STABILITY OF THE EW VACUUM, HIGGS BOSON, AND NEW PHYSICS [a]

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The possibility that the Standard Model (SM) is valid up to the Planck scale M_P, i.e. that new physics occurs only around M_P, is nowadays largely explored. For a metastable EW vacuum, we show that new physics interactions can have a great impact on its lifetime, and, differently from previous analyses, they cannot be neglected. Therefore, contrary to usual believes, the stability phase diagram of the SM depends on new physics. This has far reaching consequences. Beyond SM theories can be tested against their prediction for the stability of the EW vacuum. Moreover, despite of some recent claims, higher precision measurements of the top and Higgs masses cannot provide any definite answer on the SM stability properties. Finally, doubts on Higgs inflation scenarios, all based on results obtained neglecting new physics interactions, are also cast.

1 Stability phase diagram (new physics neglected)

The Higgs effective potential $V_{eff}(\phi)$ bends down for values of ϕ larger than v, the location of the electroweak (EW) minimum (an instability due to top loop-corrections), and develops a new minimum at $\phi^{(2)}_{min} >> v$. Depending on Standard Model (SM) parameters, in particular on the top and Higgs masses, M_t and M_H, the second minimum can be higher or lower than the EW one. In the first case the EW vacuum is stable, in the second one it is metastable and we have to consider its lifetime τ.

While several different scenarios for Beyond Standard Model (BSM) physics are considered, the possibility for the SM to be valid up to the Planck scale M_P, meaning that new physics only occurs at scales around M_P, is not excluded and is the object of several investigations. In the usual analysis, it is argued that new physics interactions at M_P have no impact on the EW vacuum stability properties, and their presence is neglected[2,3,4,5].

Under this assumption, the stability phase diagram of the SM in the $M_H - M_t$ plane turns out as shown in the left panel of fig.1. The plane is divided into three different sectors. An *absolute stability* region, where $V_{eff}(v) < V_{eff}(\phi^{(2)}_{min})$, a *metastability* region, where $V_{eff}(\phi^{(2)}_{min}) < V_{eff}(v)$, but $\tau > T_U$, and an *instability* region, where $V_{eff}(\phi^{(2)}_{min}) < V_{eff}(v)$ and $\tau < T_U$, where T_U is the

[a] based on work done in collaboration with E. Messina[1]

age of the universe. The dashed line separates the stability and the metastability sectors and is obtained for M_H and M_t such that $V_{eff}(v) = V_{eff}(\phi^{(2)}_{min})$. The dashed-dotted line separates the metastability and the instability regions and is obtained for M_H and M_t such that $\tau = T_U$.

For $M_t \sim 173.1$ GeV and $M_H \sim 126$ GeV, the SM lies within the metastability region (black dot in the left panel of fig.1). This observation leads to the so called metastability scenario, that consists of the following proposal. Even though the EW vacuum is not the absolute minimum of $V_{eff}(\phi)$, if $\tau > T_U$, our universe may well be sitting on such a metastable (false) vacuum.

There is another observation related to the position of the SM "point" in this figure. When the errors (not shown in fig.1) in the determination of M_H and M_t are taken into account, it turns out that within 2.5-3 σ, the SM could be sitting on the dashed line, i.e. it could reach the stability region. This case is named "critical", as λ at M_P would reach the value $\lambda(M_P) \sim 0$, and also the beta function would be, $\beta(\lambda(M_P)) \sim 0$. This "near-criticality" is considered by some authors the most important message from the experimental data on the Higgs boson[5].

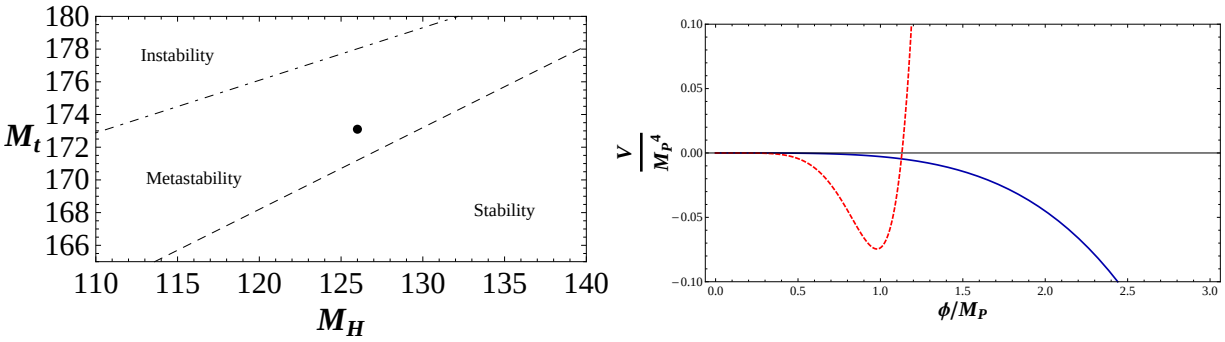

Figure 1 – Left panel: Stability phase diagram obtained neglecting the presence of new interactions at the Planck scale. The $M_H - M_t$ plane is divided in three sectors, stability, metastability, and instability regions (see text). The dot indicates $M_H \sim 126$ GeV and $M_t \sim 173.1$ GeV. Right panel: Effective potential $V^{new}_{eff}(\phi)$ (red line) in the presence of the higher dimension operators ϕ^6 and ϕ^8, with $\lambda_6 = -2$ and $\lambda_8 = 2.1$. For comparison, the blue line is for $V_{eff}(\phi)$ ($\lambda_6 = 0$ and $\lambda_8 = 0$).

The above analysis, however, has some caveats. For the central values $M_t \sim 173.1$ GeV and $M_H \sim 126$ GeV, for instance, the new minimum forms at $\phi^{(2)}_{min}$ much larger than M_P, $\phi^{(2)}_{min} \sim 10^{31}$ GeV ! Despite of these (quite untrastable) results, it is argued that new physics at the Planck scale should stabilize the potential, bringing the new minimum around M_P, and that the computation of τ can be performed with the unmodified potential $V_{eff}(\phi)$, as the impact of new physics interactions should be negligible.

Moreover, as the instability of the effective potential occurs for very large values of ϕ, $V_{eff}(\phi)$ is well approximated by keeping only the quartic term[6], $V_{eff}(\phi) \sim \frac{\lambda_{eff}(\phi)}{4}\phi^4$, where $\lambda_{eff}(\phi)$ depends on ϕ essentially as the running quartic coupling $\lambda(\mu)$ depends on the running scale μ. For large values of μ, $\lambda(\mu)$ becomes negative and almost constant. Therefore, τ is computed by considering first the bounce solution to the euclidean equation of motion for the potential $\frac{\lambda}{4}\phi^4$ with negative λ, and then taking into account the fluctuations around the bounce[7,8].

In the following we show that new physics interactions at the Planck scale can dramatically change the lifetime of the metastable EW vacuum from $\tau >> T_U$ to $\tau << T_U$.

2 Lifetime of the EW vacuum and new physics

The tunnelling rate Γ, inverse lifetime time τ, is given by[7,8] (for the sake of simplicity, we write the formula with the contribution of the scalar sector of the SM only, the inclusion of the other contributions being straightforward)

$$\Gamma = \frac{1}{\tau} = T_U^3 \frac{S[\phi_b]^2}{4\pi^2} \left| \frac{\det' \left[-\partial^2 + V''(\phi_b)\right]}{\det \left[-\partial^2 + V''(v)\right]} \right|^{-1/2} e^{-S[\phi_b]} \qquad (1)$$

where $\phi_b(r)$ is the $O(4)$ bounce solution to the euclidean equation of motion ($r = \sqrt{x_\mu x_\mu}$), $S[\phi_b]$ the action for the bounce, and $\left[-\partial^2 + V''(\phi_b)\right]$ the fluctuation operator around the bounce (V'' is the second derivative of V with respect to ϕ). The prime in the det$'$ means that the zero modes are excluded, and $\frac{S[\phi_b]^2}{4\pi^2}$ comes from the translational zero modes.

New physics interactions at the Planck scale appear as higher order operators multiplied by appropriate inverse powers of M_P. In order to study the possible impact of new physics at M_P, we now add to the SM Higgs potential two higher dimension operators ϕ^6 and ϕ^8,

$$V(\phi) \;=\; \frac{\lambda}{4}\phi^4 + \frac{\lambda_6}{6}\frac{\phi^6}{M_P^2} + \frac{\lambda_8}{8}\frac{\phi^8}{M_P^4}\,. \tag{2}$$

In the right panel of fig.1, the resulting effective potential $V_{eff}^{new}(\phi)$ (red line) for a specific example with natural values of λ_6 and λ_8, $\lambda_6 = -2$ and $\lambda_8 = 2.1$, is plotted. For comparison, we also plot $V_{eff}(\phi)$ (blue line). This example is well suited for our analysis. First of all, we have explicitly realized the stabilization of the effective potential around the Planck scale through the action of new physics operators as required in[2] (see above). At the same time, we have a "bona fide" potential that we can use to check whether or not the usual assumption[2,3,4,5] that in the evaluation of the EW vacuum lifetime new physics interactions can be neglected is correct. As we shall see, they cannot be neglected and the stability phase diagram of fig.1 has to be revised.

With $\lambda_6 = 0$ and $\lambda_8 = 0$, the euclidean equation of motion for the bounce can be solved analytically and we have

$$\phi_b^{(1)}(r) = \sqrt{\frac{2}{|\lambda|}}\frac{2R}{r^2+R^2}\;, \tag{3}$$

R being the size of the bounce. The action is degenerate with R, $\;S[\phi_b^{(1)}] = \frac{8\,\pi^2}{3|\lambda|}$, the degeneracy being lifted by quantum fluctuations. The latters select only one bounce with given size, R_1. With $M_H \sim 126$ GeV and $M_t \sim 173.1$ GeV, $R_1 \sim 8 \times 10^{-18}\, GeV^{-1}$.

With $\lambda_6 \neq 0$ and $\lambda_8 \neq 0$, the euclidean equation of motion cannot be solved analytically. However, we can easily find the bounce numerically. Let us call $\phi_b^{(2)}(r)$ this solution. Due to the presence of the higher order terms, the degeneracy in this case is lifted already at the tree level, and the size is $R_2 \sim \frac{5.06}{M_P}$. For our scopes, it is important to note that, for $R >> 1/M_P$, the bounce $\phi_b^{(1)}(r)$ is also an approximate solution of the theory with non vanishing λ_6 and λ_8.

If, according to the usual analysis, we now neglect the new physics interactions and compute the EW vacuum lifetime with $\lambda_6 = 0$ and $\lambda_8 = 0$, the tunnelling rate Γ_0 turns out to be

$$\Gamma_0 = \frac{1}{\tau_0} = \frac{1}{T_U}\left[\frac{S[\phi_b^{(1)}]^2}{4\pi^2}\,\frac{T_U^4}{R_M^4}\,e^{-S[\phi_b^{(1)}]}\right] \times \left[e^{-\Delta S_1}\right]\,, \tag{4}$$

where $S[\phi_b^{(1)}] = \frac{8\,\pi^2}{3|\lambda|}$, R_1 and T_U are as before, and ΔS_1 is the loop contribution.

If, on the contrary, we take into account ϕ^6 and ϕ^8, both $\phi_b^{(2)}(r)$ and the quasi solution $\phi_b^{(1)}(r)$ have to be considered, and for Γ_{np} we get (np= new physics),

$$\begin{aligned}\Gamma_{np} = \frac{1}{\tau_{_{np}}} \;&=\; \frac{1}{T_U}\left[\frac{S[\phi_b^{(1)}]^2}{4\pi^2}\,\frac{T_U^4}{{R_1}^4}\,e^{-S[\phi_b^{(1)}]}\right] \times \left[e^{-\Delta S_1}\right] \\ &+\; \frac{1}{T_U}\left[\frac{S[\phi_b^{(2)}]^2}{4\pi^2}\,\frac{T_U^4}{{R_2}^4}\,e^{-S[\phi_b^{(2)}]}\right] \times \left[e^{-\Delta S_2}\right]\,.\end{aligned} \tag{5}$$

Inserting the obtained numerical values, $S[\phi_b^{(1)}] \sim 1833$, $S[\phi_b^{(2)}] \sim 82$, $R_1 \sim 8 \times 10^{-18}\, GeV^{-1}$, and $R_2 \;\sim \frac{5}{M_P}$, even neglecting, for a moment, the one-loop ΔS_i contributions, we find from Eq. (4) (where $\lambda_6 = 0$, $\lambda_8 = 0$) and from Eq. (5) (where $\lambda_6 = -2$, $\lambda_8 = 2.1$)

$$\tau_0 \sim 10^{555}\, T_U \qquad , \qquad \tau_{_{np}} \sim 10^{-208}\, T_U\;. \tag{6}$$

Needless to say, Eq. (6) clearly shows that new physics interactions at the Planck scale can have a dramatic impact on the EW vacuum lifetime. Moreover, from Eqs. (5) we see that the contribution to $\tau_{_{np}}$ coming from $\phi_b^{(1)}$ is exponentially suppressed. It is the bounce $\phi_b^{(2)}$, the one that we miss when we switch off the new physics interactions, that dominates!

The reason for such an impact of new physics on $\tau_{_{np}}$ is easy to understand. New physics interactions appear in terms of higher dimensional operators, and we could naively expect their contribution to be suppressed. However, the tunnelling is a non-perturbative phenomenon. We first compute the bounce (tree level) and then the quantum fluctuations (loop corrections) on the top of it. The suppression in terms of inverse M_P powers (power counting theorem) concerns the loop corrections, not the selection of the saddle point (tree level). The latter is intrinsically non-perturbative. In Eq.(2) we have a new potential, and then a new saddle point.

The inclusion of the ΔS_i does not change the above results significantly. For completeness, we write the values of $\tau_{_0}$ and $\tau_{_{np}}$ with the ΔS_i included : $\tau_{_0} \sim 10^{588}$, $\tau_{_{np}} \sim 10^{-189}$.

3 Phenomenological consequences and conclusions

The lifetime of the EW vacuum, as we have seen, strongly depends on new physics, and the stability phase diagram of fig.1 has to be revised. From the phenomenological point of view, this poses constraints on theories beyond the SM. Any acceptable UV completion of the SM should not provide for τ results of the kind obtained in the above example. In other words, our analysis provides a "BSM stability test": a BSM theory is acceptable if it provides either a stable EW vacuum or a metastable one, with lifetime larger than the age of the universe. In the past it was thought that, given M_H and M_t, the stability, metastability or instability of the EW vacuum could be established with no reference to the specific UV completion of the SM (stability phase diagram of fig.1). Clearly, our analysis can be repeated even when the new physics scale lies below the Planck scale (GUT scale, for instance).

The "near-criticality" suggestion[5], $\lambda(M_P) \sim 0$ and $\beta(\lambda(M_P)) \sim 0$, is also very much challenged by our results. The inclusion of new physics interactions can easily screw up these relations. The same is true for the Higgs inflation scenario of[9], heavily based on the validity of the SM up to the Planck scale and on the criticality assumption[10]. Other Higgs inflation scenarios, based on the possibility for the SM Higgs potential to develop a minimum at energies $\sim 10^{16}$ GeV, where inflation could have started in a metastable state[11], are also subject, for the same reasons, to the same criticisms.

Finally, precision measurements of the top mass, that according to the phase diagram in fig.1 should tell us whether or not the SM moves towards the stability line (the above discussed criticality), cannot give any answer to this question. As it should be clear by now, the knowledge of M_t and M_H is not sufficient to decide of the EW vacuum stability status.

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