GEOMETRICAL SCALING AND ITS BREAKING IN HIGH ENERGY COLLISIONS [a]

M. PRASZALOWICZ
M. Smoluchowski Institute of Physics, Jagiellonian University
4 Reymonta str., 30-059 Krakow, Poland

We report on recent analyses of different pieces of data, which exhibit Geometrical Scaling (GS) and its breaking. GS is a consequence of the existence of an intermediate energy scale, called saturation momentum, and allows to relate data at different energies, of different systems and also at different multiplicities and/or centralities.

In this talk we give a short overview of searches for the presence of Geometrical Scaling in hadronic collisions. For details we refer the reader to the original publications. Let's start from the formula for the cross-section for inclusive gluon production [1] in a collision $1+2 \to g+X$:

$$\frac{d\sigma}{dyd^2p_{\rm T}} = \frac{3\pi}{2p_{\rm T}^2}\int d^2\vec{k}_{\rm T}\,\alpha_{\rm s}(k_{\rm T}^2)\varphi_1(x_1,\vec{k}_{\rm T}^2)\varphi_2(x_2,(\vec{k}-\vec{p})_{\rm T}^2). \tag{1}$$

Here $\varphi_{1,2}$ are unintegrated gluon densities and $x_{1,2}$ are gluon momenta fractions needed to produce a gluon of transverse momentum $p_{\rm T}$ and rapidity y:

$$x_{1,2} = e^{\pm y}p_{\rm T}/\sqrt{s}\,. \tag{2}$$

Note that unintegrated gluon densities have dimension of area. This is at best seen from the very simple parametrization propsed by Kharzeev and Levin [2] or by Golec-Biernat and Wüsthoff [3] in the context of Deep Inelastic Scattering (DIS):

$$\varphi(k_{\rm T}^2) = S_\perp \begin{cases} 1 & \text{for} \quad k_{\rm T}^2 < Q_{\rm s}^2 \\ Q_{\rm s}^2/k_{\rm T}^2 & \text{for} \quad k_{\rm T}^2 < p_{\rm T}^2 \end{cases} \quad \text{or} \quad \varphi(k_{\rm T}^2) = S_\perp \frac{3}{4\pi^2}\frac{k_{\rm T}^2}{Q_{\rm s}^2}\exp\left(-k_{\rm T}^2/Q_{\rm s}^2\right). \tag{3}$$

Here $S_\perp$ is the transverse size given by inelastic cross-section (or its part) for the minimum bias inclusive multiplicity or in the case of DIS $S_\perp = \sigma_0$ is the dipole-proton cross-section for large dipoles. Another feature of the unintegrated glue (3) is the fact that φ depends on the ratio

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$k_{\rm T}^2/Q_{\rm s}^2(x)$ rather than on $k_{\rm T}^2$ and x separately. This is called Geometrical Scaling [4] and has been for the first time proposed in the context of DIS. Here

$$Q_{\rm s}^2(x) = Q_0^2\,(x/x_0)^{-\lambda} = Q_0^2(e^{\pm y}\,p_{\rm T}/W)^{-\lambda} \tag{4}$$

is the saturation scale. Taking [b] $x_0 = 10^{-3}$ we have $W = \sqrt{s} \times 10^{-3}$ in formula (4).

Assuming particles 1 and 2 to be identical and $y \sim 0$ (central rapidity) which corresponds to $x_1 \simeq x_2$ (denoted in the following as x) and suppressing $\alpha_{\rm s}$ we arrive at:

$$\frac{d\sigma}{dyd^2p_{\rm T}} = S_\perp^2\mathcal{F}(\tau) \qquad \text{or} \qquad \frac{1}{S_\perp}\frac{dN}{dyd^2p_{\rm T}} = \mathcal{F}(\tau) \tag{5}$$

where $\tau = p_{\rm T}^2/Q_{\rm s}^2(x)$ is scaling variable and dN/dy stands for multiplicity density. Eq.(5) implies that particle spectra at different energies should coincide if plotted in terms τ. In other words they exhibit GS [5].

We can integrate now (5) over $d^2p_{\rm T}$ using

$$dp_{\rm T}^2 = \frac{2Q_0^2}{2+\lambda}\left(W^2/Q_0^2\right)^{\frac{\lambda}{2+\lambda}}\tau^{-\frac{\lambda}{2+\lambda}}d\tau$$

arriving at

$$\frac{dN}{dy} = S_\perp\int\mathcal{F}(\tau)d^2p_{\rm T} = S_\perp\bar{Q}_{\rm s}^2\frac{2\pi}{2+\lambda}\int\mathcal{F}(\tau)\tau^{-\frac{\lambda}{2+\lambda}}d\tau = \frac{1}{\kappa}S_\perp\,\bar{Q}_{\rm s}^2 \tag{6}$$

where $1/\kappa$ is a universal, energy independent integral of $\mathcal{F}$, and

$$\bar{Q}_{\rm s}^2 = Q_0^2\left(W^2/Q_0^2\right)^{\frac{\lambda}{2+\lambda}} \tag{7}$$

is an *average* saturation scale, which can be thought of as a solution of the equation

$$Q_{\rm s}^2(\bar{Q}_{\rm s}^2/W^2) = \bar{Q}_{\rm s}^2.$$

It follows that

$$\bar{Q}_{\rm s}^2 = \frac{\kappa}{S_\perp}\frac{dN}{dy}. \tag{8}$$

Equation (8) means that the average saturation scale is proportional to the gluon density per unit transverse area. One should keep in mind the distinction between saturation scales (4) and (8), since they are interchangeably used in the literature. The theory behind gluon saturation (for a review see Refs. [6,7] and references therein) is Color Glass Condensate [8,9,10].

The existence of GS in pp collisions as given by eq.(5) has been indeed observed in the data [5] and reported at Moriond 2012 [11]. An efficient way to study GS is to form ratios $R_{W_1,W_2}(\tau) = dN/dydp_{\rm T}|_{W_1}(\tau)/\,dN/dydp_{\rm T}|_{W_2}(\tau)$ which, according to (5) should be equal 1 over wide range of τ [12]. This requirement allows to find the optimal value of λ which in the case of the LHC data is equal to 0.27, which is a bit smaller than in DIS [13]. It has been shown [13] that in DIS GS extends up to rather large $x_{\rm max} \approx 0.08$.

Surprisingly GS scaling works also for the $p_{\rm T}$ spectra in heavy ion collisions at RHIC energies [12]. In the case of heavy ions the saturation momentum scales as $Q_{A\,{\rm s}}^2 = A^{1/3}Q_{\rm s}^2$ and the scaling variable is therefore $\tau_A = p_{\rm T}^2/Q_{A\,{\rm s}}^2$. This is illustrated in Fig. 1 where charged particles spectra in AuAu and CaCa collisions as measured by PHOBOS are plotted in terms of $p_{\rm T}$ and $\sqrt{\tau_A}$. Recently GS for the photons produced in different systems (AA, dA and pp), at different energies and at different centralties (*i.e.* at different $S_\perp$) has been reported [14].

For $y > 0$ two Bjorken x's (2) can be quite different: $x_1 > x_2$. Therefore by increasing y one can eventually reach $x_1 > x_{\rm max}$ and violation of GS is expected. To show this [15] we have used pp

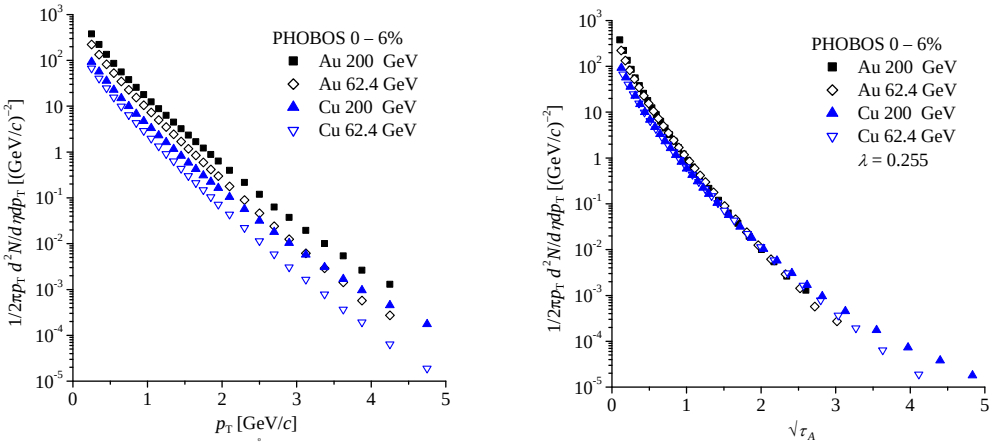

Figure 1 – Transverse momentum spectra measured by PHOBOS in Au-Au and Cu-Cu most central collisions as functions of p_T (right) and $\sqrt{\tau_A}$ (left) (from Ref.[12])

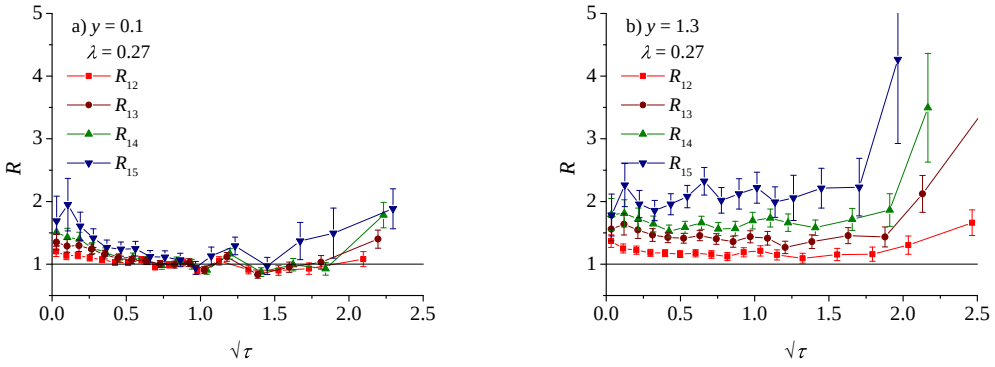

Figure 2 – Ratios R_{1k} as functions of $\sqrt{\tau}$ for $\lambda = 0.27$ and for different rapidities a) $y = 0.7$ and b) $y = 1.3$. With increase of rapidity, gradual closure of the GS region can be seen (from Ref.[15]).

data from NA61/SHINE experiment [16] which measured particle spectra at different rapidities $y = 0.1 - 3.5$ and at 5 scattering energies $W_{1,\ldots,5} = 17.28,\ 12.36,\ 8.77,\ 7.75$, and 6.28 GeV.

In Fig. 2.a we plot ratios $R_{1i} = R_{W_1,W_i}$ for π^- spectra in rapidity region $y = 0.1$ for $\lambda = 0.27$. Here the GS window extends down to the smallest energy because $x_{\max}$ is as large as 0.08. Nevertheless one can see that the quality of GS is the worst for the smallest energy W_5. By increasing y some points fall outside the GS region due to the fact that $x_1 \geq x_{\max}$, and finally for $y \geq 1.7$ geometrical scaling is no longer seen. This is shown in Fig. 2.b.

In a situation where two (or more) external energy scales are present, like p_T and particle mass m (for identified particles), one can form two independent ratios with Q_s what implies violation or at least modification of GS. We have argued that in the case of identified particles GS is still present [17] if another scaling variable is used in which p_T is replaced by $\tilde{m}_T = m_T - m = \sqrt{m_T^2 + p_T^2} - m$. This scaling variable is connected with the fact that accurate fits are obtained by means of Tsallis-like parametrization [18,19,20] where particle multiplicity distribution takes the following form (see *e.g.* Ref.[21]):

$$\frac{1}{p_T}\frac{d^2N}{dydp_T} = C\frac{dN}{dy}\left[1 + \frac{m_T - m}{nT}\right]^{-n}. \tag{9}$$

[b]The precise value of Q_0 and x_0 is not important in the following. Only the value of exponent λ will be determined.

Coefficient C ensures proper normalization of (9). Here n and T are free fit parameters that depend on particle species. Formula (9) admits GS solution [17], provided that n is a constant (with possible corrections that would allow for the energy dependence of n seen in the data) and $T \sim \bar{Q}_s$ of eq.(7) which has a power-like energy dependence.

In summary we can say that by now the existence of the saturation scale is undoubtedly well established. Geometrical Scaling follows as a natural consequence. One can use GS to relate different pieces of data with an accuracy much higher than originally expected. New results from the LHC at higher energies will be important for further studies of the details GS and of the underlying theory of dense gluonic system.

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