

# QCD in 2D Flux Tube

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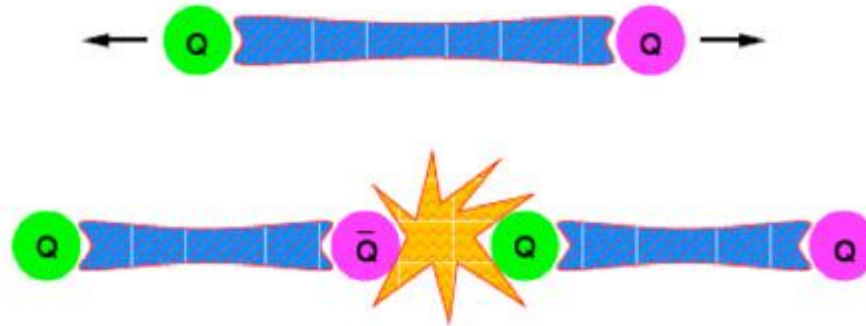
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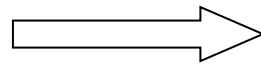
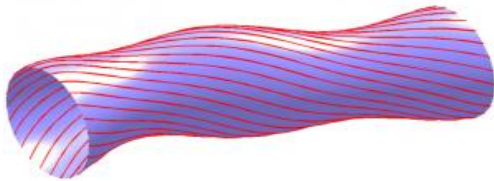
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# 1.Introduction



3D (3+1)



“3D” (3+1)



“3D” (2+ (1+1))



## 2. The main goal

$$\mathcal{A}(4D)[\Psi(4D, x), F_{\mu\nu}^a(4D, x), g_{4D}] \Rightarrow \mathcal{A}(2D)[\Psi(2D, x^0, x^3), F_{\mu\nu}^a(2D, x^0, x^3), g_{2D}] \quad (1)$$

$$\mathcal{A}(4D) = \int d^4x \left\{ \text{Tr} \left[ \frac{1}{2} [\bar{\Psi}(4D, x) \gamma^\mu(4D) \Pi_\mu(4D) \Psi(4D, x) - \bar{\psi}(4D, x) m(x) \Psi(4D, x)] \right. \right. \\ \left. \left. - \frac{1}{2} [\bar{\Psi}(4D, x) \gamma^\mu(4D) \overleftarrow{\Pi}_\mu(4D) \Psi(4D, x) + \bar{\Psi}(4D, x) m(x) \Psi(4D, x)] \right] - \frac{1}{4} F_{\mu\nu}^a(4D, x) F_a^{\mu\nu}(4D, x) \right\}. \quad (2)$$

$$\Pi_\mu(4D) = i\partial_\mu + g_{4D} T_a A_\mu^a(4D, x) = p_\mu + g_{4D} T_a A_\mu^a(4D, x); \quad (3)$$

$$\overleftarrow{\Pi}_\mu(4D) = i\overleftarrow{\partial}_\mu - g_{4D} T_a A_\mu^a(4D, x) = \overleftarrow{p}_\mu - g_{4D} T_a A_\mu^a(4D, x);$$

$$F_{\mu\nu}^a(4D, x) = \partial_\mu A_\nu^a(4D, x) - \partial_\nu A_\mu^a(4D, x) + g_{4D} f_{bc}^a A_\mu^b(4D, x) A_\nu^c(4D, x) \\ \equiv \partial_\mu A_\nu^a(4D, x) - \partial_\nu A_\mu^a(4D, x) - ig_{4D} [A_\mu^b(4D, x), A_\nu^c(4D, x)]^a. \quad (4)$$

Gauge transformation

$$\delta A_\mu^a(4D, x) = f_{bc}^a \varepsilon^b(x) A_\mu^c(4D, x) - \frac{1}{g_{4D}} \partial_\mu \varepsilon^a(x) \quad (5)$$

### 3. The key approximation

The longitudinal dominance and transverse confinement

$$\max \{|A_1^a|; |A_2^a|\} \ll \min \{|A_0^a|; |A_3^a|\} \Leftrightarrow |A_1^a|, |A_2^a| \equiv 0$$

However!!!

$$A_0^a = A_0^a(x^0, \mathbf{x}), \quad A_3^a = A_3^a(x^0, \mathbf{x})$$

The Lorentz Gauge

$$\partial_\mu A^\mu(x^0, \vec{x}) = 0$$



# 4.Compactification 4D -> 2D

We transform

(i) a fermion field

$$\Psi(4D, x) \equiv \begin{pmatrix} \varphi(x^0; \mathbf{x}) \\ \chi(x^0; \mathbf{x}) \end{pmatrix} \equiv \begin{pmatrix} \varphi_1(x^0; \mathbf{x}) \\ \varphi_2(x^0; \mathbf{x}) \\ \chi_1(x^0; \mathbf{x}) \\ \chi_2(x^0; \mathbf{x}) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} G_1(\mathbf{r}_\perp) (f_+(x^0; x^3) + f_-(x^0; x^3)) \\ -G_2(\mathbf{r}_\perp) (f_+(x^0; x^3) - f_-(x^0; x^3)) \\ G_1(\mathbf{r}_\perp) (f_+(x^0; x^3) - f_-(x^0; x^3)) \\ G_2(\mathbf{r}_\perp) (f_+(x^0; x^3) + f_-(x^0; x^3)) \end{pmatrix}, \quad (6)$$

$$\int dx^1 dx^2 (|G_1(\mathbf{r}_\perp)|^2 + |G_2(\mathbf{r}_\perp)|^2) = 1. \quad (7)$$

(ii) a gauge field

$$A_\mu^a(4D, x^0, x^3, \mathbf{r}_\perp) = \sqrt{|G_1(\mathbf{r}_\perp)|^2 + |G_2(\mathbf{r}_\perp)|^2} A_\mu^a(2D, x^0, x^3). \quad (8)$$

The coupling constant and gauge transformation become

$$g_{2D} = \int dx^1 dx^2 g_{4D} [|G_1(\mathbf{r}_\perp)|^2 + |G_2(\mathbf{r}_\perp)|^2]^{3/2} \quad (9)$$

$$\delta A_\mu^a(2D, x^0, x^3) = f_{bc}^a \varepsilon^b(x^0, x^3) A_\mu^c(2D, x^0, x^3) \quad (10)$$

## 5. 2D - action functional

$$\mathcal{A}(2D) = \int d^2X \left\{ \text{Tr} \left[ \frac{1}{2} [\bar{\Psi}(2D, X) \gamma^k(2D) \Pi_k(2D) \Psi(2D, X) - \bar{\Psi}(2D, X) m_{qT} \Psi(2D, X)] \right. \right. \\ \left. \left. - \frac{1}{2} [\bar{\Psi}(2D, X) \gamma^k(2D) \overleftarrow{\Pi}_k(2D) \Psi(2D, X) + \bar{\Psi}(2D, X) m_{qT} \Psi(2D, X)] \right] \right. \\ \left. - \frac{1}{4} F_{\mu\nu}^a(2D) F_a^{\mu\nu}(2D) + \frac{1}{2} m_{gT}^2 [A_a^\mu(2D) A_\mu^a(2D)] \right\}, \quad (11)$$

where  $\{\mu, \nu\} = 0, 3$ , and

$$\Pi_\mu(2D) = i\partial_\mu + g_{2D} T_a A_\mu^a(2D, x) = p_\mu + g_{2D} T_a A_\mu^a(2D, x),$$

$$\overleftarrow{\Pi}_\mu(2D) = i\overleftarrow{\partial}_\mu - g_{2D} T_a A_\mu^a(2D, x) = \overleftarrow{p}_\mu - g_{2D} T_a A_\mu^a(2D, x).$$

$$\Psi(X) \equiv \Psi(2D, X) = \begin{pmatrix} f_+(X) \\ f_-(X) \end{pmatrix}, \quad X = (x^0; x^3),$$

$$\gamma^0(2D) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^3(2D) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad g_{\mu\nu}(2D) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$m_{qT} = \int dx^1 dx^2 \left\{ m(\mathbf{r}_\perp) (|G_1(\mathbf{r}_\perp)|^2 - |G_2(\mathbf{r}_\perp)|^2) + (G_1^*(\mathbf{r}_\perp)(p_1 - ip_2)G_2(\mathbf{r}_\perp)) - (G_1(\mathbf{r}_\perp)(p_1 + ip_2)G_2^*(\mathbf{r}_\perp)) \right\} \quad (13)$$

$$m_{gT}^2 = \frac{1}{2} \int dx^1 dx^2 \left[ \{\partial_1[|G_1(\mathbf{r}_\perp)|^2 + |G_2(\mathbf{r}_\perp)|^2]^{1/2}\}^2 + \{\partial_2[|G_1(\mathbf{r}_\perp)|^2 + |G_2(\mathbf{r}_\perp)|^2]^{1/2}\}^2 \right] \quad (14)$$

## 6. The Dirac fields in (1+1) space-time

$$\{i\gamma^\mu (\partial_\mu - ig_{2D} \cdot A_\mu^a(x)T_a) - m_{qT}\} \Psi(2D, X) = 0, \quad \mu = 0, 3, \quad (15)$$

$$A_\mu^a(2D, X) = (A_0^a(X), -A_3^a(X)), \quad X = (x^0; x^3).$$

(i) Formal solution

$$\begin{aligned} \Psi(2D, X) = & \int_{-\infty}^{+\infty} \frac{d\omega}{2\sqrt{L}} \sum_p \frac{m_{qT}}{\sqrt{\omega^2 + p\omega}} \exp(-iP_\mu x^\mu) a(p, \omega) [\delta(\omega - \varepsilon(p)) + \delta(\omega + \varepsilon(p))] \begin{pmatrix} \frac{\omega+p}{m_{qT}} + 1 \\ \frac{\omega+p}{m_{qT}} - 1 \end{pmatrix} \\ & \times \{T_{l(M_0;M)} \exp\} \left\{ ig_{2D} T_a \int dx^\mu A_\mu^a \right\}, \end{aligned} \quad (16)$$

where  $a(p, \omega)$  is an operator of creation of any particle (a particle or antiparticle).

## (ii) Fermion current

$$J_a^\mu = g_{2D} \text{Tr} \{ \bar{\Psi}(x) \gamma^\mu T_a \Psi(x') \}; \quad x' \rightarrow x. \quad (17)$$

$$J_a^\mu(x) = \frac{g_{2D}^2}{2\pi} N_f \left[ A_a^\mu - \partial^\mu \frac{1}{\partial^\lambda \partial_\lambda} \partial_\nu A_a^\nu \right] \quad (18)$$

### Gauge transformation

$$\delta J_a^\mu = \varepsilon_b f_a^{bc} J_c^\mu \quad (19)$$

$$\delta A_a^\mu = \varepsilon_b f_a^{bc} A_c^\mu$$

$$\delta \mathcal{L}_{m_g T} = 0.$$





# 7. Equation of motion for the 2D Gauge Fields

(In the Lorentz gauge)

$$\begin{aligned} \mathcal{A}(2D) = \int d^2X \left\{ \frac{i}{2} [\bar{\Psi}(2D, X) \gamma^k(2D) \partial_k(2D) \Psi(2D, X) - \bar{\Psi}(2D, X) m_{gT} \Psi(X)] \right. \\ \left. - \frac{i}{2} [\bar{\Psi}(2D, X) \gamma^k(2D) \overleftarrow{\partial}_k(2D) \Psi(2D, X) + \bar{\Psi}(2D, X) m_{gT} \Psi(X)] \right. \\ \left. - \frac{1}{4} F_{\mu\nu}^a(2D) F_a^{\mu\nu}(2D) + \frac{1}{2} M_{gT}^2 A_\nu^a(2D) A_a^\nu(2D) \right\}, \end{aligned} \quad (20)$$

$$\begin{aligned} M_{gT}^2 &= \frac{1}{2} \int dx^1 dx^2 \left[ \{\partial_1[|G_1(\mathbf{r}_\perp)|^2 + |G_2(\mathbf{r}_\perp)|^2]^{1/2}\}^2 + \{\partial_2[|G_1(\mathbf{r}_\perp)|^2 + |G_2(\mathbf{r}_\perp)|^2]^{1/2}\}^2 \right] + \frac{g_{2D}^2 N_f}{2\pi} \\ &\equiv m_{gT}^2 + m_{gfT}^2 \geq 0. \end{aligned} \quad (21)$$

(i) The motion equation

$$\square A_a^\nu(2D, X) = M_{gT}^2 A_a^\nu(2D, X) \quad (22)$$

(ii) Its solution

$$A_a^\nu(2D, X) = \sum_k \frac{e_a^\nu M_{qT}}{\sqrt{(k^2 + M_{qT}^2)^3}} \left\{ \exp(-ikX) b_a(k, \nu) + \exp(+ikX) \bar{b}_a^\dagger(k, \nu) \right\} \quad (23)$$
$$k^\mu = (k^0; \mathbf{k}), \quad E(k) \equiv k^0 = +\sqrt{k^2 + M_{qT}^2}.$$

(iii) Colorless particles

$$A_{\text{color-singlet}}^\nu = \frac{1}{\sqrt{8}} \sum_a A_a^\nu |8, a\rangle \quad (24)$$

$$\square A_{\text{color-singlet}}^\nu(2D, X) = M_{gT}^2 A_{\text{color-singlet}}^\nu(2D, X).$$

## 8. Transverse motion in a tube

$$\mathcal{F} = \mathcal{A}(4D) + \frac{\lambda}{2} \int dx^1 dx^2 (|G_1(\vec{r}_\perp)|^2 + |G_2(\vec{r}_\perp)|^2) \int dx^0 dx^3 (\bar{\psi}(2D, X) \psi(2D, X)) \quad (25)$$

$$\begin{aligned} (p_1 + ip_2)G_1(\vec{r}_\perp) &= (m(\vec{r}_\perp) + \lambda)G_2(\vec{r}_\perp), \\ (p_1 - ip_2)G_2(\vec{r}_\perp) &= (\lambda - m(\vec{r}_\perp))G_1(\vec{r}_\perp), \\ (p_1 + ip_2)G_2^*(\vec{r}_\perp) &= (m(\vec{r}_\perp) - \lambda)G_1^*(\vec{r}_\perp), \\ (p_1 - ip_2)G_1^*(\vec{r}_\perp) &= -(m(\vec{r}_\perp) + \lambda)G_2^*(\vec{r}_\perp). \end{aligned} \quad (26)$$

$$\lambda = \lambda^* = m_q T \quad (!) \quad (27)$$

## 9.The Gaussian tube

$$G_1(\vec{r}_\perp) = e^{-i\frac{g}{2}} \frac{e^{-\frac{r_\perp^2}{2R_T^2}}}{R_T \sqrt{2\pi}} ; \quad G_2(\vec{r}_\perp) = -i e^{+i\frac{g}{2}} \frac{e^{-\frac{r_\perp^2}{2R_T^2}}}{R_T \sqrt{2\pi}}$$

$$g_{2D} = \frac{g_{4D}}{R_T} \sqrt{\frac{2}{9\pi}}$$

$$m_{fgT}^2 = \frac{2 g_{4D}^2}{9\pi R_T^2}$$

$$m_{gT}^2 = 2 R_T^{-2}$$

$$m_{gT} = \frac{g_{4D}}{R_T} \sqrt{\frac{2}{g_{4D}^2} + \frac{2}{9\pi}}$$

# 10.The confinement potential

$$m(\vec{r}_\perp) = \zeta r_\perp, \quad \zeta > 0$$

$$\begin{pmatrix} H_0 & +i\zeta e^{+i\varphi} \\ -i\zeta e^{-i\varphi} & H_0 \end{pmatrix} \begin{pmatrix} G_+(r_\perp) \\ G_-(r_\perp) \end{pmatrix} = \lambda^2 \begin{pmatrix} G_+(r_\perp) \\ G_-(r_\perp) \end{pmatrix};$$

$$H_0 = p_\perp^2 + m^2(r_\perp).$$

$$G_+ \Rightarrow \uparrow, \quad G_- \Rightarrow \downarrow$$

$$J, \quad \sigma = \pm 1/2$$

## J=0, the ground state

$$\xi = \zeta r_{\perp}^2$$

$$\lambda^2 = 4\zeta \left( n_r + \frac{1}{2} \right); \quad n_r = 0, 1, 2, \dots$$

$$G_- = \sqrt{\frac{\zeta n_r!}{2\Gamma(n_r + 3/2)}} \frac{e^{(-i\varphi/2)}}{\sqrt{2\pi}} \xi^{1/4} e^{-\xi/2} \left( L_{n_r}^{1/2}(\xi) + \frac{\sqrt{n_r + 1/2}}{\xi^{1/2}} L_{n_r}^{-1/2}(\xi) \right)$$

$$G_+ = -i \sqrt{\frac{\zeta n_r!}{2\Gamma(n_r + 3/2)}} \frac{e^{(+i\varphi/2)}}{\sqrt{2\pi}} \xi^{1/4} e^{-\xi/2} \left( L_{n_r}^{1/2}(\xi) - \frac{\sqrt{n_r + 1/2}}{\xi^{1/2}} L_{n_r}^{-1/2}(\xi) \right)$$

$$\lambda^2 = 4\zeta (n_r + 1);$$

$$G_- = \sqrt{\frac{\zeta n_r!}{2\Gamma(n_r + 3/2)}} \frac{e^{(-i\varphi/2)}}{\sqrt{2\pi}} \xi^{1/4} e^{-\xi/2} \left( L_{n_r}^{1/2}(\xi) + \frac{\sqrt{n_r + 1}}{\xi^{1/2}} L_{n_r+1}^{-1/2}(\xi) \right)$$

$$G_+ = -i \sqrt{\frac{\zeta n_r!}{2\Gamma(n_r + 3/2)}} \frac{e^{(+i\varphi/2)}}{\sqrt{2\pi}} \xi^{1/4} e^{-\xi/2} \left( L_{n_r}^{1/2}(\xi) - \frac{\sqrt{n_r + 1}}{\xi^{1/2}} L_{n_r+1}^{-1/2}(\xi) \right)$$

$$(|\Lambda| \gg 1)$$

$$\lambda^2 = 4\zeta \left( n_r + \frac{|\Lambda|}{2} \right);$$

$$G_- = \sqrt{\frac{\zeta n_r!}{2\Gamma(n_r + |\Lambda|/2)}} \frac{e^{(-i\varphi|\Lambda|)}}{\sqrt{2\pi}} \xi^{|\Lambda|/2} e^{-\xi/2} \left( L_{n_r}^{|\Lambda|/2}(\xi) \right)$$

$$G_+ = -i \sqrt{\frac{\zeta n_r!}{2\Gamma(n_r + |\Lambda|/2)}} \frac{e^{(-i\varphi|\Lambda|)}}{\sqrt{2\pi}} \xi^{|\Lambda|/2} e^{-\xi/2} \left( L_{n_r}^{|\Lambda|/2}(\xi) \right)$$

# Spectra

$$(\Lambda = 0)$$

$$n_r = 7/2 \text{ _____}$$

$$n_r = 5/2 \text{ _____}$$

$$n_r = 3/2 \text{ _____}$$

$$n_r = 1/2 \text{ _____}$$

$$n_r = 3 \text{ _____}$$

$$n_r = 2 \text{ _____}$$

$$n_r = 1 \text{ _____}$$

$$(|\Lambda| \gg 1)$$

$$N = 3 + |\Lambda|/2 \text{ _____}$$

$$N = 2 + |\Lambda|/2 \text{ _____}$$

$$N = 1 + |\Lambda|/2 \text{ _____}$$



# 10.Conclusion

1. The 4D  $\rightarrow$  2D compactification is realized in terms of the action principle in the SU(N) gauge field theory.
2. The exact gauge invariant 2D-Lagrangian is derived. On a basis of the obtained Lagrangian, the gauge invariant fermion current and the transverse mass of a fermion are calculated.
3. The derived current is found to be proportional to the amplitude of the gauge field, similar to the case of 2D QED (J.Schwinger,1962).
4. The obtained mass for SU(N) field theory depends strongly on the 2D coupling constant, number of flavors, as well as on the transverse mass of the fermion.
5. The results allow us to calculate  $p_T$  - distribution of hadrons produced in collisions of heavy ions of high energies.

# 12.References

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and so on...

Thank you very much for your  
attention!