Asymptotic Energy Dependence of Hadronic Total Cross Sections from Lattice QCD

Enrico Meggiolaro

Dipartimento di Fisica "Enrico Fermi", Università di Pisa,
and I.N.F.N., Sezione di Pisa

Rencontres de Moriond 2013
QCD and High Energy Interactions
La Thuile, March 9th–16th, 2013

- M. Giordano, E. Meggiolaro, N. Moretti, JHEP **09** (2012) 031
- M. Giordano, E. Meggiolaro, work in progress . . .

Outline

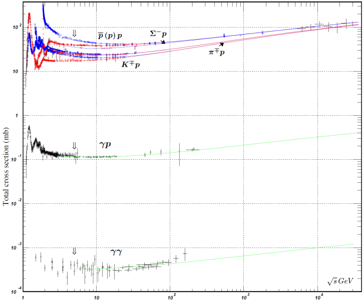
1. Hadronic scattering: theory and experiment
   - Experimental overview
   - Nonperturbative approach to soft high-energy scattering: the Wilson-loop correlator
   - Lattice data vs. analytical models
2. Hadronic total cross sections from lattice QCD
   - A new analysis of Wilson-loop correlators
   - How a Froissart-like total cross section can be obtained
   - New analysis of the lattice data
3. Conclusions and outlook

1. Hadronic scattering: theory and experiment
   - Experimental overview
   - Nonperturbative approach to soft high-energy scattering: the Wilson-loop correlator
   - Lattice data vs. analytical models

2. Hadronic total cross sections from lattice QCD
   - A new analysis of Wilson-loop correlators
   - How a Froissart-like total cross section can be obtained
   - New analysis of the lattice data

3. Conclusions and outlook

Experimental overview



(figure taken from [PDG, 2010])

Experimental data support

$$\sigma_{tot}(s) \sim B \log^2 s$$

with **universal** $B \simeq 0.3\,\mathrm{mb}$, independent of the colliding hadrons (up to $\sqrt{s} = 7\,\mathrm{TeV}$ [TOTEM, 2011])

Consistent with Froissart bound [Froissart, 1961] (unitarity + mass gap)

$$\sigma_{tot}^{(hh)}(s) \leq \frac{\pi}{m_\pi^2} \log^2\left(\frac{s}{s_0}\right)$$

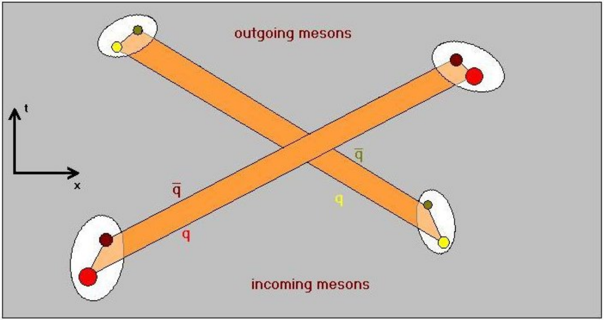
Arguments supporting universality of B (eikonal unitarisation, *Color Glass Condensate*, ...), but the situation is still unsettled ...

Nonperturbative approach to soft high-energy scattering

- Study of the problem of hadron-hadron *soft* high-energy elastic scattering ($\sqrt{|t|} \lesssim 1\,\mathrm{GeV} \ll \sqrt{s}$) $\Rightarrow \sigma_{\rm tot}$ *via optical theorem*:

$$\sigma^{(hh)}_{\rm tot} \sim \frac{1}{s} \mathrm{Im} \mathcal{M}_{(hh)}(s, t = 0)$$

- $\sqrt{|t|} \lesssim 1\,\mathrm{GeV} \Rightarrow$ a nonperturbative (NP) approach is necessary
- Nachtmann's functional-integral approach [Nachtmann, 1991]



Nonperturbative approach to soft high-energy scattering
Meson-meson elastic scattering

- Elastic meson-meson scattering amplitude from dipole-dipole scattering [Dosch *et al.*, 1994]

$$\mathcal{M}_{(hh)}(s,t) = \langle\!\langle \mathcal{M}_{(dd)}(s,t;1,2) \rangle\!\rangle \,,$$

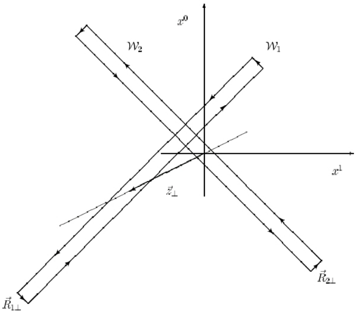
$$\langle\!\langle f(1,2) \rangle\!\rangle \equiv \int d^2\vec{R}_{1\perp} |\psi_1(\vec{R}_{1\perp})|^2 \int d^2\vec{R}_{2\perp} |\psi_2(\vec{R}_{2\perp})|^2 f(\vec{R}_{1\perp},\vec{R}_{2\perp})$$

$$\mathcal{M}_{(dd)}(s,t=-|\vec{q}_\perp|^2;\vec{R}_{1\perp},\vec{R}_{2\perp}) = -i\ 2s \int d^2\vec{z}_\perp e^{i\vec{q}_\perp\cdot\vec{z}_\perp} \mathcal{C}_M(\chi \simeq \log\left(\frac{s}{m^2}\right);\vec{z}_\perp,\vec{R}_{1\perp},\vec{R}_{2\perp})$$

- Wilson-loop correlation function

$$\mathcal{G}_M(\chi;T;\vec{z}_\perp,\vec{R}_{1\perp},\vec{R}_{2\perp}) = \frac{\langle \mathcal{W}_{\mathcal{C}_1}\mathcal{W}_{\mathcal{C}_2}\rangle}{\langle \mathcal{W}_{\mathcal{C}_1}\rangle\langle \mathcal{W}_{\mathcal{C}_2}\rangle} - 1, \quad \mathcal{C}_M \equiv \lim_{T\to\infty} \mathcal{G}_M$$

$$\mathcal{W}_{\mathcal{C}}[A] \equiv \frac{1}{N_c} \mathrm{tr}\,\mathcal{P}\exp\{-ig\oint_{\mathcal{C}} A_\mu(X)dX^\mu\}$$



$$p_1 \cdot p_2 = m^2 \cosh \chi$$

$$\chi \underset{s\to\infty}{\simeq} \log\left(\frac{s}{m^2}\right)$$

- Longitudinal sides $\Rightarrow q - \bar{q}$ trajectories ($\tau \in [-T, T]$)

$$\mathcal{C}_1 \to X^{(\pm 1)}(\tau) = z + \frac{p_1}{m}\tau \pm \frac{R_1}{2}$$

$$\mathcal{C}_2 \to X^{(\pm 2)}(\tau) = \frac{p_2}{m}\tau \pm \frac{R_2}{2}$$

$$\frac{p_{1,2}}{m} = \left(\cosh\frac{\chi}{2}, \pm\sinh\frac{\chi}{2}, \vec{0}_\perp\right)$$

- Closed at $\tau = \pm T$ by straight "links" in the transverse plane

$$R_i = (0, 0, \vec{R}_{i\perp}) \quad z = (0, 0, \vec{z}_\perp)$$

- Correlation functions in Minkowski space can be reconstructed from Euclidean correlation functions by proper **analytic-continuation relations** [EM, 1997–2007; Giordano, EM, 2006, 2009]:

$$\begin{aligned} \mathcal{G}_M(\chi; T) &= \mathcal{G}_E(\theta \to -i\chi; T \to iT) \\ \mathcal{C}_M(\chi) &= \mathcal{C}_E(\theta \to -i\chi) \end{aligned}$$

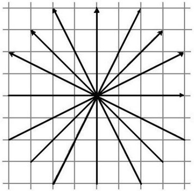
where $\theta \in (0, \pi)$ is the angle formed by the two trajectories in Euclidean space, i.e.:

$$\frac{p_{1,2E}}{m} = \left(\cos\frac{\theta}{2}, \pm \sin\frac{\theta}{2}, \vec{0}_\perp \right), \quad p_{1E} \cdot p_{2E} = m^2 \cos\theta,$$

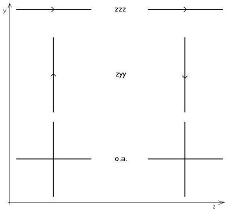
- Opens the way to investigations with nonperturbative techniques:
  - Instanton Liquid Model [Shuryak, Zahed, 2000; Giordano, EM, 2010]
  - AdS/CFT Correspondence [Janik, Peschanski, 2000]
  - Stochastic Vacuum Model [Shoshi, Steffen, Dosch, Pirner, 2003]
  - Lattice Gauge Theory [Giordano, EM, 2008; 2010]

Lattice data: setup

- longitudinal plane



- transverse plane



- Wilson action for $SU(3)$ pure-gauge theory (*quenched* QCD)
  $S = \beta \sum_{n,\mu<\nu} \{1 - \frac{1}{3}\mathrm{Re}\mathrm{Tr}\, U_{\mu\nu}(n)\}$
  - 16^4 hypercubic lattice, periodic b.c.
  - $\beta \equiv 6/g^2 = 6.0 \Rightarrow a \simeq 0.1\,\mathrm{fm}$
- Parameters of the correlators:
  - angles: $\cot\theta = 0, \pm 1/2, \pm 1, \pm 2$
  - transverse side $= 1a$
  - transverse distance da, $d = 0, 1, 2$
- Transverse-plane configurations:
  - "*zzz*" : $\vec{d}_\perp \parallel \vec{r}_{1\perp} \parallel \vec{r}_{2\perp}$
  - "*zyy*" : $\vec{d}_\perp \perp \vec{r}_{1\perp} \parallel \vec{r}_{2\perp}$
  - "*ave*" : average over orientations (relevant to meson-meson scatt.)
- The limit $T \to \infty$ is performed using the longest available loops

Lattice data vs. analytical models

Analytical results

Stochastic Vacuum Model (SVM)	$\mathcal{C}_E = \frac{2}{3} e^{-\frac{1}{3} K_{\rm SVM} \cot\theta} + \frac{1}{3} e^{\frac{2}{3} K_{\rm SVM} \cot\theta} - 1$
Perturbation Theory (PT)	$\mathcal{C}_E = K_{\rm PT} \cot^2\theta$
Instanton Liquid Model (ILM)	$\mathcal{C}_E = \frac{K_{\rm ILM}}{\sin\theta}$
ILM + PT (ILMp)	$\mathcal{C}_E = \frac{K_{\rm ILMp1}}{\sin\theta} + K_{\rm ILMp2} \cot^2\theta$
AdS/CFT correspondence	$\mathcal{C}_E = e^{\frac{K_{\rm AdS}}{\sin\theta} + K'_{\rm AdS} \cot\theta + K''_{\rm AdS} \cos\theta \cot\theta} - 1$

Summary of the $\chi^2_{\rm d.o.f.}$ for a best fit with the indicated function:

$\chi^2_{\rm d.o.f.}$	$d=0$		$d=1$			$d=2$		
	ave	*zyy/zzz*	*ave*	*zyy*	*zzz*	*ave*	*zyy*	*zzz*
SVM	–	51	–	12	16	–	2.2	1.5
PT	34	53	13	13	16	4.5	2.2	1.5
ILM	94	114	45	15	14	1.45	0.35	0.45
ILMp	9.4	20	1.8	0.92	0.54	0.19	0.12	0.13
AdS/CFT	–	40	–	0.63	1	–	0.065	0.14

1. Hadronic scattering: theory and experiment
   - Experimental overview
   - Nonperturbative approach to soft high-energy scattering: the Wilson-loop correlator
   - Lattice data vs. analytical models

2. Hadronic total cross sections from lattice QCD
   - A new analysis of Wilson-loop correlators
   - How a Froissart-like total cross section can be obtained
   - New analysis of the lattice data

3. Conclusions and outlook

A new analysis of Wilson-loop correlators

Preliminary comments

- None of the given models compares satisfactorily with the lattice data
- Different predictions for total cross sections, not in agreement with experiment
  - SVM, PT, ILM, ILMp $\Rightarrow \sigma_{\rm tot} \underset{s\to\infty}{\longrightarrow} \text{constant}$
  - AdS/CFT $\Rightarrow \sigma_{\rm tot} \underset{s\to\infty}{\sim} s^{\frac{1}{3}}$ (no mass gap in a CFT $\Rightarrow$ no Froissart bound) [Giordano, Peschanski, 2010]
- Fits to more general functions can be performed, but care is needed, because of the analytic continuation: admissible fitting functions must be constrained by physical requirements, first of all **unitarity** ...
- Therefore, we are going to introduce, and partially justify, *new* parameterisations of the correlation function that, in order:
  1. fit well the data
  2. satisfy the unitarity condition after analytic continuation
  3. lead to total cross sections rising as $B \log^2 s$ in the high-energy limit (as experimental data seem to suggest)

Exponential form of the correlator and unitarity constraint

Assumption:

$$\mathcal{C}_E = \exp\{K_E\} - 1$$

with K_E real (as $\mathcal{C}_E$ is real)

Well justified assumption:

- large-N_c, $\mathcal{C}_E \sim \mathcal{O}(1/N_c^2)$ so $\mathcal{C}_E + 1 \geq 0$ certainly true at large-N_c
- satisfied by all the known models (SVM, ILM, AdS/CFT, PT, ...)
- $\mathcal{C}_E + 1 \to 1$ at large $|\vec{z}_\perp|$, so certainly true at large impact parameter
- confirmed by lattice data

Minkowskian correlator after analytic continuation : $\mathcal{C}_M = \exp\{K_M\} - 1$

Unitarity constraint: $\mathcal{M}_{(hh)}(s,t) = -i\ 2s \int d^2\vec{z}_\perp e^{i\vec{q}_\perp \cdot \vec{z}_\perp} A(s, |\vec{z}_\perp|)$

$$|A(s,|\vec{z}_\perp|) + 1| \leq 1 \ , \quad \text{where : } \ A(s,|\vec{z}_\perp|) = \langle\langle \mathcal{C}_M(\chi; \vec{z}_\perp, 1, 2) \rangle\rangle$$

Sufficient condition $\left(\langle\langle f(1,2)\rangle\rangle \equiv \int d^2\vec{R}_{1\perp} |\psi_1(\vec{R}_{1\perp})|^2 \int d^2\vec{R}_{2\perp} |\psi_2(\vec{R}_{2\perp})|^2 f(\vec{R}_{1\perp}, \vec{R}_{2\perp})\right)$:

$$|\mathcal{C}_M(\chi; \vec{z}_\perp, \vec{R}_{1\perp}, \vec{R}_{2\perp}) + 1| \leq 1 \ , \quad \text{i.e.,} \quad \mathrm{Re}\, K_M \leq 0 \ , \quad \forall \vec{z}_\perp, \vec{R}_{1\perp}, \vec{R}_{2\perp}$$

Large-distance behaviour of K_E

In a confining theory such as QCD, one expects at large $|\vec{z}_\perp|$

$$\mathcal{C}_E \sim \left(\sum\right) e^{-\mu|\vec{z}_\perp|}$$

with mass scales μ related to the masses of the particles (*glueballs*?) exchanged between the two loops:

- $M_{0^{++}} \simeq 1.7$ GeV, $M_{2^{++}} \simeq 2.6$ GeV, $M_{1^{+-}} \simeq 3$ GeV, ...
  [Morningstar & Peardon, 1999; Gregory *et al.*, 2012]

Therefore, one also expects

$$K_E \sim \left(\sum\right) e^{-\mu|\vec{z}_\perp|}$$

Instead, for a non-confining (conformal) field theory, different behaviours like powers of $1/|\vec{z}_\perp|$ are typical ...

How a Froissart-like total cross section can be obtained

1. Assume that after analytic continuation the leading term of the Minkowskian correlator is

$$\mathcal{C}_M = \exp\{K_M\} - 1 \sim \exp\left(i\,\beta\,f(\chi)\,e^{-\mu|\vec{z}_\perp|}\right) - 1 \qquad (\star)$$

with $\beta = \beta(\vec{R}_{1\perp}, \vec{R}_{2\perp})$ and $f(\chi) \equiv e^{\eta(\chi)}$ *real* function $\to +\infty$ for $\chi \to +\infty$.
The unitarity condition implies: $\mathrm{Re} K_M \le 0 \Leftrightarrow \mathrm{Im}\,\beta \ge 0$

2. Optical theorem

$$\sigma_{\mathrm{tot}}^{(hh)} \sim \frac{4\pi}{\mu^2}\mathrm{Re}\langle\!\langle J(\eta,\beta)\rangle\!\rangle\,, \qquad J(\eta,\beta) \equiv \int_0^\infty dy\, y[1 - \exp(i\beta e^{\eta - y})]$$

Expression $(\star)$ holds only for large $|\vec{z}_\perp| \gtrsim z_0$, but can be extended to $|\vec{z}_\perp| = 0$, the difference being a constant in χ due to the unitarity bound

How a Froissart-like total cross section can be obtained

3. Setting $z = -i\beta e^{\eta}$

$$\frac{\partial J(\eta,\beta)}{\partial\eta} = -\sum_{n=1}^{\infty}\frac{(-z)^n}{n!n} = E_1(z) + \log(z) + \gamma\,, \quad \text{for } |\arg(z)| < \pi$$

$E_1(z)$: Schlömilch exponential integral, γ: Euler-Mascheroni constant
$E_1(z) \sim e^{-z}/z$ at large $|z|$, for $\mathrm{Re}\, z \geq 0 \Leftrightarrow \mathrm{Im}\beta \geq 0$

In the large-χ limit, we have

$$\sigma_{\rm tot}^{(hh)} \sim \frac{4\pi}{\mu^2}\langle\!\langle \frac{1}{2}\eta^2 + \eta(\log|\beta| + \gamma) + \ldots\rangle\!\rangle$$

4. Choosing $f(\chi) \equiv e^{\eta} = \chi^p e^{n\chi}$, i.e., $\eta = n\chi + p\log\chi$ $\left[\chi \simeq \log(\frac{s}{m_1 m_2})\right]$

$$\sigma_{\rm tot}^{(hh)} \sim B\log^2 s\,, \qquad B = \frac{2\pi n^2}{\mu^2}$$

Giordano, EM, Moretti, JHEP **09** (2012) 031

How a Froissart-like total cross section can be obtained

Froissart-like behaviour

$$\sigma_{\rm tot}^{(hh)} \sim B \log^2 s\,, \quad B = \frac{2\pi n^2}{\mu^2} \qquad \text{if} \quad K_M \sim i\,\beta\,\chi^p e^{n\chi}\, e^{-\mu|\vec{z}_\perp|}$$

- B is **universal**, independent of the wave functions and of the masses of the mesons: $\chi \simeq \log(\frac{s}{m_1 m_2}) = \log(\frac{s}{s_0}) + \log(\frac{s_0}{m_1 m_2})$
- B is unaffected by the small-$|\vec{z}_\perp|$ behaviour

Extensions [Giordano, EM, work in progress]:

- for $K_M \sim i \sum_k \beta_k \chi^{p_k} e^{n_k \chi}\, |\vec{z}_\perp|^\alpha e^{-\mu_k|\vec{z}_\perp|}$, same behaviour with

$$B \to 2\pi \max_k \left(\frac{n_k}{\mu_k}\right)^2$$

- One expects that particles with *mass* M and *spin* J, exchanged between the two loops, contribute with $\mu = M$ and $n = J - 1$ $(e^{n\chi} \to s^{J-1})$ ...

New analysis of the lattice data

Try to fit the data with functional forms that satisfy unitarity after analytic continuation and that lead to rising total cross sections

- Use averaged correlators, that are "closer" to the meson-meson amplitude in impact-parameter space

$$\mathcal{C}^{ave}_{E,M} = <\mathcal{C}_{E,M}> = <\exp\{K_{E,M}\}> \ -1 = \exp\{K^{ave}_{E,M}\} - 1$$

$< \bullet >= \int d^2\hat{R}_{1\perp} \int d^2\hat{R}_{2\perp} \bullet$ is a positive and normalised measure

- K^{ave}_M and K^{ave}_E related by analytic continuation: $K^{ave}_M(\chi) = K^{ave}_E(\theta = -i\chi)$
- Unitarity constraint: $\mathrm{Re}\, K^{ave}_M \leq 0$
- By construction $\mathcal{C}^{ave}_E(\pi - \theta) = \mathcal{C}^{ave}_E(\theta)$: only *crossing symmetric* (i.e., C-even $\Rightarrow$ *Pomeron*) contribution

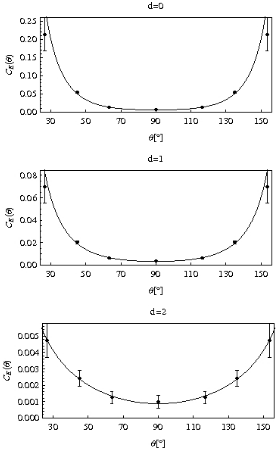
Parameterisation I

First strategy: combine known QCD results and variations thereof

Example: exponentiate two-gluon exchange and one-instanton contribution, plus a term that can yield a rising cross section

$$K_E = \frac{K_1}{\sin\theta} + K_2 \cot^2\theta + K_3 \cos\theta \cot\theta$$

$$K_M = i\frac{K_1}{\sinh\chi} + iK_3 \cosh\chi \coth\chi - K_2 \coth^2\chi$$



- Unitarity condition: $K_2 \geq 0$ (satisfied within the errors)
- Leading term: $K_3 \cos\theta \cot\theta \to iK_3 \frac{e^\chi}{2}$ $(n = 1 \to J = 2)$

fit parameters

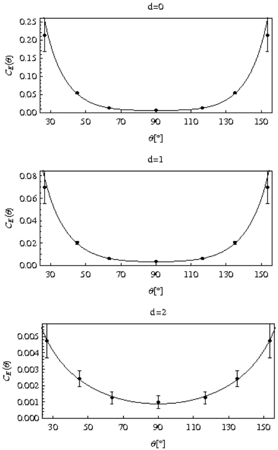
Parameterisation II

Second strategy: adapt to QCD results obtained in related models

Example: AdS/CFT expression, plus $\theta \cot\theta$ term to make the expression crossing symmetric (i.e., C-even)

$$K_E = \frac{K_1}{\sin\theta} + K_2\left(\tfrac{\pi}{2} - \theta\right)\cot\theta + K_3 \cos\theta \cot\theta$$

$$K_M = i\frac{K_1}{\sinh\chi} + iK_2 \tfrac{\pi}{2} \coth\chi + iK_3 \cosh\chi \coth\chi - \chi K_2 \coth\chi$$



- Unitarity condition: $K_2 \geq 0$ (satisfied within the errors)
- Leading term: $K_3 \cos\theta \cot\theta \to iK_3 \frac{e^\chi}{2}$ $(n = 1 \to J = 2)$

fit parameters

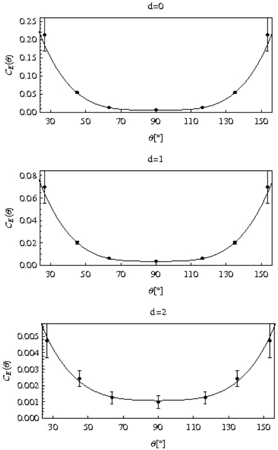
Our best parameterisation:

Exponentiate one-instanton contribution, plus a term that can yield a rising cross section

$$K_E = \frac{K_1}{\sin\theta} + K_2(\tfrac{\pi}{2} - \theta)^3 \cos\theta$$

$$K_M = i\frac{K_1}{\sinh\chi} + iK_2 \cosh\chi(\tfrac{3}{4}\pi^2\chi - \chi^3)$$

$$+ K_2 \cosh\chi(\tfrac{\pi^3}{8} - \tfrac{3}{2}\pi\chi^2)$$



- Unitarity condition: $K_2 \geq 0$ (satisfied within the errors)
- Leading term: $K_2(\frac{\pi}{2} - \theta)^3 \cos\theta \to -iK_2\chi^3 \frac{e^\chi}{2}$ $(n = 1 \to J = 2)$

fit parameters

Summary of the $\chi^2_{\mathrm{d.o.f.}}$ for a best fit with the indicated function:

$\chi^2_{\mathrm{d.o.f.}}$	$d=0$		$d=1$			$d=2$		
	ave	*zyy/zzz*	*ave*	*zyy*	*zzz*	*ave*	*zyy*	*zzz*
SVM	–	51	–	12	16	–	2.2	1.5
PT	34	53	13	13	16	4.5	2.2	1.5
ILM	94	114	45	15	14	1.45	0.35	0.45
ILMp	9.4	20	1.8	0.92	0.54	0.19	0.12	0.13
AdS/CFT	–	40	–	0.63	1	–	0.065	0.14
Corr 1	2.81	12.9	1.25	0.66	0.34	0.05	0.07	0.16
Corr 2	0.55	7.88	0.31	0.55	0.27	0.05	0.07	0.15
Corr 3	0.17	3.89	0.11	0.77	0.16	0.10	0.10	0.12

back 1 back 2 back 3

Corr 1	d=0	d=1	d=2
K_1	$5.85(42) \cdot 10^{-3}$	$3.07(37) \cdot 10^{-3}$	$8.7(3.1) \cdot 10^{-4}$
K_2	$9.60(98) \cdot 10^{-2}$	$2.44(49) \cdot 10^{-2}$	$-5.3(84.5) \cdot 10^{-5}$
K_3	$-7.8(1.3) \cdot 10^{-2}$	$-1.37(72) \cdot 10^{-2}$	$1.7(1.9) \cdot 10^{-3}$
$\chi^2_{\text{d.o.f.}}$	2.81	1.25	0.05
Corr 2	d=0	d=1	d=2
K_1	$6.03(42) \cdot 10^{-3}$	$3.26(38) \cdot 10^{-3}$	$8.7(3.2) \cdot 10^{-4}$
K_2	$4.63(46) \cdot 10^{-1}$	$1.33(25) \cdot 10^{-1}$	$-1.2(54.2) \cdot 10^{-4}$
K_3	$-4.54(50) \cdot 10^{-1}$	$-1.26(28) \cdot 10^{-1}$	$1.7(6.7) \cdot 10^{-3}$
$\chi^2_{\text{d.o.f.}}$	0.55	0.31	0.05
Corr 3	d=0	d=1	d=2
K_1	$6.02(36) \cdot 10^{-3}$	$3.46(29) \cdot 10^{-3}$	$1.07(20) \cdot 10^{-3}$
K_2	$1.29(5) \cdot 10^{-1}$	$4.47(27) \cdot 10^{-2}$	$2.11(73) \cdot 10^{-3}$
$\chi^2_{\text{d.o.f.}}$	0.17	0.11	0.10

Table: Parameters (with their errors) for the Correlators 1, 2 , and 3, obtained from best fits to the averaged lattice data, and the corresponding $\chi^2_{\text{d.o.f.}}$, for the transverse distances $d = 0, 1, 2$

New analysis of the lattice data

Remarks

- In the three cases, the resulting total cross section is **universal**

$$\sigma_{\rm tot}^{(hh)} \sim B \log^2 s$$

- Estimate of B through a fit of the coefficient of the leading term with an exponential: fair agreement with exp. value $B_{\rm exp} \simeq 0.3$ mb

	μ (GeV)	$\lambda = \frac{1}{\mu}$ (fm)	$B = \frac{2\pi}{\mu^2}$ (mb)
Corr 1	4.64(2.38)	$0.042^{+0.045}_{-0.014}$	$0.113^{+0.364}_{-0.037}$
Corr 2	3.79(1.46)	$0.052^{+0.032}_{-0.014}$	$0.170^{+0.277}_{-0.081}$
Corr 3	3.18(98)	$0.062^{+0.028}_{-0.015}$	$0.245^{+0.263}_{-0.100}$

- Result applies directly to meson-meson scattering, experimental data mainly available for baryon-baryon and meson-baryon
  - prediction of universality for meson-meson scattering
  - Wilson-loop approach can be extended to baryons by adopting a quark-diquark picture: analysis carries over unchanged
- "Quenched" data, but $\sigma_{\rm tot}^{(hh)}$ expected to depend on bosonic sector of QCD $\Rightarrow$ results should not change much with dynamical fermions

Conclusions and outlook

- We have provided a framework to investigate the issue of total cross sections on the lattice by means of numerical simulations
- We have found parameterisations of the lattice data that yield a good fit and at the same time a Froissart-like total cross section rising as $B \log^2 s$ in the high-energy limit, with a **universal** B
- The comparison of our results with the experiments is rather good, even if the errors are quite large

Open issues [Giordano, EM, work in progress]:

- Which is the relation between the mass-scale μ and $M_{Glueballs}$? Analytical expression of the large impact-parameter behaviour of the correlator ... $\mu_{exp} = \sqrt{2\pi/B_{exp}} \simeq 2.85$ GeV $\simeq M_{2^{++}}$?
- Rigorous formulation in case of baryon-baryon scattering
- Of course, more precise lattice data (on larger lattices, at larger distances, for more angles, ...), possibly including also dynamical fermion effects, would be useful ...

References I

- K. Nakamura *et al.* (Particle Data Group),
  J. Phys. G **37** (2010) 075021
- G. Antchev *et al.* (TOTEM collaboration),
  Eur. Phys. Lett. **96** (2011) 21002
- K. Igi and M. Ishida,
  Phys. Rev. D **66** (2002) 034023
  J.R. Cudell *et al.* (COMPETE collaboration),
  Phys. Rev. D **65** (2002) 074024
  K. Igi and M. Ishida,
  Phys. Lett. B **622** (2005) 286
  M.M. Block and F. Halzen,
  Phys. Rev. D **72** (2005) 036006
  (*Errata*, **72** (2005) 039902)
  M. Ishida and K. Igi,
  Phys. Lett. B **670** (2009) 395
  M. Ishida and K. Igi,
  Prog. Theor. Phys. Suppl. **187** (2011) 297
  M.M. Block and F. Halzen,
  Phys. Rev. Lett. **107** (2011) 212002
- M. Froissart,
  Phys. Rev. **123** (1961) 1053
  A. Martin,
  Il Nuovo Cimento **42A** (1966) 930
  L. Lukaszuk and A. Martin,
  Il Nuovo Cimento **52A** (1967) 122
- I.Y. Pomeranchuk,
  Sov. Phys. JETP 7 (1958) 499
  Zh. Eksp. Teor. Fiz. 34 (1958) 775
- L.L. Jenkovszky, B.V. Struminsky and A.N. Wall,
  Yad. Fiz. **46** (1987) 1519
  J. Finkelstein, H.M. Fried, K. Kang and C.-I. Tang,
  Phys. Lett. B **232** (1989) 257
- G. Basar, D.E. Kharzeev, H.-U. Yee, and I. Zahed,
  Phys. Rev. D **85** (2012) 105005
- E. Ferreiro, E. Iancu, K. Itakura and L. McLerran,
  Nucl. Phys. A **710** (2002) 373
  L. Frankfurt, M. Strikman and M. Zhalov,
  Phys. Lett. B **616** (2005) 59
- O. Nachtmann,
  Ann. Phys. **209** (1991) 436
- H.G. Dosch, E. Ferreira, A. Krämer,
  Phys. Rev. D **50** (1994) 1992
- O. Nachtmann,
  in *Perturbative and Nonperturbative aspects of Quantum Field Theory*, edited by H. Latal and W. Schweiger
  (Springer-Verlag, Berlin, Heidelberg, 1997)
- E.R. Berger and O. Nachtmann,
  Eur. Phys. J. C **7** (1999) 459
- A.I. Shoshi, F.D. Steffen and H.J. Pirner,
  Nucl. Phys. A **709** (2002) 131
- S. Donnachie, G. Dosch, P. Landshoff and O. Nachtmann,
  Pomeron Physics and QCD

References II

- U. Amaldi, M. Jacob, G. Matthiae,
  Ann. Rev. Nucl. Part. Sci. **26** (1976) 385
  R. Castaldi and G. Sanguinetti,
  Ann. Rev. Nucl. Part. Sci. **35** (1985) 351
  M.M. Block and R.N. Cahn,
  Rev. Mod. Phys. **57** (1985) 563
- E. Meggiolaro,
  Z. Phys. C **76** (1997) 523
  Eur. Phys. J. C **4** (1998) 101
  Nucl. Phys. B **625** (2002) 312
- E. Meggiolaro,
  Nucl. Phys. B **707** (2005) 199
- M. Giordano, E. Meggiolaro,
  Phys. Rev. D **74** (2006) 016003
- E. Meggiolaro,
  Phys. Lett. B **651** (2007) 177
- M. Giordano and E. Meggiolaro,
  Phys. Rev. D **78** (2008) 074510
- M. Giordano and E. Meggiolaro,
  Phys. Lett. B **675** (2009) 123
- M. Giordano and E. Meggiolaro,
  Phys. Rev. D **81** (2010) 074022
- M. Giordano, E. Meggiolaro, N. Moretti,
  JHEP **09** (2012) 031
- E. Shuryak, I. Zahed,
  Phys. Rev. D **62** (2000) 085014
- R.A. Janik, R. Peschanski,
  Nucl. Phys. B **565** (2000) 193
- A.I. Shoshi, F.D. Steffen, H.G. Dosch, H.J. Pirner,
  Phys. Rev. D **68** (2003) 074004
- C.J. Morningstar and M. Peardon,
  Phys. Rev. D **60** (1999) 034509
  E. Gregory, A. Irving, B. Lucini, C. McNeile, A. Rago,
  C. Richards and E. Rinaldi,
  JHEP **10** (2012) 170
- J.E. Bresenham,
  IBM Sys. Jour. **4** (1965) 25
- M. Giordano and R. Peschanski,
  JHEP **05** (2010) 037

Experimental overview

- Froissart bound [1961]

$$\sigma_{\rm tot} \le \frac{\pi}{m_\pi^2} \log^2\left(\frac{s}{s_0}\right)$$

- Pomeranchuk theorem [1958]

$$\frac{\sigma_{ab} - \sigma_{\bar{a}b}}{\sigma_{ab}} \underset{s\to\infty}{\to} 0$$

- Current high-energy parameterisation [PDG, 2010]:

$$\sigma^{ab} = Z^{ab} + B\log^2\left(\frac{s}{s_0}\right) + Y_1^{ab}\left(\frac{s_1}{s}\right)^{\eta_1} - Y_2^{ab}\left(\frac{s_1}{s}\right)^{\eta_2}$$

$$\sigma^{\bar{a}b} = Z^{ab} + B\log^2\left(\frac{s}{s_0}\right) + Y_1^{ab}\left(\frac{s_1}{s}\right)^{\eta_1} + Y_2^{ab}\left(\frac{s_1}{s}\right)^{\eta_2}$$

- **Universal** asymptotic behaviour:

[COMPETE collab., 2002; Igi, Ishida, 2002–2011; Block, Halzen, 2005–2011]

$$\sigma_{\rm tot} \underset{s\to\infty}{\sim} B\log^2\left(\frac{s}{s_0}\right) \quad \text{with} \quad B \simeq 0.3 \text{ mb}$$

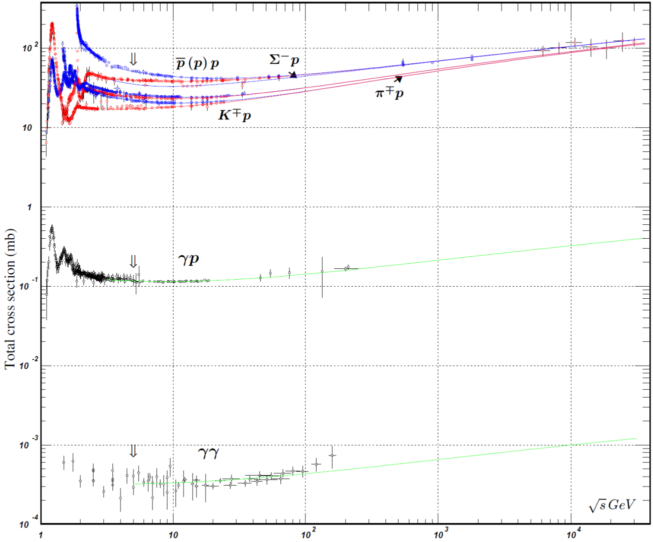


Figure: Summary of hadron-hadron, $p\gamma$ and $\gamma\gamma$ total cross sections as functions of $\sqrt{s}$ with the corresponding plots obtained from a best fit [PDG, 2010]

Nonperturbative approach to soft high-energy scattering

Euclidean formulation of the Wilson-loop correlator

- Nonperturbative techniques available in Euclidean space ⇒ Euclidean correlation functions

$$\mathcal{G}_E(\theta; T; \vec{z}_\perp, \vec{R}_{1\perp}, \vec{R}_{2\perp}) = \frac{\langle \widetilde{\mathcal{W}}_{\widetilde{\mathcal{C}}_1} \widetilde{\mathcal{W}}_{\widetilde{\mathcal{C}}_2} \rangle_E}{\langle \widetilde{\mathcal{W}}_{\widetilde{\mathcal{C}}_1} \rangle_E \langle \widetilde{\mathcal{W}}_{\widetilde{\mathcal{C}}_2} \rangle_E} - 1, \quad \mathcal{C}_E \equiv \lim_{T\to\infty} \mathcal{G}_E$$

$$\widetilde{\mathcal{W}}_{\widetilde{\mathcal{C}}}[A_E] \equiv \frac{1}{N_c} \mathrm{tr}\, \mathcal{P} \exp\{-ig \oint_{\widetilde{\mathcal{C}}} A_{E\mu}(X_E) dX_{E\mu}\}$$

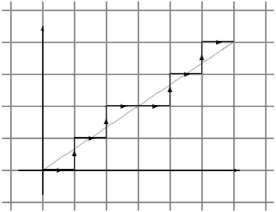
- Paths in Euclidean space (taking X_{E0} to be the "Euclidean time")

$$\widetilde{\mathcal{C}}_1 : X_E^{(\pm 1)}(\tau) = z_E + \frac{p_{1E}}{m}\tau \pm \frac{R_{1E}}{2}, \quad \widetilde{\mathcal{C}}_2 : X_E^{(\pm 2)}(\tau) = \frac{p_{2E}}{m}\tau \pm \frac{R_{2E}}{2}$$

$$\frac{p_{1,2E}}{m} = \left(\cos\frac{\theta}{2}, \pm\sin\frac{\theta}{2}, \vec{0}_\perp\right), \quad R_{iE} = (0, 0, \vec{R}_{i\perp}), \quad z_E = (0, 0, \vec{z}_\perp)$$

$$p_{1E} \cdot p_{2E} = m^2 \cos\theta, \quad \theta \in (0, \pi)$$

Wilson-loop construction on the lattice



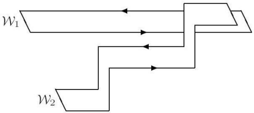
- Rotation invariance breaking $\Rightarrow$ approximation for tilted Wilson loops
- Bresenham prescription [Bresenham, 1965]: lattice path that minimizes the distance from the continuum path
- $\widetilde{\mathcal{W}}_L(\vec{l}_{\|}, \vec{r}_{\perp}; n)$: center in n, sides l ($\|$ plane) and r ($\perp$ plane)

- Lattice Wilson-loop correlators $[d = (0, 0, \vec{d}_{\perp})]$

$$\mathcal{G}_L(\vec{l}_{1\|}, \vec{l}_{2\|}; \vec{d}_{\perp}, \vec{r}_{1\perp}, \vec{r}_{2\perp}) = \frac{\langle \widetilde{\mathcal{W}}_L(\vec{l}_{1\|}, \vec{r}_{1\perp}; d) \widetilde{\mathcal{W}}_L(\vec{l}_{2\|}, \vec{r}_{2\perp}; 0) \rangle_E}{\langle \widetilde{\mathcal{W}}_L(\vec{l}_{1\|}, \vec{r}_{1\perp}; d) \rangle_E \langle \widetilde{\mathcal{W}}_L(\vec{l}_{2\|}, \vec{r}_{2\perp}; 0) \rangle_E} - 1$$

- Rotation invariance restored in the continuum limit $[2L_i = |\vec{l}_{i\|}|]$

$$\mathcal{G}_L(\vec{l}_{1\|}, \vec{l}_{2\|}; \vec{d}_{\perp}, \vec{r}_{1\perp}, \vec{r}_{2\perp}) \underset{a \to 0}{\simeq} \mathcal{G}_E(\theta; aL_1, aL_2; a\vec{d}_{\perp}, a\vec{r}_{1\perp}, a\vec{r}_{2\perp}) + \mathcal{O}(a)$$



- Rotation invariance breaking $\Rightarrow$ approximation for tilted Wilson loops
- Bresenham prescription [Bresenham, 1965]: lattice path that minimizes the distance from the continuum path
- $\widetilde{\mathcal{W}}_L(\vec{l}_{\parallel}, \vec{r}_{\perp}; n)$: center in n, sides l ($\parallel$ plane) and r ($\perp$ plane)

- Lattice Wilson-loop correlators $[d = (0, 0, \vec{d}_{\perp})]$

$$\mathcal{G}_L(\vec{l}_{1\parallel}, \vec{l}_{2\parallel}; \vec{d}_{\perp}, \vec{r}_{1\perp}, \vec{r}_{2\perp}) = \frac{\langle \widetilde{\mathcal{W}}_L(\vec{l}_{1\parallel}, \vec{r}_{1\perp}; d) \widetilde{\mathcal{W}}_L(\vec{l}_{2\parallel}, \vec{r}_{2\perp}; 0) \rangle_E}{\langle \widetilde{\mathcal{W}}_L(\vec{l}_{1\parallel}, \vec{r}_{1\perp}; d) \rangle_E \langle \widetilde{\mathcal{W}}_L(\vec{l}_{2\parallel}, \vec{r}_{2\perp}; 0) \rangle_E} - 1$$

- Rotation invariance restored in the continuum limit $[2L_i = |\vec{l}_{i\parallel}|]$

$$\mathcal{G}_L(\vec{l}_{1\parallel}, \vec{l}_{2\parallel}; \vec{d}_{\perp}, \vec{r}_{1\perp}, \vec{r}_{2\perp}) \underset{a\to 0}{\simeq} \mathcal{G}_E(\theta; aL_1, aL_2; a\vec{d}_{\perp}, a\vec{r}_{1\perp}, a\vec{r}_{2\perp}) + \mathcal{O}(a)$$

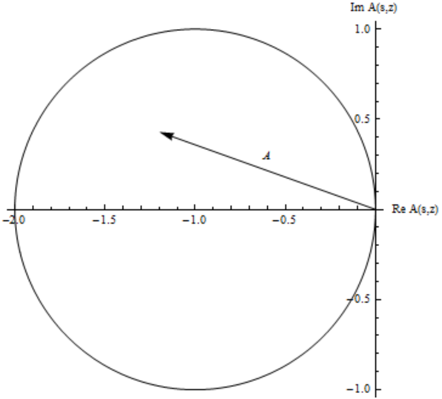


Figure: The Argand plot for $A(s, |\vec{z}_\perp|) = \langle\langle \mathcal{C}_M(\chi, \vec{z}_\perp; 1, 2) \rangle\rangle$

[Block, Cahn, 1985; Berger, Nachtmann, 1999; Shoshi, Steffen, Pirner, 2002]

A nontrivial example: onium scattering in $\mathcal{N} = 4$ SYM

$\mathcal{N} = 4$ SYM: replace mesons with "onia", wave packets of colourless dipoles, and describe "onium-onium" scattering in terms of dipoles

Large N_c, strong coupling: AdS/CFT correspondence

$\mathcal{C}_E$ at large $z = |\vec{z}_\perp|$ from a supergravity calculation [Janik, Peschanski, 2000]

$$\mathcal{C}_E^{(\mathrm{AdS/CFT})} = \exp\left\{[K_S + K_D]\frac{1}{\sin\theta} + K_B \cot\theta + K_G \frac{(\cos\theta)^2}{\sin\theta}\right\} - 1$$

$K_X = K_X(z) \sim$ exchange of supergravity field X between the string worldsheets

At large z, $K_G(z) \sim \frac{\bar{K}_G}{z^6}$; after $\theta \to -i\chi$, $\chi \to \infty$,

$$\mathcal{C}_M^{(\mathrm{AdS/CFT})} \sim \exp\left\{\frac{i\bar{K}_G}{z^6}\frac{e^\chi}{2}\right\} - 1 \Rightarrow \sigma_{\mathrm{tot}} \propto s^{\frac{1}{3}}$$

Rising σ_{tot} for "onium-onium" scattering [Giordano, Peschanski, 2010]