Propagation of nonlinear gauge theory and corresponding phenomenology in deconfined strong interaction matter

LI Shi-Yuan
School of Physics, Shandong University, Jinan 250100, PR China

We summarize the shock wave propagation of non-linear gauge theories, and point out that the results are the same for Abelian and Non-Abelian gauge theories. Then we argue that the deconfied strong interaction matter can be employed to study the Non-Abelian non-linear propagation. We discuss the examples for various external strong fields and investigate the way to measure the predicted polarizations

Non-linear field theory models have attracted a lot of attentions. Soliton is one example. The other interesting topic is shock wave propagation. There are yet a lot of questions to be addressed. Assuming that the classical nonlinear Lagrangian is real, i.e., Hermitian after quantizied. In the Heisenberg picture, the solution of the non-linear equation of motion of the field operators do not respect the superposition principle, and hence the Green functions. On the other hand, since the Hamiltonian is Hermitian, it still is possible to construct the linear Hilbert space of the states with the eigenstates of the Hamiltonian as bases. How to relate the field operators and the states from the Hilbert space is a complex question to address the 'wave (field) and particle (state) correspondence'. In this paper we only encounter the simple example that the field propagation select the polarization states.

A feasible framework to deal with the shock wave propagation of the non-linear theory is the Hadamard method [1]. The main point is to calculate the discontinuity in space-time to study the wave vector (dispersion relation) as well as the polarization. This method has the advantage that one need not to linearize the equation of motion, so that can be applied to the case that the propagation can not be treated as perturbation of the more strong slowly-varying background/external field. From this method, it is also very straightforward to see that the formulations of the propagation is the same for Abelian as well as non-Abelian gauge theory. The key point is that the discontinuity is only non-zero for second-order derivative of the vector gauge field.

In fact, a thorough investigation for the most general non-linear Lagrangian for the Abelian case is also needed. I.e., to write down the most generally framework to see how to set the configurations to get the specified propagation phenomenon. Here we first list the frameworks and results. We start from the general Lagrangian density $L = L(F, G)$, where F and G are the two local gauge invariants defined in terms of the tensor field $F_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu]$, and its dual

$$F*_{\alpha\beta} = \frac{1}{2}\eta_{\alpha\beta}{}^{\sigma\tau}F_{\sigma\tau}, \tag{1}$$

as

$$F = F^{\mu\nu}F_{\mu\nu} \tag{2}$$

$$G = F^{\mu\nu}F*_{\mu\nu}\,. \tag{3}$$

In terms of the electric $\vec{E}$ and magnetic $\vec{B}$ field strengths we have $F = -2(E^2 - B^2)$ and $G = -4\vec{E} \cdot \vec{B}$. Here the covariant derivative is $D_\mu = \partial_\mu - igA_\mu$, with g the coupling constant. In the above formulations, if applied to the non-Abelian case, the vector/tensor field such as $A_\mu, F_{\mu\nu}$, as well as the components $\vec{E}$ and $\vec{B}$ are understood as also a N-dimensional vector of the inner joint-representation space. The contraction of the field tensor are also done for the inner space. However, for the Non-Abelian case, there could be more invariants than F and G [2]. Here we will not discuss such a complexity, for our following example does not depend on all the other non-Abelian invariants. The field equation can be obtained from the least action principle. In addition, $F_{\mu\nu}$ satisfies the Bianchi identity $F*^{\mu\nu}{}_{,\nu} = 0$.

The method of field discontinuities can be briefly stated as follows [1,3]. Consider a differentiable inextendible oriented borderless hyper-surface Σ, defined locally by $\phi(x^\mu) = 0$, where ϕ is a real differentiable scalar field which locally is a function of the spacetime coordinates $x^\mu = (t, \vec{x})$. Let U^+ be the spacetime points whose coordinates satisfy $\phi(x^\mu) > 0$, and similarly U^- be such that $\phi(x^\mu) < 0$. Let P be any given point of Σ. For each sufficiently small $r > 0$, let $V_r(P)$ be a neighborhood of P which consists of the spacetime points Q whose Euclidean distance from P is $[(t_Q - t_P)^2 + ||\vec{x}_Q - \vec{x}_P||^2]^{(1/2)}$ smaller than r. Let $P^+ \in U^+ \cap V_r(P)$ and $P^- \in U^- \cap V_r(P)$ be any two neighbor points from P arbitrarily chosen at opposite sides of Σ. Let f be any given tensor field defined at $V_r(P)$. The Hadamard discontinuity at P of f across Σ is defined as

$$[f]_\Sigma(P) \doteq \lim_{r \to 0^+} [f(P^+) - f(P^-)]. \tag{4}$$

Suppose f such that $[f]_\Sigma = 0$ for each $P \in \Sigma$. Following Hadamard [1], we have $[f_{,\lambda}]_\Sigma(P) = k_\lambda \bar{f}(P)$, where $k_\lambda = \phi_{,\lambda}|_P$ is the normal vector to Σ at P and $\bar{f}$ is a tensor field defined at Σ with the same rank and the same algebraic symmetries as those of f.

We assume the tensor field $F_{\mu\nu}$ to be smooth in each $U^\pm$, but merely continuous at Σ (that is to say, the $F_{\mu\nu}$ are continuous functions at Σ but their derivatives may present discontinuities at Σ). The Hadamard discontinuities at Σ of the equation of motion and the Bianchi identity lead to [3]

$$f_{\beta\lambda} k^\lambda + \frac{2}{L_F} N_\beta{}^{\mu\nu\rho} f_{\nu\rho} k_\mu = 0 \tag{5}$$

and

$$f_{\alpha\beta} = \epsilon_\alpha k_\beta - \epsilon_\beta k_\alpha, \tag{6}$$

where the quantities $f_{\alpha\beta}$ are related to the derivatives of $F_{\alpha\beta}$ on Σ by $[F_{\alpha\beta,\lambda}]_\Sigma = f_{\alpha\beta} k_\lambda$ and ϵ_μ is the polarization vector $[A_{\mu,\alpha\beta}]_\Sigma = e_\mu k_\alpha k_\beta$. Here

$$N^{\mu\nu\alpha\beta} \doteq L_{FF} F^{\mu\nu} F^{\alpha\beta} + L_{GG} F*^{\mu\nu} F*^{\alpha\beta} + L_{FG} \left(F^{\mu\nu} F*^{\alpha\beta} + F*^{\mu\nu} F^{\alpha\beta} \right). \tag{7}$$

Use is being made here of the notation $L_{X^1 X^2 \cdots X^n} = \partial^n L / \partial X^1 \partial X^2 \cdots \partial X^n$ previously introduced [3], where each X^i is one of the two invariants F or G upon which the Lagrangian L arbitrarily depends.

We set $k_\lambda = \omega V_\lambda + q_\lambda$ as the wave 4-vector, where $V_\lambda = \delta^0_\lambda$ is the 4-velocity of the observer which decomposes $F^{\mu\nu}$ into 'electric' and 'magnetic fields'. The components of this 4-vector k_λ are thus the frequency ω and the wave vector $\vec{q} = q\hat{q}$. Taking together Eqs. (5) and (6), we obtain the general eigenvalue equation [4,3]

$$Z^\mu{}_\nu \epsilon^\nu = 0, \tag{8}$$

where

$$Z^\mu{}_\nu \doteq k^2 \delta^\mu{}_\nu + \frac{4}{L_F} N^{\mu\alpha}{}_{\nu\beta} k_\alpha k^\beta. \tag{9}$$

Nontrivial solutions of Eq. (8) can be found only if det $| Z_{\mu\nu} | = 0$, the well known generalized Fresnel equation, and yields

$$\alpha (k^2)^2 + \beta f^2 k^2 + \gamma (f^2)^2 = 0, \tag{10}$$

where $f^2 = F^{\alpha\mu} F_\alpha{}^\nu k_\mu k_\nu$, and

$$\alpha = L_F^2 + 2L_F(GL_{FG} - FL_{GG}) - (L_{FF}L_{GG} - L_{FG}^2)G^2, \quad (11)$$

$$\beta = 4L_F(L_{FF} + L_{GG}) - 8(L_{FF}L_{GG} - L_{FG}^2)F, \quad (12)$$

$$\gamma = 16(L_{FF}L_{GG} - L_{FG}^2). \quad (13)$$

The phase velocity $v \doteq \omega/q$ of the waves can be obtained from a fourth order equation, we could find up to four solutions for the phase velocity, in the same wave direction.

In what follows we study a particular model. Such a model appears in different contexts in the literature, for instance as the effective Lagrangian density for quantum electrodynamics.

$$L_{NL} = -\frac{1}{4} b_0 F \log \frac{F}{\lambda^2}, \quad (14)$$

where b_0 and λ are constants. Particularly λ can be chosen in order to split the above model in a Maxwellian part plus a nonlinear contribution. When the F is construted by the $SU(3)$ gauge field, this can be a good effective theory of QCD (see, e.g., [2]).

For a most interesting example, let us assume constant external electric $\vec{E} = E\hat{x}$ and magnetic $\vec{B} = B\hat{y}$ fields, much larger than their wave counterparts. The calculations [5] show there could be trirefregence. If in a gluon plasma, such external colour field configuration can be set, we can try to observe the similar novel behaviour of gluons. However, such an external field configuration is very difficult to prepare in QCD case. Since now what can be studied is only the bulk of hot matter produced in heavy ion collisions. In such an collision, as the lowest order approximation we adopt the following symmentric average

$$\overline{E_i} = 0, \qquad \overline{H_i} = 0, \qquad \overline{E_i\,H_j - H_i\,E_j} = 0, \quad (15)$$

$$\overline{E_i\,E_j} = -\frac{1}{3}E^2\,\eta_{ij}, \quad (16)$$

$$\overline{H_i\,H_j} = -\frac{1}{3}H^2\,\eta_{ij}, \quad (17)$$

We find that it behaves as a polarizer [6], since a physical propagation can only be the 'ordinary ray', with the specified polarization determined by the colour fields.

In the hadronization process, the polarization can be tranferred into hadrons. Λ is a good example. From the SU(6) quark model, one finds that in Λ the (ud) quark must be a scalar, hence the spin of the s quark is just the spin of the Λ. When the gluon splits into quark pair, because of helicity conservation, the specified helicity state of the gluon determines the helicity states of the quark and antiquark, e.g., $|1,1> \to |\frac{1}{2}\frac{1}{2}>$. As mentioned above, only the ordinary ray of the gluon field can propagate, and the polarization vector can be calculated via Eq. (8) when the direction and the colour field are definite so that $Z^{\mu\nu}$ in Eq. (8) is determined. Since any polarization vector can be expanded by the two helicity state of the gluon, then we easily deduce that the spin state of the quark pair produced from the gluon can be determined. This means that a quark which comes from the gluon moves in a specific direction must be in a definite spin state. If the quark is an s quark and combine a diquark to hadronize as a Λ, its polarization is definite.

However, in each collision, for different direction, the spin state is different. Further more, for different collision, the colour fields are different. For an average among all events and all particles the results vanishing. To construct SO(3) scalar, one experimental observable for the polarization correlation has been suggested [6]. This can be extracted from the ideal case of two Λ particles with the same polarization $\vec{P}$, with $P = |\vec{P}|$ representing the polarization rate. The conventional way to measure $\vec{P}$ of a single Λ is by measuring the direction vector (denoted as $\hat{p}$) of the momenta of the daughter particles, e.g., proton or pion from the Λ decay, at the rest frame of Λ. Then the angular distribution,

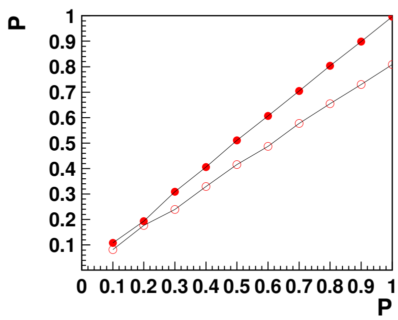
$$\frac{dN}{dcos\theta} \sim 1 + \alpha\hat{p}\cdot\vec{P} = 1 + \alpha P cos\theta, \tag{18}$$

can give the information on the polarization. Here α is the hyperon decay parameter. From this equation we see that if the direction of $\vec{P}$ is random, the average of all $\Lambda's$ gives zero, then P is not able to be measured. However, for the two $\Lambda's$ with the same polarization, in the rest frame of *each* Λ, *respectively*, the direction vectors $\hat{p}_1$ and $\hat{p}_2$ can be measured. Then we calculate the expectation value $<\hat{p}_1\cdot\hat{p}_2>$, which results in

$$P = 3\sqrt{<\hat{p}_1\cdot\hat{p}_2>}/\alpha. \tag{19}$$

In reality, we can not expect in an event, there could be easily find two Λ move exactly parallel. So that we can only try to find two particle with the nearest moving directions. In such a case, employing the formulation right hand side of Eq. (19), we can not get the exact polarization value but an approximation P'. However, imagining the case of a heavy ion collision with the center of mass energy infinite, the multiplicity is also infinite, so that we can have $P' \to P$.

At RHIC and LHC, the center of mass energy is not infinite but very high, we will find the P' is not a bad approximation. Fig. 1 demonstrates this fact. This shows the results of RHIC energy. To mimic the true data, we employ a generator developed by the Shandong University group. Here we did not show the comparing of the kinematic details of the model and real data, but only requiring the averaged multiplicity and multiplicity distribution of this model consistent with the data.



In the concrete calculations, we only employ the pairs of $\Lambda's$ with the smallest relative angles which are all smaller than $\frac{\pi}{4}$. We find that such a restriction will keep enough statistics. To have enough large multiplicity, We only investigate the most central 5% collision. This also guarantees that the asymmetries are largely reduced.

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