left) from Nuovo Cim., C034S1:31-40, 2011 [5] and relative contributions to $a_\mu^{had}$ (right).

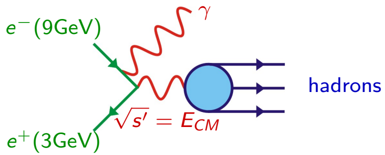
# The *BABAR* Experiment

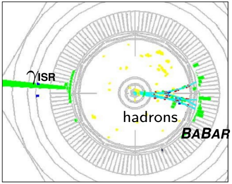



## Experimental specifications

Energy: $\sqrt{s} = 10.6\,\text{GeV} \quad (E_{e^-} = 9.0\,\text{GeV}, E_{e^+} = 3.1\,\text{GeV})$,
Luminosity: $\mathcal{L} = 454\,\text{fb}^{-1} \quad (\Upsilon(4S))$

# Initial State Radiation (ISR) events at *BABAR*

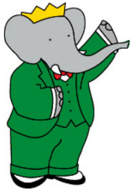


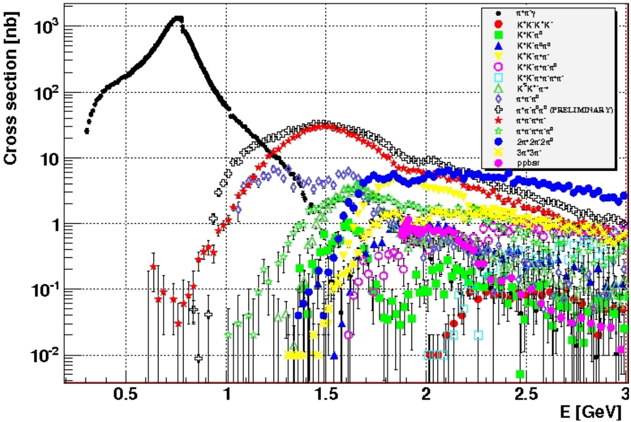




## ISR selection

- Detected high energy photon: $E_\gamma > 3$GeV
  $\rightarrow$ defines $E_{CM}$ & provides strong background rejection
- Event topology: $\gamma_{ISR}$ back-to-back to hadrons
  $\rightarrow$ high acceptance
- Kinematic fit including $\gamma_{ISR}$
  $\rightarrow$ very good energy resolution ($4 - 15$MeV)
- $e^+e^-$-boost into the laboratory reference frame
  $\rightarrow$ high efficiency at production threshold of hadronic system
- Continuous measurement from threshold to $\sim$4.5GeV
  $\rightarrow$ provides common, consistent systematic uncertainties

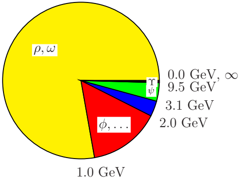
# Most important channels

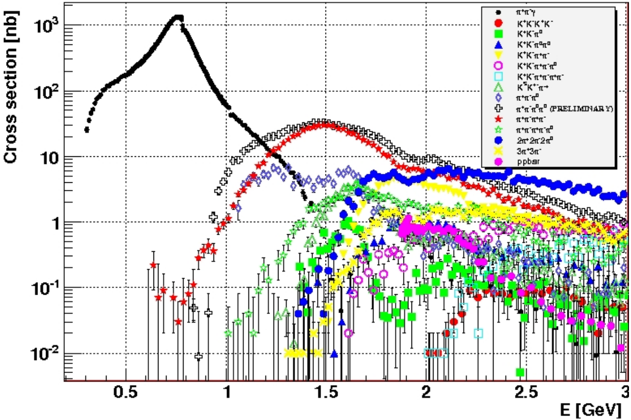





Cross Sections of the single channels measured at *BABAR* (from Nucl.Phys.Proc.Suppl., 207-208:133-136, 2010 [4]).

# Most important channels




Right panel: Cross Sect. of single channels (from Nucl.Phys.Proc.Suppl., 207-208:133-136, 2010 [4]).
Left panel: Relative contributions to $a_\mu^{had}$ (from Nuovo Cim., C034S1:31-40, 2011 [5]).

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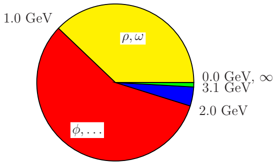
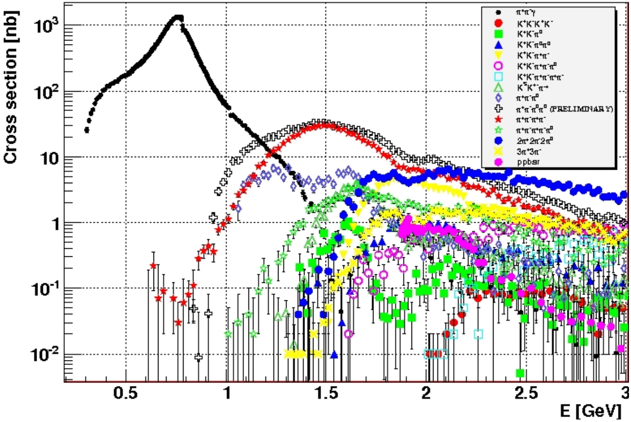



Right panel: Cross Sect. of single channels (from Nucl.Phys.Proc.Suppl., 207-208:133-136, 2010 [4]).
Left panel: Relative contributions to $\delta a_\mu^{had}$ (from Nuovo Cim., C034S1:31-40, 2011 [5]).

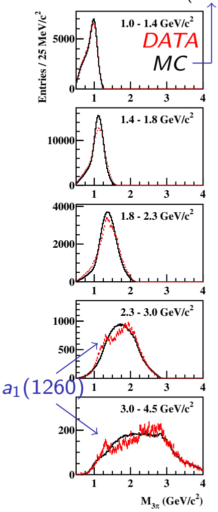
# $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$

Phys. Rev. D**85**, 112009 (2012), based on 454 $\text{fb}^{-1}$

supersedes our previous publication,
based on 89 $\text{fb}^{-1}$ of the data:
Phys. Rev. D**71**, 052001 (2005).

# Internal structure in various $E_{CM}$ energy slices

(EPJ C18, 497 (2001))




**First column (4 entries/event):**

$a_1(1260)$

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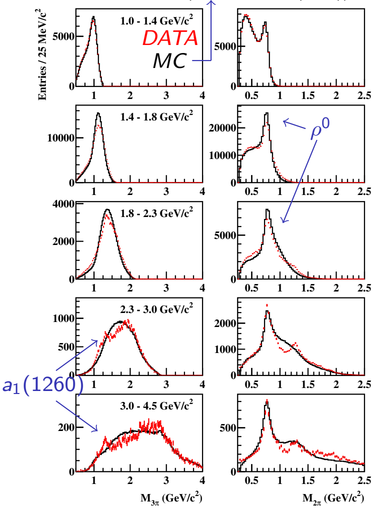


### First column (4 entries/event):

$a_1(1260)$

### Second column (4 entries/event):

strong $\rho^0$ contribution
e.g. for $M_{4\pi} > 1.4\,\mathrm{GeV}/c^2$:
$1/4th$ of entries in $\rho^0$ peak
$\rho^0\rho^0$ is forbidden
$\to \rho^0$ in each event!

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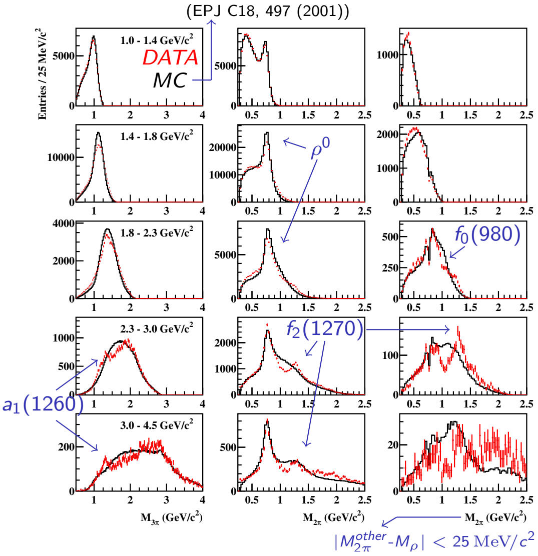


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$\rho^0\rho^0$ is forbidden
$\to \rho^0$ in each event!

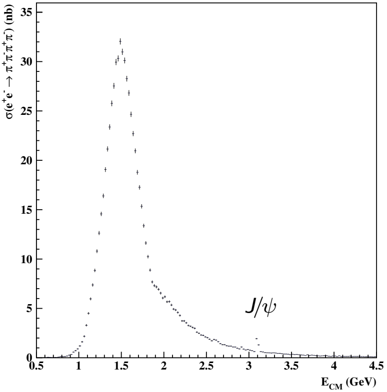
Third column (1 entry/event):

$2\pi$ lie within $\rho^0$ mass
$\to$ other $\pi^+\pi^-$'s mass plotted

$f_2(1270)$, $a_1(1260)$, $f_0(980)\ldots$?

$\to$ Partial Wave Analysis needed

# Cross section for $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$




- Systematic uncertainties
  2.4% in peak region (1.1-2.8 GeV)
  11% (0.6-1.1 GeV)
  4% (2.8-4.0 GeV)
- $J/\psi$ visible

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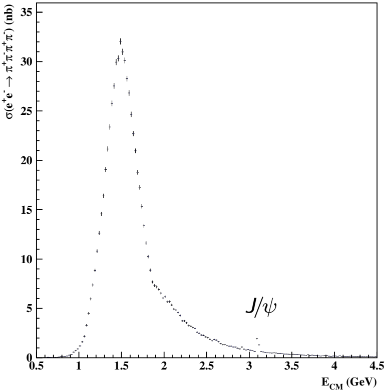


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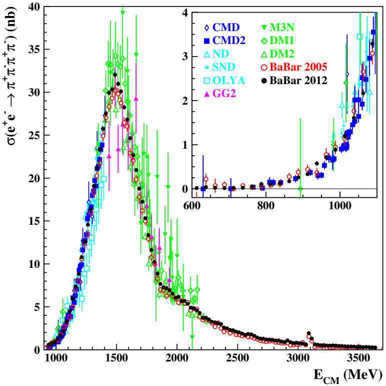


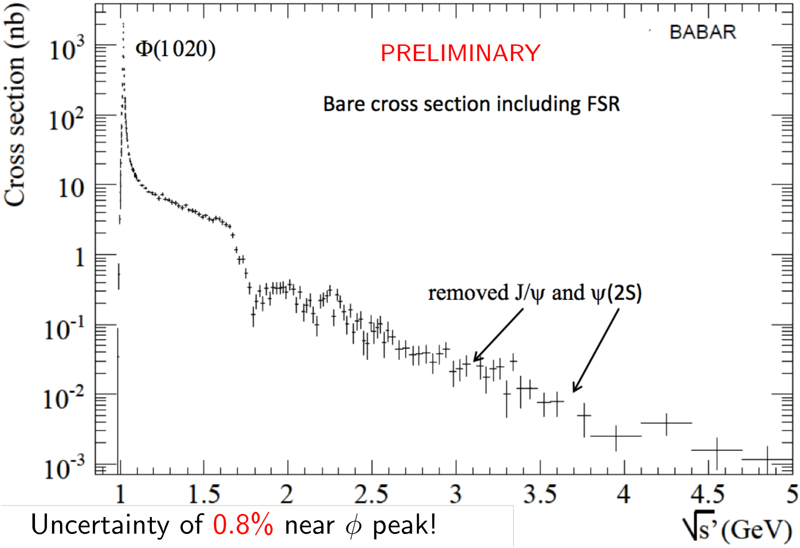

- $< 1.4\,\mathrm{GeV}$: agreement with previous *BABAR* results, SND and CMD-2 data
- $> 1.4\,\mathrm{GeV}$: highest precision (DM2, 20%)

# $e^+e^- \to K^+K^-$

PRELIMINARY

$\to$ about to be submitted to PRD, based on $232\,\text{fb}^{-1}$

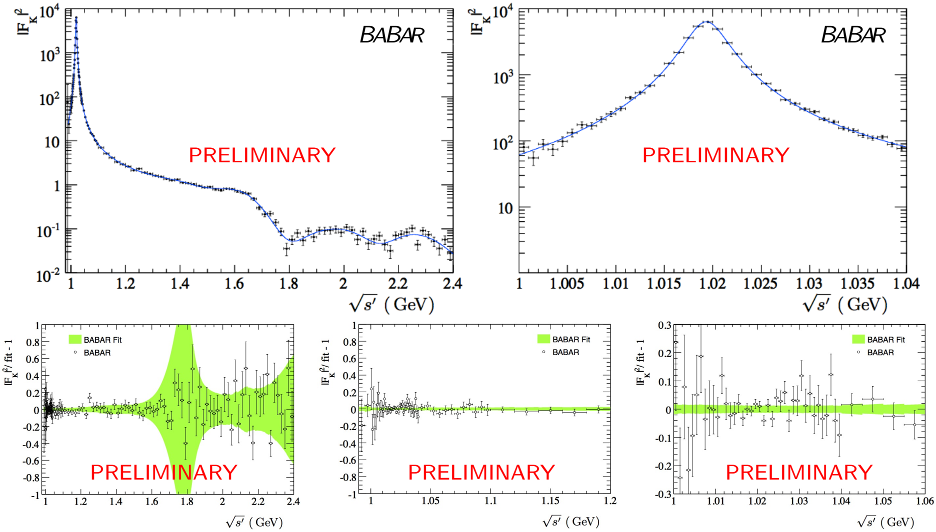
# Cross section $\sigma(e^+e^- \to K^+K^-)$



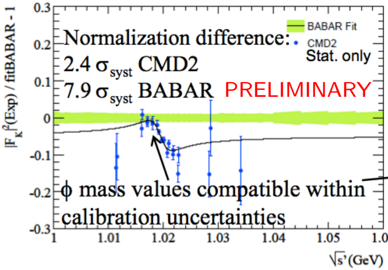

Uncertainty of 0.8% near $\phi$ peak!

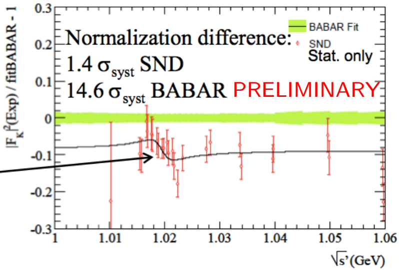
$\rightarrow$ luminosity from $\mu\mu\gamma$ and $K$-ID

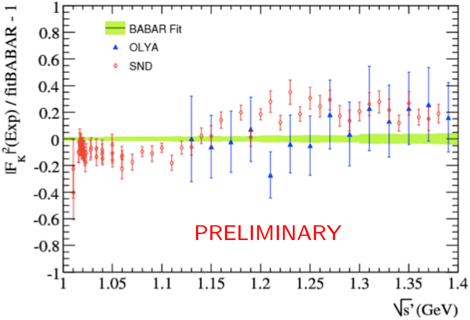
# A phenomenological fit to the form factor

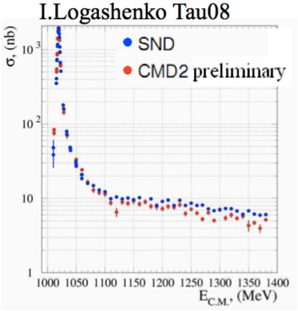


# Comparison to other experiments



# The $\Phi$ parameters

$m_\Phi$ and $\Gamma_\Phi$ obtained from the fit of the form factor

*BABAR*

$$m_\Phi = 1019.51 \pm 0.02(\pm 0.11)\,\mathrm{MeV}$$
$$\Gamma_\Phi = 4.29 \pm 0.04(\pm 0.07)\,\mathrm{MeV}$$

PDG

$$m_\Phi = 1019.455 \pm 0.020\,\mathrm{MeV}$$
$$\Gamma_\Phi = 4.26 \pm 0.04\,\mathrm{MeV}$$

$\to$ good agreement

From integrated $\Phi$ peak: $\Gamma_\Phi^{ee} \times \mathcal{B}(\Phi \to K^+K^-) = \frac{\alpha^2\beta^3(s,m_K)}{324}\frac{m_\Phi^2}{\Gamma_\Phi} a_\Phi^2 C_{FS}$

*BABAR*:

$\Gamma_\Phi^{ee} \times \mathcal{B}(\Phi \to K^+K^-) = 0.6344 \pm 0.0059_{exp} \pm 0.0028_{fit} \pm 0.0015_{cal}\,\mathrm{keV}(1.1\%)$

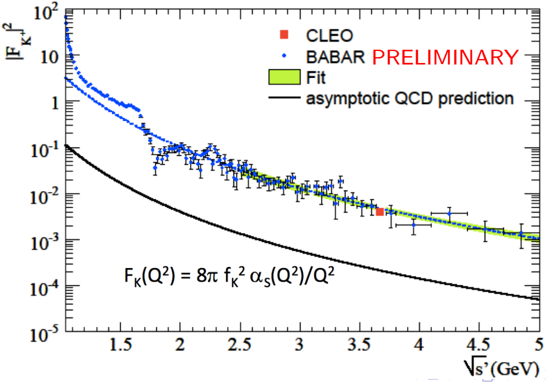
CMD2:

$\Gamma_\Phi^{ee} \times \mathcal{B}(\Phi \to K^+K^-) = 0.605 \pm 0.002 \pm 0.013\,\mathrm{keV}(2.1\%)$

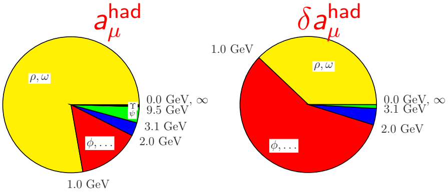
# Charged kaon form factor at large $Q^2$

Predictions based on QCD in asymptotic regime (Chernyak, Brodsky-Lepage, Farrar-Jackson)

- Power law: $F_K \sim \alpha_S(Q^2)Q^{-n}$ with n=2
  $\rightarrow$in good agreement with the data (2.5-5 $\mathrm{GeV}$ $n = 2.10 \pm 0.23$)
- HOWEVER: data on $|F_K|^2$ factor $\sim$ 20 above prediction!
- No trend in data up to 25 $\mathrm{GeV}^2$ for approaching the asymp. QCD prediction



# Impact on $g_\mu - 2$

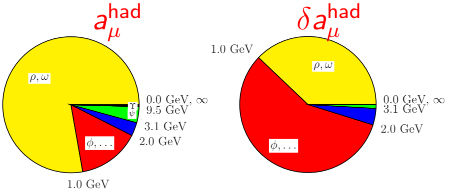



[PR 477, 1 (2009).]

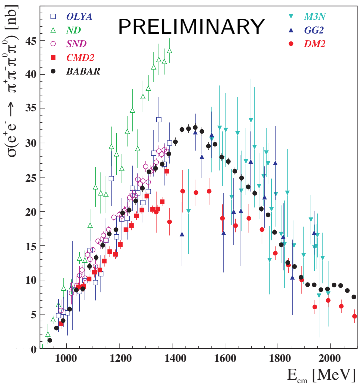
$$a_\mu^{had}(K^+K^-) = 216.3 \pm 2.7 \pm 6.8$$
$$\downarrow$$
$$a_\mu^{had}(K^+K^-) = 229.5 \pm 1.4 \pm 2.2$$
calculation only based on *BABAR* 2013 data!

$$a_\mu^{had}(\pi^+\pi^-\pi^+\pi^-) = 133.5 \pm 1.0 \pm 5.2$$
$$\downarrow$$
$$a_\mu^{had}(\pi^+\pi^-\pi^+\pi^-) = 136.4 \pm 0.3 \pm 3.6$$
calculation only based on *BABAR* 2012 data!

# Impact on $g_\mu - 2$




[PR 477, 1 (2009).]

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(all $a_\mu$ units in $10^{-11}$)

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$$\downarrow$$
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calculation only based on *BABAR* 2012 data!

dominant contribution to $\delta a_\mu^{had}$:
$$a_\mu^{had}(\pi^+\pi^-\pi^0\pi^0) = 180.1 \pm 0.3 \pm 12.4$$
$$\Downarrow$$
*BABAR* analysis in progress

# Summary

- ISR physics has proven to be a very productive field even years after the end of data taking at the B-factories
- Precision measurements of $\sigma(e^+e^- \to 4\pi)$ and $\sigma(e^+e^- \to 2K)$ have greatly improved $a_\mu^{\mathrm{SM}}$
- More hadronic final states (e.g. $2\pi 2\pi^0$) in preparation
- $g_\mu - 2$ puzzle needs to be solved
  - ⋆ Data from new experiments (e.g. BES-III)
  - ⋆ Light-By-Light scattering needs to be studied
  - ⋆ $E$989 at Fermilab and J-PARC $g$-2/EDM

Thank you!

Any questions?

# Backup slides

# References I

[1] T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio.
Complete Tenth-Order QED Contribution to the Muon $g-2$.
*Phys.Rev.Lett.*, 109:111808, 2012.

[2] G. Bennett et al.
Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL.
*Phys.Rev.*, D73:072003, 2006.

[3] M. Davier, A. Hoecker, B. Malaescu, and Z. Zhang.
Reevaluation of the Hadronic Contributions to the Muon $g-2$ and to $\alpha(M_Z)$.
*Eur.Phys.J.*, C71:1515, 2011.

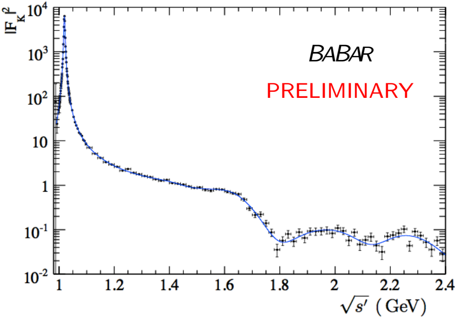
# References II


[4] A. Hafner.
Exclusive hadronic cross sections measured via ISR from BaBar.
*Nucl.Phys.Proc.Suppl.*, 207-208:133–136, 2010.

[5] F. Jegerlehner.
Electroweak effective couplings for future precision experiments.
*Nuovo Cim.*, C034S1:31–40, 2011.

[6] N. Saito.
A novel precision measurement of muon g-2 and EDM at J-PARC.
*AIP Conf.Proc.*, 1467:45–56, 2012.

# Breit-Wigner fit function




$$
\begin{aligned}
F_K(s) &= (a_\phi \cdot BW_\phi(s) + a_{\phi'} \cdot BW_{\phi'}(s) + a_{\phi''} \cdot BW_{\phi''}(s))/3 \\
&+ (a_\rho \cdot BW_\rho(s) + a_{\rho'} \cdot BW_{\rho'}(s) + a_{\rho''} \cdot BW_{\rho''}(s) + a_{\rho'''} \cdot BW_{\rho'''}(s))/2 \\
&+ (a_\omega \cdot BW_\omega(s) + a_{\omega'} \cdot BW_{\omega'}(s) + a_{\omega''} \cdot BW_{\omega''}(s) + a_{\omega'''} \cdot BW_{\omega'''}(s))/6 \\
\text{with} \quad & a_\phi + a_{\phi'} + a_{\phi''} = 1 \\
& a_\rho + a_{\rho'} + a_{\rho''} + a_{\rho'''} = 1 \\
& a_\omega + a_{\omega'} + a_{\omega''} + a_{\omega'''} = 1
\end{aligned}
$$

# Basic method

Definition of $g$:

$$\vec{\mu} = g \frac{e}{2m} \vec{s} \,.$$

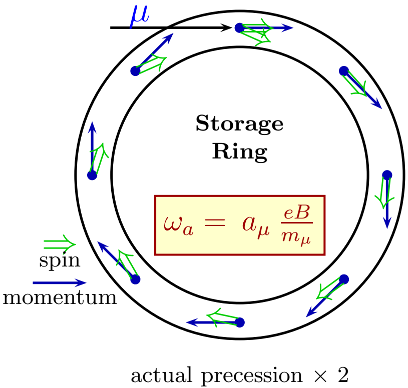
Motion in magnetic field:

$$\vec{\omega}_c = \frac{e\vec{B}}{m\gamma} \,,$$

$$\vec{\omega}_l = \frac{e\vec{B}}{m\gamma} + a\frac{e\vec{B}}{m} \,,$$

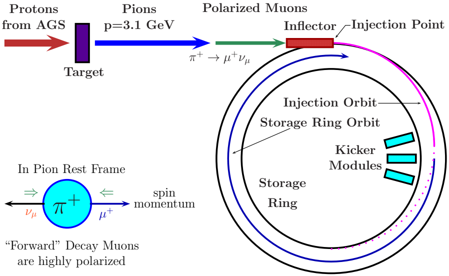
$$\Rightarrow \vec{\omega}_a = a\frac{e\vec{B}}{m} \,.$$

$$(a = (g-2)/2)$$




Spin precession. [5]

# Realization in detail




From $\pi^+$ production to $\mu^+$ decay. [5]

## Electric field necessary for focussing: BMT equation

$$\vec{\omega}_a = \frac{e}{m_\mu}\left( a_\mu \vec{B} - \left[ a_\mu - \frac{1}{\gamma^2 - 1} \right] \vec{v} \times \vec{E} \right) .$$

$\vec{\omega}_a$ is independent of $\vec{E}$ for $\gamma = 29.3 \Leftrightarrow E_\mu = \gamma m_\mu = 3.1\,\mathrm{GeV}$.

# Direct Measurement of $(g_\mu - 2)$

## Experiment E821 at Brookhaven National Laboratory

The result [2]:

$$a_\mu = (116\,592\,089 \pm 54_{stat} \pm 33_{syst}) \cdot 10^{-11}$$

# New Direct Measurement of $(g_\mu - 2)$

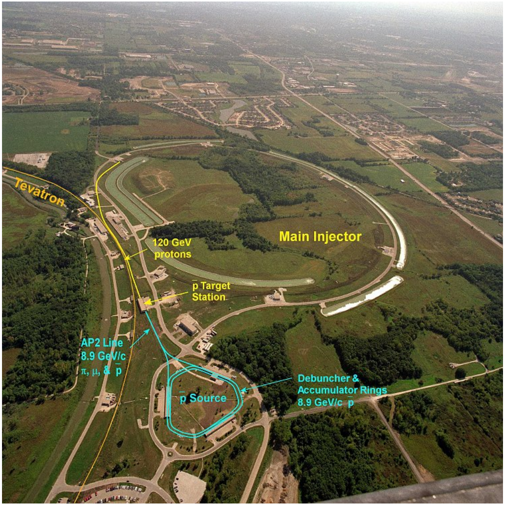
## Experiment E989 at Fermilab

The goal:

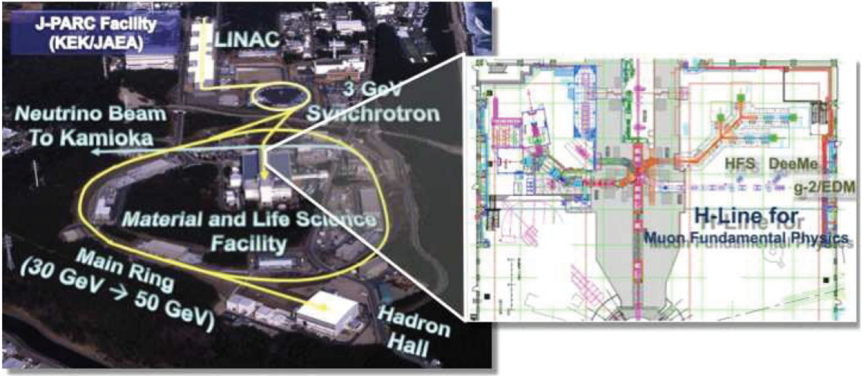
Reduce uncertainty by factor 4 (!)

# New Direct Measurement of $(g_\mu - 2)$

## Experiment E989 at Fermilab



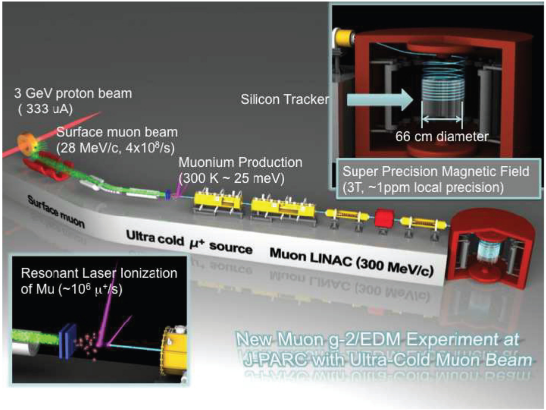
# Ultra-cold muon experiment at J-PARC MLF from [6]




## The goal:

Uncertainty of $\sim 10 \cdot 10^{-11}$ (!)

# Ultra-cold muon experiment at J-PARC MLF from [6]




## New Method:

Produce muons from ionization of muonium, store them and track decay.