

Glueball Decay Patterns from the Witten-Sakai-Sugimoto model

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Longstanding, still elusive prediction of QCD: Glueballs

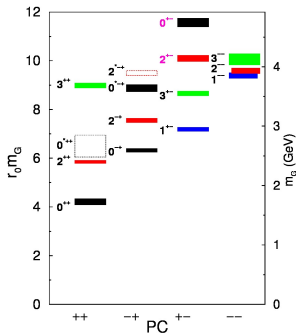
- Spectrum of *bare* glueballs (prior to mixing with $q\bar{q}$ states) more or less known from lattice:

$$m_{0^{++}} \sim 1.7 \text{ GeV}$$

$$m_{2^{++}} \sim 2.4 \text{ GeV}$$

$$m_{0^{-+}} \sim 2.6 \text{ GeV}$$

Morningstar & Peardon hep-lat/9901004



- Interactions, decays of glueballs and mixing with $q\bar{q}$ however largely unknown: → no conclusive identification of any glueball in meson spectrum

- most discussed scenario for lightest scalar glueball:

various phenomenological models describe $f_0(1500)$ or $f_0(1710)$ isoscalar 0^{++} mesons alternatingly as $\sim 50\text{-}70\%$ or $\sim 75\text{-}90\%$ G(lueball)

[G and two isoscalar $q\bar{q}$ states $u\bar{u} + d\bar{d}$ and $s\bar{s}$ can be shared by $f_0(1370)$, $f_0(1500)$, $f_0(1710)$]

Holographic QCD

Glueballs to be identified through *decay patterns*, but not (yet) available from lattice
→ seeking help from holographic (large- N_c) QCD:

Witten-Sakai-Sugimoto model (Witten 1997, Sakai & Sugimoto 2004)

- almost parameter-free top-down construction from type-IIA superstrings:
 - N_c D4 branes provide nonconformal dual of 4+1-dim. super-Yang-Mills
 - KK compactification on circle in x_4
→ nonsupersymmetric 3+1-dim. Yang-Mills theory at low energies
 - N_f chiral quarks from N_f D8- $\overline{\text{D8}}$ probe branes (Sakai & Sugimoto 2004ff)
- reproduces many aspects of low-energy QCD
 - chiral symmetry breaking $U(N_f)_L \times U(N_f)_R \rightarrow U(N_f)_V$ with Witten-Veneziano mass² $\propto 1/N_c$
 - effective Lagrangian: nonlinear σ -model with Skyrme term plus vector and axial-vector mesons similar to HLS models
 - correct WZW terms
- fitting M_{KK} and 't Hooft coupling $\lambda|_{M_{KK}}$ yields quantitative predictions, often with only 10-30% errors:

Quantitative predictions

- Parameter-free: mass spectrum of vector and axial-vector mesons

$$m(\rho^*)/m(\rho), m(a_1)/m(\rho), m(a_1^*)/m(\rho) \text{ correct within } \lesssim 20\%$$

- Other predictions, depending on value of 't Hooft coupling λ at scale M_{KK} with

① $m_\rho \approx 776 \text{ MeV}$ fixes $M_{\text{KK}} = 949 \text{ MeV}$

② $f_\pi^2 = \frac{\lambda N_c}{54\pi^4} M_{\text{KK}}^2$ gives $\lambda = g_{\text{YM}}^2 N_c \approx 16.63$ [Sakai&Sugimoto 2005-7]

(matching instead large- N_c lattice result [Bali et al. 2013] for $m_\rho/\sqrt{\sigma}$ gives $\lambda \approx 12.55$)

give (for $N_c = 3$ and $\lambda = 16.63 \dots 12.55$):

- LO decay rate of ρ meson $\sim \lambda^{-1} N_c^{-1}$

$$\Gamma_{\rho \rightarrow 2\pi}/m_\rho = 0.1535 \dots 0.2034 \quad (\text{exp.: } 0.191(1))$$

- decay rate for $\omega \rightarrow 3\pi$ (from Chern-Simons part of D8 action) $\sim \lambda^{-4} N_c^{-2}$

$$\Gamma_{\omega \rightarrow 3\pi}/m_\omega = 0.0033 \dots 0.0102 \quad (\text{exp.: } 0.0097(1))$$

- Witten-Veneziano mass $m_{\eta_0} = \frac{\sqrt{N_f/N_c}}{3\sqrt{3}\pi} \lambda M_{\text{KK}} \approx 967 \dots 730 \text{ MeV}$ for $N_f = 3$;
with explicit quark mass terms and fitted $m_\pi, m_K \rightarrow m_\eta, m_{\eta'}$ within 10%

Glueballs from supergravity background

- ∃ scalar and tensor glueballs corresponding to 5D dilaton Φ and graviton G_{ij}
- Csaki, Ooguri, Oz & Terning 1999

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Type-IIA supergravity compactified on x_4 -circle many more modes:
 Constable & Myers 1999; Brower, Mathur & Tan 2000

Mode Sugra fields J^{PC}	S_4 G_{44} 0^{++}	T_4 Φ, G_{ij} $0^{++}/2^{++}$	V_4 C_1 0^{-+}	N_4 B_{ij} 1^{+-}	M_4 C_{ij4} 1^{--}	L_4 G_{α}^{α} 0^{++}
n=0	7.30835	22.0966	31.9853	53.3758	83.0449	115.002
n=1	46.9855	55.5833	72.4793	109.446	143.581	189.632
n=2	94.4816	102.452	126.144	177.231	217.397	277.283
n=3	154.963	162.699	193.133	257.959	304.531	378.099
n=4	228.709	236.328	273.482	351.895	405.011	492.171

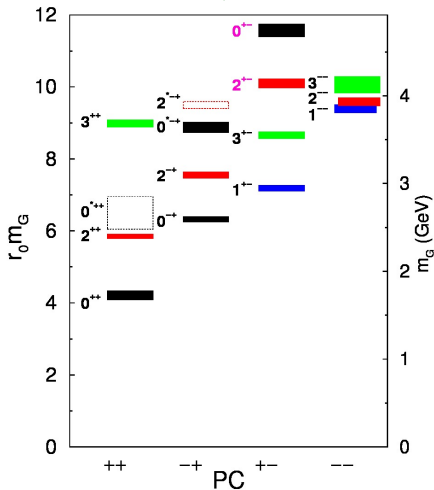
Lowest mode **not** from dilaton, but from “exotic polarization” – in 11D notation:

$$\delta g_{44} = -\frac{r^2}{L^2} f H(r) G(x), \quad \delta g_{\mu\nu} = \frac{r^2}{L^2} \left[\frac{1}{4} H(r) \eta_{\mu\nu} - \left(\frac{1}{4} + \frac{3R^6}{5r^6 - 2R^6} \right) H(r) \frac{\partial_\mu \partial_\nu}{M^2} \right] G(x)$$

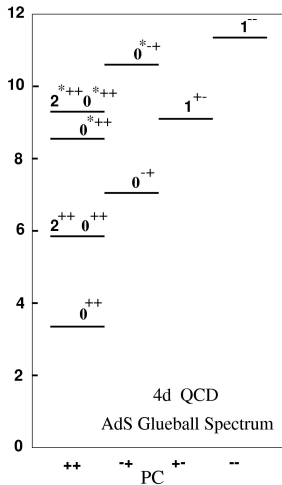
$$\delta g_{11,11} = \frac{r^2}{L^2} \frac{1}{4} H(r) G(x), \quad \delta g_{rr} = -\frac{L^2}{r^2} f^{-1} \frac{3R^6 H(r) G(r)}{5r^6 - 2R^6}, \quad \delta g_{r\mu} = \frac{90r^7 R^6 H(r) \partial_\mu G(x)}{M^2 L^2 (5r^6 - 2R^6)^2}$$

Lattice glueballs vs. supergravity glueballs

Morningstar & Peardon hep-lat/9901004:



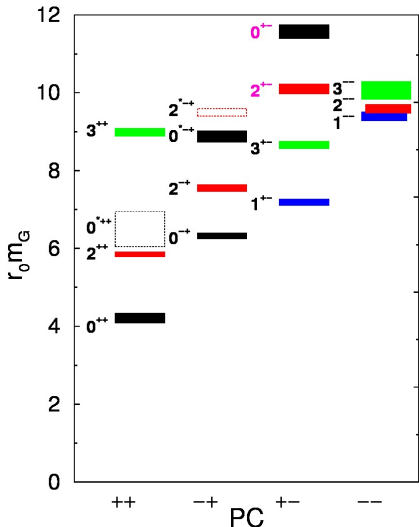
Brower, Mathur & Tan 2000:



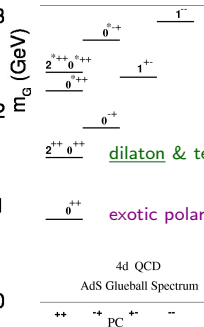
when mass scales matched on $2^{++} \rightarrow$ seemingly good qualitative agreement!
but somewhat stretched and slightly too many 0^{++}

Lattice vs. supergravity glueballs in Sakai-Sugimoto model

Sakai-Sugimoto model: glueball masses $\propto M_{KK} = 949$ MeV fixed by m_ρ



now good match lowest $0^{++} \leftrightarrow$ dilaton



dilaton & tensor 1487 MeV

exotic polarization 855 MeV

4d QCD
AdS Glueball Spectrum

Lattice vs. supergravity glueballs in Sakai-Sugimoto model

Should exotic polarization (δG_{44} with x_4 the compactified direction of SYM_{4+1}) be excluded as lowest glueball mode?

- possibly not part of spectrum of holographic QCD in limit $M_{\text{KK}} \rightarrow \infty, \lambda \rightarrow 0$ (already asked by Constable & Myers)
- simpler bottom-up AdS/QCD have dilaton mode as dual for lowest glueball

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- next lowest scalar mode ~ 1487 MeV is (predominantly) dilaton mode (induces metric perturbations other than δG_{44})

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- next lowest scalar mode ~ 1487 MeV is (predominantly) dilaton mode (induces metric perturbations other than δG_{44})

Nonrealistic degeneracy of dilatonic 0^{++} and tensor 2^{++} suggests that supergravity approximation insufficient for masses

Take good results for (dimensionless) mesonic Γ/m as encouragement for calculation of relative width

Glueball- $\bar{q}q$ couplings in Sakai-Sugimoto model

Gravitational modes stable in confined background, but can calculate *effective action for glueball- $\bar{q}q$ interactions*

done for lowest (exotic) mode by

Hashimoto, Tan & Terashima, Phys.Rev. D77 (2008) 086001 [arXiv:0709.2208]

revisited and extended to other modes by

Brünner, Parganlija & AR, Phys.Rev. D91 (2015) 106002 [arXiv:1501.07906]

For example: Vertices of one glueball and two (massless) pions for “exotic” mode:

$$S_{G_E\pi\pi} = \text{Tr} \int d^4x \frac{1}{2} \partial_\mu \pi \partial_\nu \pi \left(\check{c}_1 \eta^{\mu\nu} - c_1 \frac{\partial^\mu \partial^\nu}{M_E^2} \right) G_E$$

for “predominantly dilatonic” mode:

$$S_{G_D\pi\pi} = \text{Tr} \int d^4x \frac{1}{2} \partial_\mu \pi \partial_\nu \pi \check{c}_1 \left(\eta^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{M_D^2} \right) G_D$$

with $\{c_1, \check{c}_1, \tilde{c}_1\} = \{62.66, 16.39, 17.23\} \times \lambda^{-1/2} N_c^{-1} M_{\text{KK}}^{-1}$

and many more: $S_{G_{\rho\rho}} \propto \lambda^{-1/2} N_c^{-1}$, $S_{G_{\rho\pi\pi}} \propto \lambda^{-1} N_c^{-3/2}$, ...

Glueball decay rates in Sakai-Sugimoto model

Results for decay into two pions:

$$\text{Exotic mode: } \Gamma_{G_E \rightarrow \pi\pi} / M_E \approx \frac{13.79}{\lambda N_c^2} \approx 0.092 \dots 0.122 \quad (M_E \approx 855 \text{MeV})$$

$$\text{Dilaton mode: } \Gamma_{D \rightarrow \pi\pi} / M_D \approx \frac{1.359}{\lambda N_c^2} \approx 0.009 \dots 0.012 \quad (M_D \approx 1487 \text{MeV})$$

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Most likely *experimental candidates* for meson with dominant scalar glueball content:
 $f_0(1500)$ or $f_0(1710)$

$$\Gamma^{(\text{ex})}(f_0(1500) \rightarrow \pi\pi) / (1505 \text{MeV}) = 0.025(3)$$

$$\Gamma^{(\text{ex})}(f_0(1710) \rightarrow \pi\pi) / (1722 \text{MeV}) \sim 0.01$$

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NB: relative width of lowest (exotic) scalar mode much larger than next ones!?

→ take as another indication that G_E should be discarded

Glueball decay rates in Sakai-Sugimoto model (cont'd)

Full decay pattern:

decay $G_D \rightarrow 4\pi$ suppressed (below 2ρ threshold): $\Gamma_{G \rightarrow 4\pi} / \Gamma_{G \rightarrow 2\pi} \sim \lambda^{-1} N_c^{-1}$,
while $f_0(1500) \rightarrow 4\pi$ dominant:

decay	Γ/M (PDG)	$\Gamma/M[G_D]$
$f_0(1500)$ (total)	0.072(5)	0.027...0.037
$f_0(1500) \rightarrow 4\pi$	0.036(3)	0.003...0.005
$f_0(1500) \rightarrow 2\pi$	0.025(2)	0.009...0.012
$f_0(1500) \rightarrow 2K$	0.006(1)	0.012...0.016
$f_0(1500) \rightarrow 2\eta$	0.004(1)	0.003...0.004

$\Rightarrow f_0(1500)$ seemingly disfavored, at least when nearly pure glue

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$f_0(1710) \rightarrow \pi\pi$ OK,

but $f_0(1710)$ decays predominantly into $K\bar{K}$!

— not reproduced by (chiral) WSS model,

but may be due to mechanism of “chiral suppression of scalar glueball decay”
(Chanowitz 2005)

Nonchiral enhancement in mass-deformed WSS?

Current quark masses can be introduced in principle through deformations of the WSS model by either world-sheet instantons [Hashimoto, Hirayama, Lin, Yee 2008; Aharony, Kutasov 2008] or with bifundamental background scalar \mathcal{T} [Bergman, Seki, Sonnenschein 2007]

both lead to

$$\int d^4x \int_{u_{\text{KK}}}^{\infty} du h(u) \text{Tr} \left(\mathcal{T}(u) \underbrace{\text{P} e^{-i \int dz A_z(z,x)}}_{U(x)=\exp i\Pi^a \lambda^a / f_\pi} + h.c. \right),$$

where $h(u)$ includes metric (glueball) fields

Choosing appropriate boundary conditions for \mathcal{T} , the quark mass matrix arises through

$$\int_{u_{\text{KK}}}^{\infty} du h(u) \mathcal{T}(u) \propto \mathcal{M} = \text{diag}(m_u, m_d, m_s),$$

→ Gell-Mann-Oakes-Renner relations

Nonchiral enhancement in mass-deformed WSS!

F. Br unner & AR, PRL 115 (2015) 131601 [1504.05815]

Holographic realization of mass terms

→ additional vertices between glueballs and pseudoscalars

- Rigorously calculable for $G_D \eta_0^2$,

$$\mathcal{L}_{G_D \eta_0 \eta_0}^{\text{chiral}} = \frac{3}{2} d_0 m_0^2 \eta_0^2 G_D, \quad d_0 \approx \frac{17.915}{\lambda^{1/2} N_c M_{\text{KK}}}$$

but not (yet) calculable for octet.

- Parametrize uncertainty by free parameter x :

$$\mathcal{L}_{G_D \pi \pi}^{\text{massive}} = \frac{3}{2} d_m G_D \mathcal{L}_m^{\mathcal{M}}, \quad d_m \equiv x d_0$$

Most symmetric choice $x = 1$ (\Leftrightarrow no $G_D \rightarrow \eta \eta'$)

→ relatively strong enhancement factor for kaons and η mesons:

$$\Gamma_{G \rightarrow PP}^{\text{chiral}} \rightarrow \Gamma_{G \rightarrow PP}^{\text{chiral}} \times \left(1 - 4 \frac{m_P^2}{M_G^2}\right)^{1/2} \left(1 + 8.480 \frac{m_P^2}{M_G^2}\right)^2$$

Comparison with $f_0(1710)$

decay	Γ/M (PDG)	$\Gamma/M[G_D]$ (chiral)	$\Gamma/M[G_D]$ (massive)
$f_0(1710)$ (total)	0.081(5)	0.059...0.076	0.083...0.106
$f_0(1710) \rightarrow 2K$	(*) 0.029(10)	0.012...0.016	0.029...0.038
$f_0(1710) \rightarrow 2\eta$	0.014(6)	0.003...0.004	0.009...0.011
$f_0(1710) \rightarrow 2\pi$	0.012($^{+5}_{-6}$)	0.009...0.012	0.010...0.013
$f_0(1710) \rightarrow 2\rho, \rho\pi\pi \rightarrow 4\pi$?	0.024...0.030	0.024...0.030
$f_0(1710) \rightarrow 2\omega$	0.010($^{+6}_{-7}$)	0.011...0.014	0.011...0.014
$f_0(1710) \rightarrow \eta\eta'$?	0	if 0 : \updownarrow
$\Gamma(\pi\pi)/\Gamma(K\bar{K})$	0.41 $^{+0.11}_{-0.17}$	3/4	0.35
$\Gamma(\eta\eta)/\Gamma(K\bar{K})$	0.48 ± 0.15	1/4	0.28

* PDG ratios for decay rates + $\text{Br}(f_0(1710) \rightarrow KK) = 0.36(12)$ [Albaladejo&Oller 2008]

- decays into 2 pseudoscalars: massive WSS perfectly compatible with PDG data!

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- decays into 2 pseudoscalars: massive WSS perfectly compatible with PDG data!
- significant decay into 4 pions (after extrapolation to beyond 2ρ threshold): falsifiable prediction of this model!

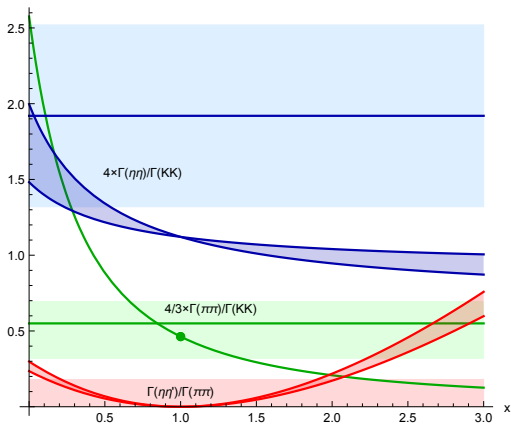
$(f_0(1710) \rightarrow 2\rho^0)$ forthcoming from CMS-TOTEM!

Constraints on $\eta\eta'$ rates for $f_0(1710)$ as \approx pure glueball

Relaxing $x = 1$: [F. Brünner & AR, PRD92, 1510.07605]

WSS model gives flavor asymmetries consistent with experimental results for $f_0(1710)$ in

as long as $\Gamma(G \rightarrow \eta\eta')/\Gamma(G \rightarrow \pi\pi) \lesssim 0.04$ (upper limit from WA102: < 0.18)



[by contrast: Frère & Heeck 1506.04766 predict $\eta\eta'$ rates above WA102 limit]

Tensor glueball decay rates in Sakai-Sugimoto model

Tensor glueball in WSS, and extrapolated to higher mass:

decay	M	$\Gamma/M[T(M)]$
$T \rightarrow 2\pi$	1487	0.013...0.018
$T \rightarrow 2K$	1487	0.004...0.006
$T \rightarrow 2\eta$	1487	0.0005...0.0007
T (total)	1487	$\approx 0.02 \dots 0.03$
$T \rightarrow 2\rho \rightarrow 4\pi$	2000	0.135...0.178
$T \rightarrow 2\omega \rightarrow 6\pi$	2000	0.045...0.059
$T \rightarrow 2\pi$	2000	0.014...0.018
$T \rightarrow 2K$	2000	0.010...0.013
$T \rightarrow 2\eta$	2000	0.0018...0.0024
T (total)	2000	$\approx 0.16 \dots 0.21$

With a mass of 2 GeV, the relative width turns out to be comparable with that of the rather broad tensor meson $f_2(1950)$, which has $\Gamma/M = 0.24(1)$.

Very narrow (unconfirmed) candidate $f_J(2220)$ not compatible with WSS

Summary – Glueballs in Witten-Sakai-Sugimoto model

After fitting just m_ρ to fix $M_{KK} = 949$ MeV

- good prediction of higher vector and axial vector mesons masses,
- reasonable prediction of glueball masses if “exotic mode” discarded

after fitting f_π or $m_\rho/\sqrt{\sigma}$ to also fix 't Hooft coupling at $\lambda = 16.63 \dots 12.55$

- good prediction of ρ and ω decay rates
- good prediction of anomalous $m'_\eta \propto N_c^{-\frac{1}{2}} \lambda M_{KK}$

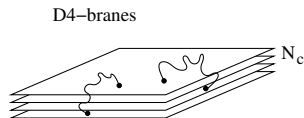
Holographic glueball decay rates:

- narrow partial width $G_D \rightarrow \pi\pi$,
quite compatible with experimental data for $f_0(1710)$ as nearly pure glueball
- much stronger decay of $f_0(1710)$ into $K\bar{K}$ need not be indicative of $s\bar{s}$ nature – well reproduced if (so far unobserved) decay into $\eta\eta'$ small
- predictions for decay to 4π , $\eta\eta'$ falsifiable by CMS-TOTEM, BESIII, ...
- tensor glueball broad if at $\gtrsim 2$ GeV
- pseudoscalar glueball again narrow (in prep.)

Witten model: Holographic nonsupersymmetric QCD

E. Witten, *Adv. Theor. Math. Phys.* 2, 505 (1998):

Type-IIA string theory with $N_c \rightarrow \infty$ $D4$ branes
dual to $4 + 1$ -dimensional super-Yang-Mills theory



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dual to **4 + 1**-dimensional super-Yang-Mills theory

supersymmetry completely broken by compactification

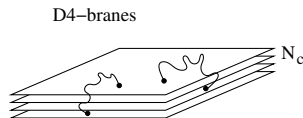
on “thermal-like” circle $x_4 \equiv x_4 + 2\pi/M_{\text{KK}}$ (Kaluza–Klein)

- antisymmetric b.c. for adjoint fermions: masses $\sim M_{\text{KK}}$
- adjoint scalars not protected by gauge symmetry: also masses $\sim M_{\text{KK}}$

→ dual to pure-gluon YM theory

3+1-dimensional at scales $\ll M_{\text{KK}}$

but supergravity approximation needs weak curvature,
cannot take limit $M_{\text{KK}} \rightarrow \infty$

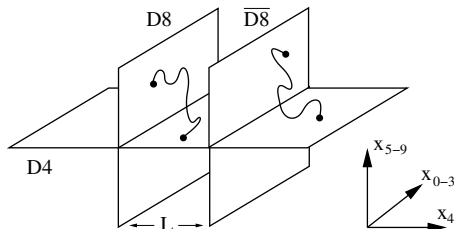


Sakai-Sugimoto model: Adding chiral quarks

T. Sakai, S. Sugimoto, *Prog. Theor. Phys.* **113**, 843 (2005)

add N_f D8- and $\overline{D8}$ -branes, separated in x_4 , $N_f \ll N_c$ (probe branes)

	0	1	2	3	4	5	6	7	8	9
D4	x	x	x	x	x					
D8/ $\overline{D8}$	x	x	x	x		x	x	x	x	x



4-8, $4-\overline{8}$ strings

→ fundamental, massless
chiral fermions

flavor symmetry

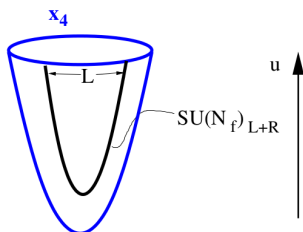
$U(N_f)_L \times U(N_f)_R$

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add N_f D8- and $\overline{D8}$ -branes, separated in x_4 , $N_f \ll N_c$ (probe branes)

	0	1	2	3	4	5	6	7	8	9
D4	x	x	x	x	x					
D8/ $\overline{D8}$	x	x	x	x		x	x	x	x	x



4-8, $4-\overline{8}$ strings

→ fundamental, massless
chiral fermions

flavor symmetry

$U(N_f)_L \times U(N_f)_R$

spontaneously broken because D8- $\overline{D8}$ have
to join in cigar-shaped topology

for now: maximal separation in x_4 (antipodal on x_4 circle): $L = \pi/M_{KK}$

Quantitative predictions

Parameter-free prediction of (axial-)vector meson mass pattern:

Isotriplet Meson	$\lambda_n = m^2/M_{KK}^2$	m/m_ρ	$(m/m_\rho)^{\text{exp.}}$	$(m/m_\rho)^{N \rightarrow \infty}$
$1^{--} (\rho)$	0.669314	1	1	1
$1^{++} (a_1)$	1.568766	1.531	1.59(5)	1.86(2)
$1^{--} (\rho^*)$	2.874323	2.072	1.89(3)	2.40(4)
$1^{++} (a_1^*)$	4.546104	2.606	2.12(3)	2.98(5)

(last column from lattice study by
Bali et al. JHEP 06, 071 (2013))

agreement within $\lesssim 20\%$

not bad, given that WSS is not yet large- N QCD (in particular at scales $\gtrsim M_{KK}$)

(near-perfect agreement for m_{a_1}/m_ρ with real QCD certainly fortuitous)

Witten-Veneziano mass term

Already in chiral model:

WSS contains (fully determined) Witten-Veneziano mass term for singlet η_0 pseudoscalar from $U(1)_A$ anomaly contributions $\sim 1/N_c$

$$m_0^2 = \frac{N_f}{27\pi^2 N_c} \lambda^2 M_{\text{KK}}^2$$

from $S_{C_1} = -\frac{1}{4\pi(2\pi l_s)^6} \int d^{10}x \sqrt{-g} |\tilde{F}_2|^2$ with $\tilde{F}_2 = \frac{6\pi u^3_{\text{KK}} M_{\text{KK}}^{-1}}{u^4} \left(\theta + \frac{\sqrt{2N_f}}{f_\pi} \eta_0 \right) du \wedge dx^4$,
where θ is the QCD theta angle and $\eta_0(x) = \frac{f_\pi}{\sqrt{2N_f}} \int dz \text{Tr} A_z(z, x)$.

With $N_f = N_c = 3$, $M_{\text{KK}} = 949$ MeV, $\lambda = 16.63 \dots 12.55$: $m_0 = 967 \dots 730$ MeV

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With $N_f = N_c = 3$, $M_{KK} = 949$ MeV, $\lambda = 16.63 \dots 12.55$: $m_0 = 967 \dots 730$ MeV

With finite quark masses η_0 and η_8 no longer mass eigenstates.

Diagonalizing (with $\mathcal{M} = \text{diag}(\hat{m}, \hat{m}, m_s)$), fixing $m_\pi = 140$ MeV and $m_K = 497$ MeV) \rightarrow

$$m_\eta = 518 \dots 476 \text{ MeV}, \quad m_{\eta'} = 1077 \dots 894 \text{ MeV}, \\ \theta_P = -14.4^\circ \dots -24.2^\circ,$$

nice ballpark!