INVARIANTS AND FLAVOUR IN THE GENERAL TWO-HIGGS DOUBLET MODEL

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We construct weak basis invariants which can give insight into the physical implications of any flavour model, written in an arbitrary weak basis (WB) in the context of 2HDM.

1 Introduction

The LHC has started the "Higgs Era" with the discovery of a scalar boson with a mass of approximately 126 GeV [1], [2], which seems to behave like the Standard Model (SM) Higgs boson. There are very good motivations to consider models with two Higgs doublets [3], [4] despite the fact that the SM is a very successful theory. Most observations in the hadronic sector are in agreement with the SM predictions apart from a few anomalies and tensions. However, in the leptonic sector the SM has to be extended in order to accommodate neutrino masses and leptonic mixing. Furthermore, accounting for the baryon asymmetry of the Universe requires new sources of CP violation thus providing a possible motivation to consider two Higgs doublet models. Supersymmetry, if discovered, will also require the existence of two Higgs doublets. The discovery of a charged Higgs at the LHC in the future would be an important step towards the experimental confirmation of the need to extend the Higgs sector of the SM.

Models with two Higgs doublets have potentially large Higgs mediated flavour changing neutral currents (FCNC). Experimentally FCNC are strongly constrained. There are several possible ways of suppressing these currents. Natural flavour conservation [5] or the imposition of alignment [6] eliminate tree level Higgs FCNC. In alternative, one may have FCNC at tree level suppressed by small entries of the Cabibbo– Kobayashi – Maskawa matrix, V_{CKM}. The first models of this type based on a symmetry were built by Branco, Grimus and Lavoura [7] and later on extended in Refs [8], [9]. This talk is based on work done in collaboration with Botella and Branco [10].

2 General Framework

The flavour structure of two Higgs doublet models is given by the Yukawa interactions:

$$L_Y = -\overline{Q_L^0}\,\Gamma_1\Phi_1 d_R^0 - \overline{Q_L^0}\,\Gamma_2\Phi_2 d_R^0 - \overline{Q_L^0}\,\Delta_1\tilde{\Phi}_1 u_R^0 - \overline{Q_L^0}\,\Delta_2\tilde{\Phi}_2 u_R^0 \; + \text{h. c.} \tag{1}$$

where Γ_i and Δ_i denote the Yukawa couplings of the lefthanded quark doublets Q_L^0 to the righthanded quarks d_R^0, u_R^0 and the Higgs doublets Φ_j. The quark mass matrices generated after spontaneous gauge symmetry breaking are given by:

$$M_d = \frac{1}{\sqrt{2}}(v_1\Gamma_1 + v_2 e^{i\alpha}\Gamma_2), \quad M_u = \frac{1}{\sqrt{2}}(v_1\Delta_1 + v_2 e^{i\alpha}\Delta_2), \tag{2}$$

where $v_i \equiv |<0|\phi_i^0|0>|$ and α denotes the relative phase of the vacuum expectation values (vevs) of the neutral components of Φ_i. The matrices M_d, M_u are diagonalized by the usual bi-unitary transformations:

$$U_{dL}^\dagger M_d U_{dR} = D_d \equiv \text{d}iag\,(m_d, m_s, m_b) \tag{3}$$

$$U_{uL}^\dagger M_u U_{uR} = D_u \equiv \text{d}iag\,(m_u, m_c, m_t) \tag{4}$$

The neutral and the charged Higgs interactions obtained from Eq. (1) are of the form

$$
\begin{aligned}
\mathcal{L}_Y(\text{quark, Higgs}) \;=\; & -\overline{d_L^0}\frac{1}{v}\,[M_d H^0 + N_d^0 R + iN_d^0 I]\,d_R^0 - \\
& -\; \overline{u_L^0}\frac{1}{v}\,[M_u H^0 + N_u^0 R + iN_u^0 I]\,u_R^0 - \\
& -\; \frac{\sqrt{2}H^+}{v}(\overline{u_L^0}N_d^0\,d_R^0 - \overline{u_R^0}N_u^{0\dagger}\,d_L^0) + \text{h.c.}
\end{aligned}
\tag{5}
$$

where $v \equiv \sqrt{v_1^2 + v_2^2} \approx 246$ GeV, and H^0, R are orthogonal combinations of the fields ρ_j, arising when one expands [11] the neutral scalar fields around their vacuum expectation values, $\phi_j^0 = \frac{e^{i\alpha_j}}{\sqrt{2}}(v_j + \rho_j + i\eta_j)$, choosing H^0 in such a way that it has couplings to the quarks which are proportional to the mass matrices, as can be seen from Eq. (5). Similarly, I denotes the linear combination of η_j orthogonal to the neutral Goldstone boson. The matrices N_d^0, N_u^0 are given by:

$$N_d^0 = \frac{1}{\sqrt{2}}(v_2\Gamma_1 - v_1 e^{i\alpha}\Gamma_2), \quad N_u^0 = \frac{1}{\sqrt{2}}(v_2\Delta_1 - v_1 e^{-i\alpha}\Delta_2) \tag{6}$$

The flavour structure of the quark sector of two Higgs doublet models is thus fully specified in terms of the four matrices M_d, M_u, N_d^0, N_u^0. The physical neutral Higgs fields are combinations of H^0, R and I. Flavour changing neutral currents are controlled by N_d^0 and N_u^0.

3 Weak Basis Invariants

The four flavour matrices M_d, M_u, N_d^0, N_u^0 contain a large redundancy of parameters which results from the fact that under a weak basis (WB) transformation they change transforming as

$$
\begin{aligned}
M_d \to M_d' = W_L^\dagger M_d W_R^d, \qquad & M_u \to M_u' = W_L^\dagger M_u W_R^u, \\
N_d^0 \to N_d^{0\prime} = W_L^\dagger N_d^0 W_R^d, \qquad & N_u^0 \to N_u^{0\prime} = W_L^\dagger N_u^0 W_R^u
\end{aligned}
\tag{7}
$$

without altering their physical content. Different Lagrangians related to each other by WB transformations describe the same physics. In view of the above redundancy, it is of great interest to construct WB invariants which can be very useful in the analysis of the physical

content of the flavour sector of two Higgs doublet models [10] by following the technique that was introduced in [12] to the study of CP violation in the SM. This technique was later generalized to many different scenarios, in particular to the study of explicit CP violation in the scalar sector of multi-Higgs doublet models prior to gauge symmetry breaking [13] as well as CP violation in the scalar sector after this breaking [14] and also taking into account both the scalar and the fermionic sector [15].

In this framework, it is clear that one can build new WB basis invariants which do not arise in the SM by evaluating traces of blocks of matrices involving the up and down quark sector, like for example $M_\gamma N_\gamma^{0\dagger}$ or $N_\gamma^0 N_\gamma^{0\dagger}$. For definiteness let us consider the WB invariant $\text{tr}(M_d N_d^{0\dagger})$ and note that its physical significance becomes transparent in the WB where M_d is diagonal, real, since in this basis the matrix N_d^0 already coincides with the couplings to the physical quarks. In this basis one has:

$$I_1 \equiv \text{tr}(M_d N_d^{0\dagger}) = m_d (N_d^*)_{11} + m_s (N_d^*)_{22} + m_b (N_b^*)_{33} \tag{8}$$

We denote N_d, the matrix N_d^0 in the basis where it couples to the physical quarks. This invariant is not sensitive to Higgs-mediated FCNC, but $\text{Im}(I_1)$ is specially important, since it probes the phases of $(N_d)_{jj}$ which contribute to the electric dipole moment of down-type quarks. Obviously, one can construct an analogous invariant for the up-quark sector, namely $\text{tr}(M_u N_u^{0\dagger})$. Let us now consider a WB invariant which is sensitive to the off-diagonal elements of N_d, namely:

$$I_2 \equiv \text{tr} \left[M_d N_d^{0\dagger}, M_d M_d^\dagger \right]^2 = -2 m_d m_s (m_s^2 - m_d^2)^2 (N_d^*)_{12} (N_d^*)_{21} - $$
$$-2 m_d m_b (m_b^2 - m_d^2)^2 (N_d^*)_{13} (N_d^*)_{31} - 2 m_s m_b (m_b^2 - m_s^2)^2 (N_d^*)_{23} (N_d^*)_{32}, \tag{9}$$

where we have kept the notation used in Eq. (8), having evaluated I_2 in the WB where M_d is real and diagonal. WB invariants are also important to study CP violation. In the SM a necessary and sufficient condition for CP invariance is the vanishing of the WB invariant [12] :

$$I_1^{CP} \equiv \text{tr} \left[H_u, H_d \right]^3 = 6i (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2) \times$$
$$\times (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2) \text{Im} Q_{uscb} \tag{10}$$

where $H_{d,u} \equiv (M_{d,u} M_{d,u}^\dagger)$, Q stands for a rephasing invariant quartet of V_{CKM}, defined by $Q_{\alpha i \beta j} \equiv V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*$ ($\alpha \neq \beta$, $i \neq j$) with $V_{CKM} \equiv U_{uL}^\dagger U_{dL}$. The fact that V_{CKM} is not the identity reflects the fact that $U_{dL} \neq U_{uL}$, i.e. that there is misalignment of the matrices H_d, H_u in flavour space. For three generations I_1^{CP} is proportional to $\det[H_u, H_d]$, introduced in Ref. [16]. In the present framework there are four matrices relevant for flavour rather than the two mass matrices of the SM, therefore we can generalise the definition of I_1^{CP} to diferent WB invariants sensitive to the misalignment of different pairs of Hermitian matrices, such as, for example $I_2^{CP} \equiv \text{tr} \left[H_u, H_{N_d^0} \right]^3$ or $I_3^{CP} \equiv \text{tr} \left[H_d, H_{N_d^0} \right]^3$, with $H_{N_d^0} \equiv N_d^0 N_d^{0\dagger}$.

In the SM the lowest order WB invariant sensitive to CP violation is given by Eq. (10) and has dimension twelve in powers of mass. The richer flavour structure of models with two Higgs doublets allows for lower order invariants sensitive to CP violation, namely, for instance:

$$I_9^{CP} \equiv \text{Im} \, \text{tr} \left[M_d N_d^{0\dagger} M_d M_d^\dagger M_u M_u^\dagger M_d M_d^\dagger \right] \tag{11}$$

In generic two Higgs doublet models one may even have lower order invariants sensitive to CP violation. However, with the imposition of a symmetry in order to suppress Higgs mediated FCNC such lower other invariants may become trivial.

Flavour symmetries and/or texture zeros reduce the number of free parameters and have physical implications. However symmetries and textures are introduced in a specific WB. Under a change of WB these will in principle cease to be apparent. In this respect the computation of weak basis invariants is a very useful tool to recognize properties related to special symmetries

or textures that may not be apparent. In Ref. [10] the summary presented here is extended and applied to two special cases: models of the type proposed by Branco, Grimus and Lavoura [7] and a special implementation of nearest – neighbour – interaction (NNI) textures [17] in the context of two Higgs doublet models based on an Abelian symmetry[18].

Acknowledgments

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