Lattice computation of $B \to D,\ D^*,\ D^{**}\ell\nu$ at finite heavy masses

Transition amplitudes and form factors

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Outline

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1. Semileptonic B decays
2. Lattice calculation of form factors

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3. Simulation setup

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2. Lattice calculation of form factors
3. Simulation setup
4. Results
5. Conclusions

Introduction

Semileptonic decays $\bar{B} \to D^{**}\,\ell\,\bar{\nu}$

Orbitally excited P wave D mesons (D^{**})

doublet	values J^P	notation
$j^P = \frac{1}{2}^+$	$\underbrace{0^+}_{\text{scalar state}}$	D_0^*
	1^+	D_1^*
$j^P = \frac{3}{2}^+$	1^+	D_1
	$\underbrace{2^+}_{\text{tensor state}}$	D_2^*

Precise knowledge of the corresponding branching ratios is important e.g. to reduce the systematic uncertainty of the CKM matrix element V_{cb}

"1/2 versus 3/2 puzzle"

Puzzling features associated with the semileptonic $b \to c$ data, for example :

"1/2 versus 3/2 puzzle"

A. Le Yaouanc, *et al. PLB 520, 25-30, 2001* *PLB 520, 59-62, 2001*

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Theory* : $\Gamma(\bar{B}_d \to D^{**}_{1/2}\,\ell\nu) \ll \Gamma(\bar{B}_d \to D^{**}_{3/2}\,\ell\nu)$

Experiment : $\Gamma(\bar{B}_d \to D^{**}_{1/2}\,\ell\nu) \gg \Gamma(\bar{B}_d \to D^{**}_{3/2}\,\ell\nu)$

* HQET, QCD sum rules, covariant quark models, etc...

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Lattice calculations of decay rates of $\bar{B} \to D^{**}\,\ell\,\bar{\nu}$ channels with "real quarks" could shed some light on this puzzle

A. LE YOUANC, *et al. PLB 520, 25-30, 2001* *PLB 520, 59-62, 2001*

Form Factors for the scalar state ($^3P_0$)

$$\langle {}^3P_0(p_{D^{**}}) \,|\, V_\mu \,|\, \bar{B}(p_B) \rangle = 0$$

$$\langle {}^3P_0(p_{D^{**}}) \,|\, A_\mu \,|\, \bar{B}(p_B) \rangle = \tilde{u}_+ \,(p_B + p_{D^{**}})_\mu + \tilde{u}_- \,(p_B - p_{D^{**}})_\mu$$

Form factors of the tensor state ($^3P_2$)

$$\langle {}^3P_2(p_{D^{**}}, \lambda) \,|\, V_\mu \,|\, B(p_B) \rangle = i\, \tilde{h}\, \epsilon_{\mu\rho\sigma\tau}\, \varepsilon^{\rho\alpha *}_{(p_{D^{**}}, \lambda)}\, p_{B\alpha}\, (p_B + p_{D^{**}})^\sigma\, (p_B - p_{D^{**}})^\tau$$

$$\langle {}^3P_2(p_{D^{**}}, \lambda) \,|\, A_\mu \,|\, B(p_B) \rangle = \boxed{\tilde{k}}\, \varepsilon^{*\,(p_{D^{**}}, \lambda)}_{\mu\rho}\, p_B^\rho + \left(\varepsilon^{*\,(p_{D^{**}}, \lambda)}_{\alpha\beta}\, p_B^\alpha p_B^\beta\right) \left[\tilde{b}_+ \,(p_B + p_{D^{**}})_\mu + \tilde{b}_- \,(p_B - p_{D^{**}})_\mu\right]$$

$\Longrightarrow$ 6 form factors : $\underbrace{\tilde{u}_+,\ \tilde{u}_-}_{^3P_0}$ and $\underbrace{\tilde{h},\ \boxed{\tilde{k}},\ \tilde{b}_+,\ \tilde{b}_-}_{^3P_2}$

Going to the Lattice

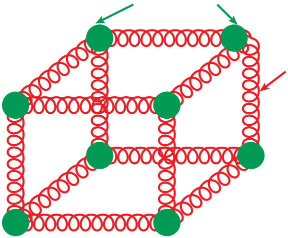
WHY

The only known way to systematically and rigorously solve the quantum theory of strong interactions

INTEREST

- No uncontrolled hypothesis, pure QCD!!
- Reach theoretical precision on par with the experimental ones

Going to the Lattice



- Discretization of space-time (Kenneth Wilson 1974)
- Hypercubic lattice $L^3 \times T$
- Lattice spacing a $\quad \vec{x} \Rightarrow a\vec{n}$
- $\int d^4x \cdots \Rightarrow a^4 \sum_n \cdots$
- $D_\mu \Psi(x) \quad \Rightarrow$ finite differences

Compute transition amplitudes, (i.e. correlation functions)
Go back to the continuum limit : $a \to 0 \quad V \to \infty$

Hadronic matrix element

Ratio

$$\boxed{\mathscr{R}(t)} = \frac{\mathscr{C}^{(3)}(t, t_i, t_f, \vec{p}_i, \vec{p}_f)}{\mathscr{C}^{(2)}_{(B)}(t - t_i, \vec{p}_f) \cdot \mathscr{C}^{(2)}_{(D)}(t_f - t, \vec{p}_i)} \cdot \sqrt{\mathscr{Z}_B} \cdot \sqrt{\mathscr{Z}_D}$$

$$\xrightarrow[t - t_i \to \infty]{t_f - t \to \infty} \langle D(\vec{p}_f) | (A_\mu, V_\mu) | B(\vec{p}_i) \rangle$$

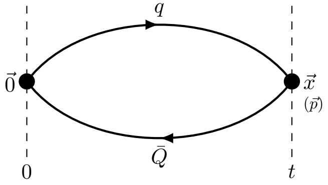
$\mathscr{Z}_M = ||\langle 0 | \mathscr{O}_M | M \rangle||^2$ obtained from the fit with $\mathscr{C}^{(2)}_{(M)}$

At large time, we observe the stable signal (plateau) which is the desired hadronic matrix element

So we need :

Three-point correlation functions $\mathscr{C}^{(3)}$ and two-point correlation functions $\mathscr{C}^{(2)}$

Meson masses



Two-point correlation functions

$$\mathscr{C}^{(2)}(T) = \langle\Omega|\mathscr{O}^{\dagger}_{t+T}\mathscr{O}_t|\Omega\rangle$$ $|\Omega\rangle$: vacuum

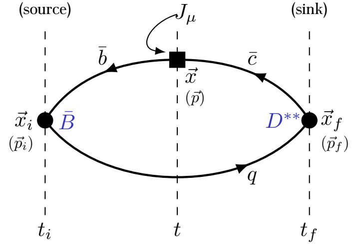
$\mathscr{O}_t$: Interpolating fields (heavy-light meson creation operators)

$$T >> 0 \approx \underbrace{|\langle 0|\mathscr{O}|\Omega\rangle|^2}_{\mathscr{Z}_D} \exp(-\underbrace{(E_0 - E_\Omega)}_{M_{meson}} T)$$ $|0\rangle$: fundamental state

meson masses are found from the behavior at large time separation of $\mathscr{C}^{(2)}(T)$

We compute :

Hadronic matrix elements



Via three-points correlation functions :

$$\mathscr{C}^{(3)}(t, t_i, t_f, \vec{p}_i, \vec{p}_f) = \sum_{\text{positions}} \langle \mathscr{O}_D^\dagger(t_f, \vec{x}_f)\ J_\mu(t, \vec{x})\ \mathscr{O}_B(t_i, \vec{x}_i)\rangle$$

$$\cdot\ e^{i(\vec{x}-\vec{x}_f)\cdot\vec{p}_f}\ \cdot\ e^{-i(\vec{x}-\vec{x}_i)\cdot\vec{p}_i}$$

Creation operators for the D mesons

Heavy-light interpolating fields

$$\mathscr{O}(t) = \bar{\psi}_Q(t, \vec{x}_Q)\ \Gamma\ \psi_q(t, \vec{x}_q)$$

Γ is one or a product of gamma matrices

allow access to $J^{PC} = 0^{-,+}, 0^{+,+}, 1^{-,-}, 1^{+,+}, 1^{+,-}$

Interpolating fields of $D^{**}(2^+)$ by means of group theory

$\gamma_1 D_1 + \gamma_2 D_2\ - 2\gamma_3 D_3$

$\gamma_1 D_1 - \gamma_2 D_2$

$\gamma_1 D_2 + \gamma_2 D_1$

$\gamma_1 D_3 + \gamma_3 D_1$

$\gamma_2 D_3 + \gamma_3 D_2$

D_i: Covariant derivative on the lattice in direction i

5 expressions related to the 5 polarization states of $J = 2$

Simulation setup

- Lattice volume $24^3 \times 48$ $(L = 24,\ T = 48)$
- Wilson twisted mass Dirac operator with two degenerate flavors

$$Q^{(\chi)} = \gamma_\mu D_\mu + m + i\mu\gamma_5 + \frac{a}{2}\Box \quad , \quad m + 4 = \frac{1}{2\kappa}$$
$$\text{avec} \quad \kappa = 0.160856$$

- "tree-level Symanzik" improved Gauge-action with $\beta = 3.9$
- Lattice spacing $a \approx 0.0855(5)$ fm, so $L = 24 \times a \approx 2.05$ fm

$a\mu_l$	$a\mu_h$	m_π in MeV	nb. of gauge configurations (ETMC)
0.0085	0.215 0.35, 0.45, 0.67	448(1)	240

Kinematics

- D^{**} rest frame $\quad p_{D^{**}} = (m_{D^{**}}, \vec{0})$
- $p_B^\mu = (E_B, p, p, p) \quad$ with $\quad p = \frac{\pi\theta_0}{L}$

  Different recoils $w = v_{D^{**}} \cdot v_B = \frac{E_B}{m_B} \quad \in \{1, 1.025, 1.05, 1.1, 1.15, 1.2, 1.3\}$

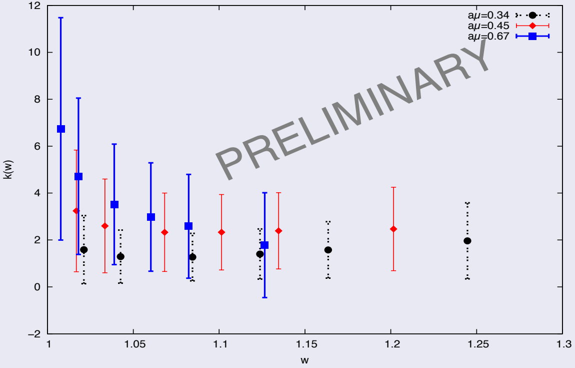
$\Longrightarrow$ Make it easier to find D^{**} creation operators and extract form factors

E.g $\tilde{k}$ formulas :

with $\quad \mathscr{T}^A_{\mu(\lambda)} \stackrel{\text{def.}}{=} \langle {}^3P_2(p_{D^{**}})|A_\mu|B(p_B)\rangle$

$$\boxed{\boxed{\tilde{k}} = -\frac{\sqrt{6}}{p}\,\mathscr{T}^A_{1(0)} = -\frac{\sqrt{6}}{p}\,\mathscr{T}^A_{2(0)} = \frac{\sqrt{6}}{2\,p}\,\mathscr{T}^A_{3(0)} = \frac{1}{p}\left[\mathscr{T}^A_{1(+2)} + \mathscr{T}^A_{1(-2)}\right] = -\frac{1}{p}\left[\mathscr{T}^A_{2(+2)} + \mathscr{T}^A_{2(-2)}\right] = \cdots}$$

Tensor state : estimation of $\tilde{k}$ from $\frac{1}{p}\left[\mathscr{T}^{A}_{1(+2)} + \mathscr{T}^{A}_{1(-2)}\right] = -\frac{1}{p}\left[\mathscr{T}^{A}_{2(+2)} + \mathscr{T}^{A}_{2(-2)}\right]$



- Large error bars
- Variation of $\tilde{k}$ as a function of w is not the same for all b masses

Goal : comparison of $\mathscr{C}^{(3)}_{(D^{**}(2^+))}$ contributing to $\tilde{k}$ with the infinite mass case

$$\mathscr{C}^{(3)}_{\text{infinite mass limit}} = p\ \tilde{k}_\infty \cdot \frac{\mathscr{C}^{(2)}_B\ \mathscr{C}^{(2)}_D}{\sqrt{\mathscr{Z}_B \mathscr{Z}_D}}$$

At the infinite mass limit : (V. Morenas *et al.*, Phys. Rev. D **56**, 5668 (1997))

$$\tilde{k}_\infty = \sqrt{3}\,\sqrt{r_{D_2^*}}\,(1+w)\,\tau_{3/2}(w) \qquad m_{D_2^*} = r_{D_2^*}\, m_B$$

$$\tau_{3/2}(w) = \tau_{3/2}(1)\left(\frac{2}{1+w}\right)^{2\sigma^2_{3/2}} \qquad \tau_{3/2}(1) \simeq 0.539 \quad \text{and} \quad \sigma^2_{3/2} \simeq 1.50$$

Example : for $w_{\text{theorical}} = 1.3$

$$\frac{\mathscr{C}^{(3)}_{\text{finite mass}}}{\mathscr{C}^{(3)}_{\text{infinite mass limit}}} = \begin{cases} a\mu = 0.34 \rightsquigarrow 1.38 \pm 0.49 \\ a\mu = 0.45 \rightsquigarrow 1.97 \pm 0.65 \\ a\mu = 0.67 \rightsquigarrow 2.88 \pm 3.82 \quad \text{larger error with higher } b \text{ mass} \end{cases}$$

Preliminary estimation of branching fractions $\mathscr{B}(B \to D_{(2^+)}) \cdot \mathscr{B}(D_{(2^+)} \to D^*\pi)$

$$\left(\frac{\mathscr{C}^{(3)}_{\text{finite mass}}}{\mathscr{C}^{(3)}_{\text{infinite mass limit}}}\right)^2 \equiv \frac{\mathscr{B}_{\text{finite mass}}}{\mathscr{B}_{\text{infinite mass limit}}}$$

At infinite mass limit :

$$\mathscr{B}(B \to D_{(2^+)}) \cdot \mathscr{B}(D_{(2^+)} \to D^*\pi) = 1.9 \times 10^{-3}$$

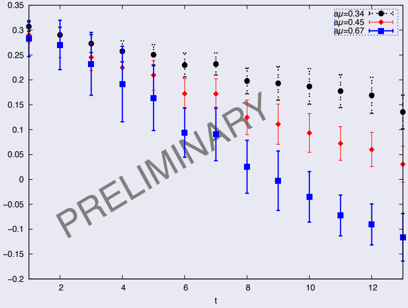
At finite mass :

$$\mathscr{B}(B \to D_{(2^+)}) \cdot \mathscr{B}(D_{(2^+)} \to D^*\pi) = \begin{cases} m_{B_{\text{GeV}}} = 2.4 & \rightsquigarrow (3.6 \pm 2.6) \times 10^{-3} \\ m_{B_{\text{GeV}}} = 2.9 & \rightsquigarrow (7.4 \pm 4.9) \times 10^{-3} \end{cases}$$

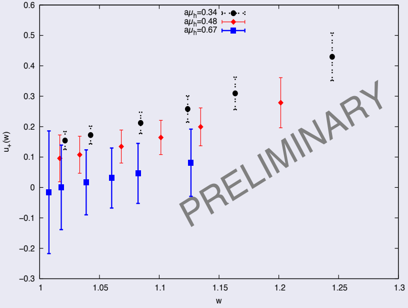
- Limited precision due to the growth of lattice spacing effect
- ....But it is feasible anyway!! So VERY Encouraging

Scalar state : Preliminary study of scalar form factors

$< D(0^+)|A_0|B >$ at zero recoil



$\tilde{u}_+(w)$



$< D(0^+)|A_0|B > \neq 0$

Difference with respect to what was found in the infinite mass limit

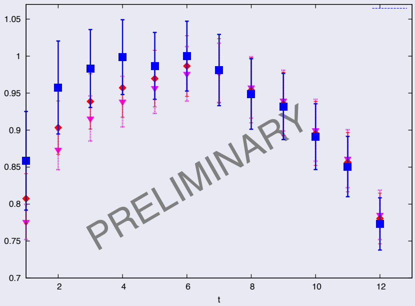
Interpolations to w for reference

Added Bonus of our work on the D^{**} :

Possibility to study $\bar{B} \to D^* \ell \bar{\nu}$

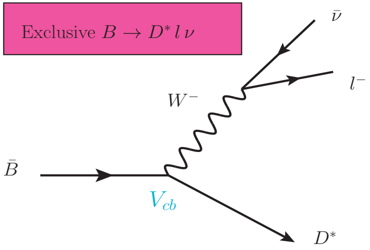
Form Factor $\mathscr{F}_0(1)$ corresponding to $\bar{B} \to D^* \ell \bar{\nu}$ at zero recoil

$\mathscr{F}_0(1)$



$$\mathscr{F}_0(1) = \frac{\langle D^*_{(M_{D^*},\vec{0})} \,|\, A_i \,|\, B_{(M_B,\vec{0})} \rangle}{2\sqrt{M_B \cdot M_{D^*}}} \cdot Z_A \qquad Z_A = 0.730(03)$$

$\mathscr{F}_0(1)$ is necessary to determine, the CKM matrix element, V_{cb}



$a\mu_b$	$\mathscr{F}_0(1)$
0.35	0.861 ±0.037
0.45	0.856 ±0.034
0.67	0.91±0.044

- $\mathscr{F}_0(1) = 0.86$ (QCD sum rules)

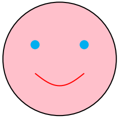
P. Gambino, T. Mannel and N. Uraltsev, arxiv :1206.2296

- Agreement with previous LQCD results
- More work to extrapolate to the continuum limit

Conclusions

- First dynamical lattice computation of the $\bar{B} \to D^{**}\ell\bar{\nu}$ form factors
  - Isolation of excited states is very delicate
  - Increase in noise when going to high $\vec{p}$ and high m_b
  - high statistics requirement
- Although results are still preliminary and need extrapolation to the continuum but they are promising.

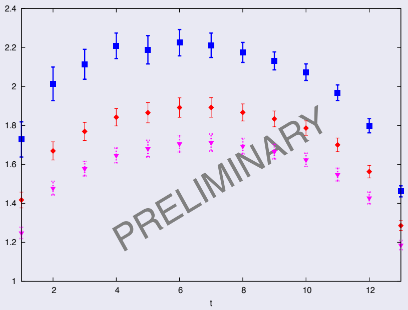
End



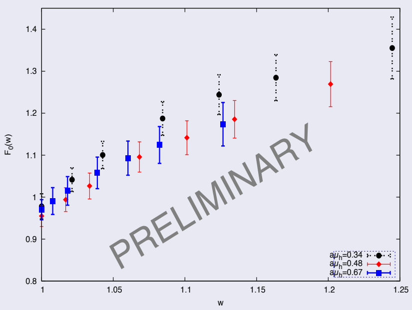
Thanks for your attention

$\bar{B} \to D\ell\bar{\nu}$ form factors

$< D(0^-)|V_0|B >$ at zero recoil



$F_0(w)$

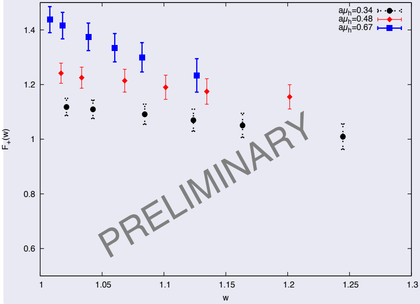


$$\langle D(k)|V_\mu|B(p)\rangle = (p+k)_\mu F_+(q^2) + q_\mu \frac{m_B^2 - m_D^2}{q^2}\left[F_0(q^2) - F_+(q^2)\right]$$

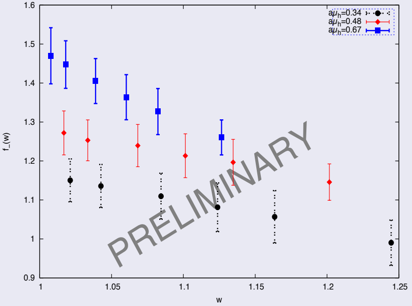
$$(q^2 = m_B^2 + m_D^2 - 2m_B\, m_D\, w)$$

$\bar{B} \to D\ell\bar{\nu}$ form factors

$F_+(w) = \tilde{f}_+(w)$



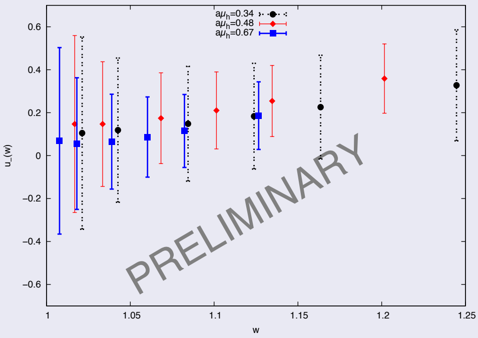
$\tilde{f}_-(w)$



$$\langle D(k)|V_\mu|B(p)\rangle = (p+k)_\mu \tilde{f}_+(q^2) + (p-k)_\mu \tilde{f}_-(q^2)$$

$\bar{B} \to D^{**}_{(0^+)} \ell \bar{\nu}$ form factors

$\tilde{u}_-(w)$



$$\langle D(k)|A_\mu|B(p)\rangle = (p+k)_\mu \tilde{u}_+(q^2) + (p-k)_\mu \tilde{u}_-(q^2)$$

$\tilde{u}_-(w)$ has a larger error bars than $\tilde{u}_+(w)$