

$\tau^- \rightarrow \pi^- \eta^{(\prime)} \nu_\tau$ decays

PABLO ROIG

*Grup de Física Teòrica, Institut de Física d'Altes Energies, Universitat Autònoma de Barcelona,
E-08193 Bellaterra, Barcelona, Spain*



1 Introduction

$\tau^- \rightarrow \pi^- \eta^{(\prime)} \nu_\tau$ decays belong to the so-called second-class current processes¹: parity conservation implies that these decays must proceed through the vector current, which has opposed G -parity to the $\pi^- \eta^{(\prime)}$ system. In the limit of exact isospin symmetry G -parity is exact and these processes are forbidden. Isospin is an approximate symmetry, slightly broken both by $m_u \neq m_d$ (in QCD) and $q_u \neq q_d$ (in QED), which results in a sizable suppression of the considered decays, which have not been measured so far. The corresponding branching ratios upper limits are $9.9 \cdot 10^{-5}$ ² and $7.2 \cdot 10^{-6}$ ³ and no second-class current process has been reported yet. This suppression motivates the study of beyond the standard model (SM) contributions to these decays^{4,5}.

Here we focus on the SM prediction of these processes, focusing on the scalar and vector form factors contributions.

2 Hadronic matrix element and decay width

Our conventions⁶ are fixed from Ref.⁷. Therefore, we have ($P = \pi/\eta/\eta'$)

$$\langle \pi^- P^0 | \bar{d} \gamma^\mu u | 0 \rangle = c_{\pi^- P}^V \left[(p_P - p_\pi)^\mu f_+^{\pi^- P}(s) - q^\mu f_-^{\pi^- P}(s) \right], \quad (1)$$

with $q^\mu = (p_P + p_\pi)^\mu$, $s = q^2$, and $c_{\pi^- \pi^0}^V = -\sqrt{2} = -c_{\pi^- \eta^{(\prime)}}^V \cdot f_0^{\pi^- P}(s)$, which can be used instead of $f_-^{\pi^- P}(s)$, is defined through

$$\langle 0 | \partial_\mu (\bar{d} \gamma^\mu u) | \pi^+ P \rangle = i(m_d - m_u) \langle 0 | \bar{d} u | \pi^+ P \rangle \equiv i \Delta_{K^0 K^+}^{QCD} c_{\pi^- P}^S f_0^{\pi^- P}(s), \quad (2)$$

with

$$c_{\pi^-\eta}^S = \sqrt{\frac{2}{3}} = c_{\pi^-\pi^0}^S, \quad c_{\pi^-\eta'}^S = \frac{2}{\sqrt{3}}, \quad \Delta_{PQ} = m_P^2 - m_Q^2. \quad (3)$$

The mass renormalization

$$m_d - m_u = \frac{\Delta_{K^0K^+}^{QCD}}{B_0} \left[1 + \frac{16c_m}{F^2 M_S^2} (c_d - c_m) m_K^2 \right], \quad (4)$$

needs to be taken into account to define $f_0^{\pi^-P}(s)$.

From eqs.(1) and (2) one gets

$$\langle \pi^- P^0 | \bar{d} \gamma^\mu u | 0 \rangle = \left[(p_P - p_\pi)^\mu + \frac{\Delta_{\pi^-P}}{s} q^\mu \right] c_{\pi^-P}^V f_+^{\pi^-P}(s) + \frac{\Delta_{K^0K^+}^{QCD}}{s} q^\mu c_{\pi^-P}^S f_0^{\pi^-P}(s). \quad (5)$$

The finiteness of the matrix element at the origin imposes

$$f_+^{\pi^-P}(0) = -\frac{c_{\pi^-P}^S}{c_{\pi^-P}^V} \frac{\Delta_{K^0K^+}^{QCD}}{\Delta_{\pi^-P}} f_0^{\pi^-P}(0), \quad (6)$$

which is obtained from

$$f_-^{\pi^-P}(s) = -\frac{\Delta_{\pi^-P}}{s} \left[\frac{c_{\pi^-P}^S}{c_{\pi^-P}^V} \frac{\Delta_{K^0K^+}^{QCD}}{\Delta_{\pi^-P}} f_0^{\pi^-P}(s) + f_+^{\pi^-P}(s) \right]. \quad (7)$$

In terms of these form factors, the differential decay width reads

$$\frac{d\Gamma(\tau^- \rightarrow \pi^- P^0 \nu_\tau)}{d\sqrt{s}} = \frac{G_F^2 M_\tau^3}{24\pi^3 s} S_{EW} |V_{ud}|^2 |f_+^{\pi^-P}(0)|^2 \left(1 - \frac{s}{M_\tau^2}\right)^2 \left\{ \left(1 + \frac{2s}{M_\tau^2}\right) q_{\pi^-P^0}^3(s) |\tilde{f}_+^{\pi^-P}(s)|^2 + \frac{3\Delta_{\pi^-P^0}^2}{4s} q_{\pi^-P}(s) |\tilde{f}_0^{\pi^-P}(s)|^2 \right\}, \quad (8)$$

$$(9)$$

where

$$\tilde{f}_{+,0}^{\pi^-P}(s) = \frac{f_{+,0}^{\pi^-P}(s)}{f_{+,0}^{\pi^-P}(0)}, \quad q_{PQ}(s) = \frac{\lambda^{1/2}(s, m_P^2, m_Q^2)}{2\sqrt{s}}. \quad (10)$$

Since the $\pi^-\eta^{(\prime)}$ vector form factors are proportional to the $\pi^-\pi^0$ vector form factor we may fix the first one at the origin from the latter, see eq.(14), using that $f_+^{\pi^-\pi^0}(0) = 1$. The proportionality constants will bring an overall suppression factor which explains the smallness of the corresponding branching fractions, in agreement with the expected vanishing in the quite accurate G-parity symmetry limit.

3 Hadronic Form factors

We have worked out the involved form factors using Chiral Perturbation Theory⁸ including resonances within the convenient antisymmetric tensor field formalism⁹, a framework which has been shown capable of providing a good description of hadronic tau decay data^{10 11 12}. The $\pi^0 - \eta - \eta'$ mixing has been parametrized by means of three Euler angles ($\epsilon^{\eta\pi}$, $\epsilon^{\eta'\pi}$ and $\theta_{\eta\eta'}$), including the small isospin breaking given by $z := \frac{f_u - f_d}{f_u + f_d}$ ¹³. We have neglected terms of $\mathcal{O}(\epsilon^2)$ in the corresponding expansions.

When the vanishing of the $f_0^{\pi^-P}(s)$ form factors at large s is required, one obtains the restriction $c_d = c_m = F/2$ ¹⁵, which yields

$$f_0^{\pi^-\pi^0}(s) = c_0^{\pi^-\pi^0} \frac{M_S^2}{M_S^2 - s}, \quad f_0^{\pi^-\eta^{(\prime)}}(s) = c_0^{\pi^-\eta^{(\prime)}} \frac{M_S^2 + \Delta_{\pi\eta^{(\prime)}}}{M_S^2 - s}. \quad (11)$$

with $c_0^{\pi^-\pi^0} = \epsilon^{\eta\pi} + \sqrt{2}\epsilon^{\eta'\pi}$, $c_0^{\pi^-\eta} = \cos\theta_{\eta\eta'} - \sqrt{2}\sin\theta_{\eta\eta'}$, $c_0^{\pi^-\eta'} = \cos\theta_{\eta\eta'} + \frac{\sin\theta_{\eta\eta'}}{\sqrt{2}}$.

We will replace $1/(M_S^2 - s)$ by $1/(M_S^2 - s - iM_S\Gamma_S(s))$, with the energy-dependent $a_0(980)$ width given by

$$\Gamma_{a_0}(s) = \Gamma_{a_0}(M_{a_0}^2) \left(\frac{s}{M_{a_0}^2} \right)^{3/2} \frac{h(s)}{h(M_{a_0}^2)}, \quad (12)$$

with

$$h(s) = \sigma_{KK}(s) + \frac{2}{3}\sigma_{\pi\eta}(s) (c_0^{\pi^-\eta})^2 \left(1 + \frac{\Delta\pi\eta}{s} \right)^2 + \frac{4}{3}\sigma_{\pi\eta'}(s) (c_0^{\pi^-\eta'})^2 \left(1 + \frac{\Delta\pi\eta'}{s} \right)^2. \quad (13)$$

In this way we are neglecting the real part of the corresponding loop functions, which will induce a small violation of analyticity (see, however, Ref. ¹⁶).

Finally, the $\pi^-\eta^{(\prime)}$ vector form factors are obtained in terms of the well-known $\pi^-\pi^0$ vector form factor

$$f_+^{\pi^-\eta}(s) = [\epsilon^{\eta\pi}\cos\theta_{\eta\eta'} - \epsilon^{\eta'\pi}\sin\theta_{\eta\eta'}] f_+^{\pi^-\pi^0}(s), \quad f_+^{\pi^-\eta'}(s) = [\epsilon^{\eta'\pi}\cos\theta_{\eta\eta'} + \epsilon^{\eta\pi}\sin\theta_{\eta\eta'}] f_+^{\pi^-\pi^0}(s). \quad (14)$$

Thus, we will have

$$f_+^{\pi^-\eta}(0) = \epsilon^{\eta\pi}\cos\theta_{\eta\eta'} - \epsilon^{\eta'\pi}\sin\theta_{\eta\eta'}, \quad f_+^{\pi^-\eta'}(0) = \epsilon^{\eta'\pi}\cos\theta_{\eta\eta'} + \epsilon^{\eta\pi}\sin\theta_{\eta\eta'}, \quad (15)$$

and the normalized form factors are all the same:

$$\tilde{f}_+^{\pi^-\eta^{(\prime)}}(s) = \tilde{f}_+^{\pi^-\pi^0}(s), \quad \tilde{f}_0^{\pi^-\pi^0}(s) = \tilde{f}_0^{\pi^-\pi^0}(s). \quad (16)$$

While $f_+^{\pi^-\eta}(0) \sim \mathcal{O}(\epsilon^{\eta\pi})$, an accidental cancellation makes $f_+^{\pi^-\eta'}(0) < \mathcal{O}[(\epsilon^{\eta\pi})^2]$: the $\tau^- \rightarrow \eta\pi^-\nu_\tau$ decays are suppressed, as it corresponds to a second class current process, but the $\tau^- \rightarrow \eta'\pi^-\nu_\tau$ decays are heavily suppressed.

4 Phenomenological analysis

For the vector form factor, we have taken $\tilde{f}_+^{\pi^-\pi^0}(s)$ using the dispersive representation of Ref. ¹¹ devised in Ref. ¹² for $\tilde{f}_+^{K\pi}(s)$. We have estimated the model dependent error by considering Belle's data ¹⁷ (whose extraction requires the knowledge of isospin-breaking corrections ^{18 19}) and the phenomenological fit made this Collaboration. This error is negligible versus the one coming from $\epsilon^{\eta\pi}$ and $\epsilon^{\eta'\pi}$. We have fixed $\Delta_{K^0K^+}^{QCD}$ ^{20 21} and determined the value of z that fulfils eqs.(6) within errors ⁶. In this way we find $z \sim -1 \cdot 10^{-3}$, $\epsilon^{\eta\pi} \sim 0.018(2)$ and $\epsilon^{\eta'\pi} = 5(1) \cdot 10^{-3}$.

In the case of the scalar form factor the error receives important contributions both from the uncertainty on the $\epsilon^{\eta^{(\prime)\pi}}$ coefficients and on $M_{a_0} = (980 \pm 20)$ MeV and $\Gamma_{a_0} = (75 \pm 25)$ MeV. We have, however, neglected the contribution of a possible a_0' resonance, which may change sizably the result, especially for the $\tau^- \rightarrow \pi^-\eta'\nu_\tau$ decays.

Under these assumptions we find $BR_+(\tau^- \rightarrow \pi^-\eta\nu_\tau) = (0.9 \pm 0.2) \cdot 10^{-5}$, $BR_0(\tau^- \rightarrow \pi^-\eta\nu_\tau) = (2.7 \pm 1.1) \cdot 10^{-5}$, which yield to $BR(\tau^- \rightarrow \pi^-\eta\nu_\tau) = (3.6 \pm 1.3) \cdot 10^{-5}$ and $BR_+(\tau^- \rightarrow \pi^-\eta'\nu_\tau) \in [10^{-11}, 10^{-9}]$, $BR_0(\tau^- \rightarrow \pi^-\eta'\nu_\tau) \in [10^{-10}, 2 \cdot 10^{-8}]$, giving $BR(\tau^- \rightarrow \pi^-\eta'\nu_\tau) \in [10^{-10}, 2 \cdot 10^{-8}]$ ^a. While our predictions for the $\pi^-\eta$ mode are larger than previous results ^{5 22 23 24 25 26 27}, our values for the $\pi^-\eta'$ mode tend to be smaller ^{26 28 29}. This is a result of our improved treatment of the $\pi^0 - \eta - \eta'$ mixing. We note in particular that, according to our findings, the $\tau^- \rightarrow \pi^-\eta\nu_\tau$ should be within discovery reach at future super-B factories.

^aErrors coming from our theoretical approach are not included. While they have been checked to be negligible for the vector form factor contribution this does not need to be the case for the scalar form factor one.

Acknowledgments

This work has been partly funded by the Spanish grant FPA2011-25948. Financial support of the organization covering my living expenses during the conference is acknowledged.

References

1. S. Weinberg, Phys. Rev. **112** (1958) 1375.
2. P. del Amo Sanchez *et al.* [BaBar Collaboration], Phys. Rev. D **83** (2011) 032002.
3. B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. D **77** (2008) 112002.
4. A. Bramon, S. Narison and A. Pich, Phys. Lett. B **196** (1987) 543.
5. S. Nussinov and A. Soffer, Phys. Rev. D **78** (2008) 033006.
6. R. Escribano, S. González-Solís and P. Roig, work in progress.
7. J. Gasser and H. Leutwyler, Nucl. Phys. B **250** (1985) 517.
8. S. Weinberg, Physica A **96** (1979) 327. J. Gasser and H. Leutwyler, Nucl. Phys. B **250** (1985) 465, Annals Phys. **158** (1984) 142.
9. G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. B **321** (1989) 311. G. Ecker, J. Gasser, H. Leutwyler, A. Pich and E. de Rafael, Phys. Lett. B **223** (1989) 425.
10. M. Jamin, A. Pich and J. Portolés, Phys. Lett. B **640**, 176 (2006), B **640**, 176 (2006). D. G. Dumm, P. Roig, A. Pich and J. Portolés, Phys. Rev. D **81** (2010) 034031, Phys. Lett. B **685**, 158 (2010). D. R. Boito, R. Escribano and M. Jamin, JHEP **1009**, 031 (2010). Z. -H. Guo and P. Roig, Phys. Rev. D **82**, 113016 (2010). O. Shekhovtsova, T. Przedzinski, P. Roig and Z. Was, Phys. Rev. D **86**, 113008 (2012), O. Shekhovtsova, I. M. Nugent, T. Przedzinski, P. Roig and Z. Was, arXiv:1301.1964 [hep-ph] and work in progress. D. G. Dumm and P. Roig, Phys. Rev. D **86**, 076009 (2012).
11. D. G. Dumm and P. Roig, arXiv:1301.6973 [hep-ph].
12. D. R. Boito, R. Escribano and M. Jamin, Eur. Phys. J. C **59** (2009) 821.
13. P. Kroll, Mod. Phys. Lett. A **20** (2005) 2667.
14. F. Ambrosino, A. Antonelli, M. Antonelli, F. Archilli, P. Beltrame, G. Bencivenni, S. Bertolucci and C. Bini *et al.*, JHEP **0907** (2009) 105.
15. M. Jamin, J. A. Oller and A. Pich, Nucl. Phys. B **587** (2000) 331, **622** (2002) 279, Phys. Rev. D **74** (2006) 074009.
16. R. Escribano, P. Masjuan and J. J. Sanz-Cillero, JHEP **1105** (2011) 094.
17. M. Fujikawa *et al.* [Belle Collaboration], Phys. Rev. D **78** (2008) 072006.
18. V. Cirigliano, G. Ecker and H. Neufeld, Phys. Lett. B **513** (2001) 361, JHEP **0208** (2002) 002.
19. F. Flores-Báez, A. Flores-Tlalpa, G. López Castro and G. Toledo Sánchez, Phys. Rev. D **74** (2006) 071301.
20. J. Bijnens, Phys. Lett. B **306** (1993) 343.
21. B. Moussallam, Nucl. Phys. B **504** (1997) 381.
22. S. Tisserant, and T. N. Truong, Phys. Lett. B **115** (1982) 264.
23. A. Pich, Phys. Lett. B **196** (1987) 561.
24. H. Neufeld and H. Rupertsberger, Z. Phys. C **68** (1995) 91.
25. N. Paver and Riazuddin, Phys. Rev. D **82** (2010) 057301.
26. M. K. Volkov and D. G. Kostunin, Phys. Rev. D **86** (2012) 013005.
27. S. Descotes-Genon, E. Kou, and B. Moussallam, arXiv:1303.2879 [hep-ph].
28. S. Nussinov and A. Soffer, Phys. Rev. D **80** (2009) 033010.
29. N. Paver and Riazuddin, Phys. Rev. D **84** (2011) 017302.