α_S from the top quark pair production cross section

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Why α_s?

- **Strong coupling constant α_s** enters in the calculation of every process that involves the strong interaction

- PDG world average (2015): **0.1181 ± 0.0011**; **~0.9%** relative uncertainty [http://pdg.lbl.gov/2016/reviews/rpp2016-rev-qcd.pdf]

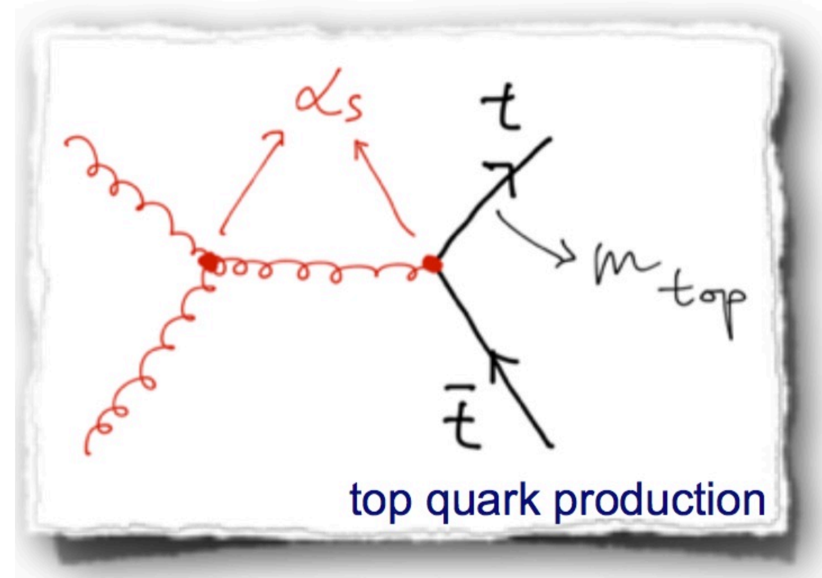
  - Relative uncertainty of the fine structure constant: $\sim 2.3 \cdot 10^{-8}\%$ [http://physics.nist.gov/cgi-bin/cuu/Value?alph]

- Uncertainty on α_s leads to non-negligible uncertainties on many observables

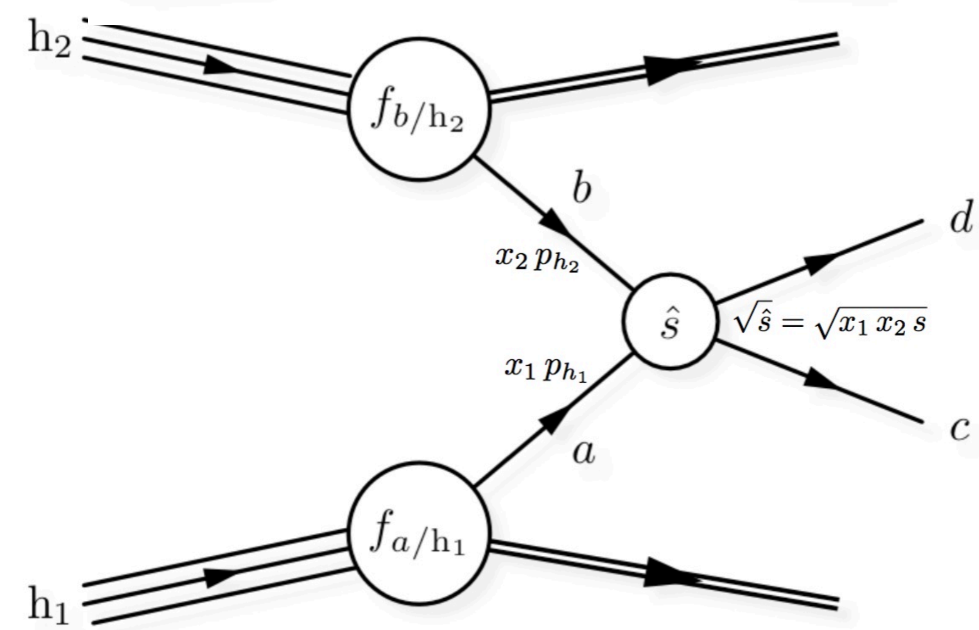
  - Notable examples: Higgs production cross sections, branching ratios

Why σ_{tt}?

Sensitivity of σ_{tt} to α_s



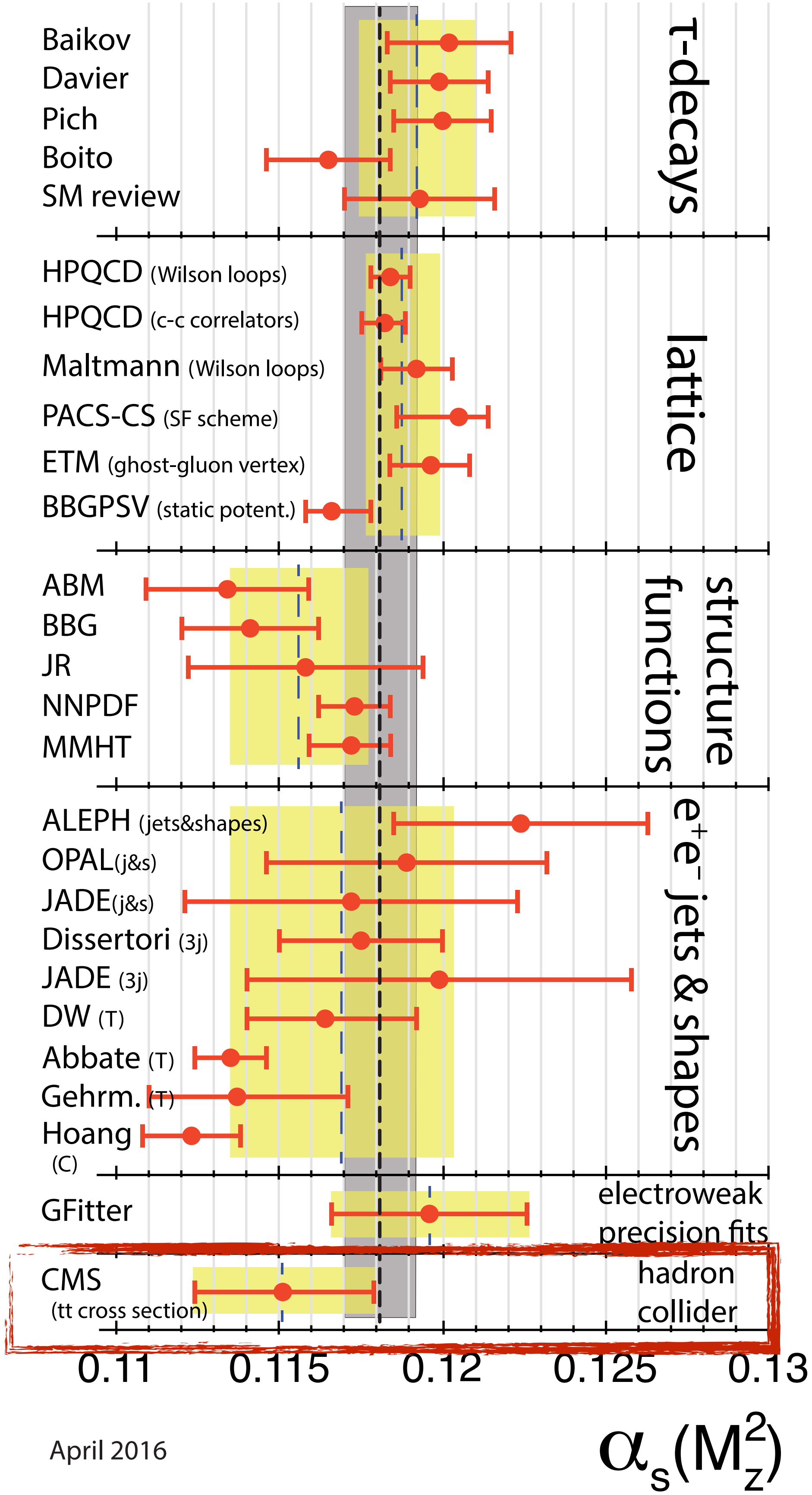
- α_s enters in the process (α_s^2 at LO)
- α_s enters in the **parton distribution functions** (PDFs) through the DGLAP evolution equations



$$d\sigma(h_1 h_2 \to cd) = \int_0^1 dx_1 dx_2 \sum_{a,b} f_{a\,/\,h_1}(x_1, \mu_F^2, \alpha_s)\; f_{b\,/\,h_2}(x_2, \mu_F^2, \alpha_s)\; d\hat{\sigma}^{(ab\to cd)}(Q^2, \mu_F^2, \alpha_s)$$

Inclusive measurement: not subject to hadronisation corrections

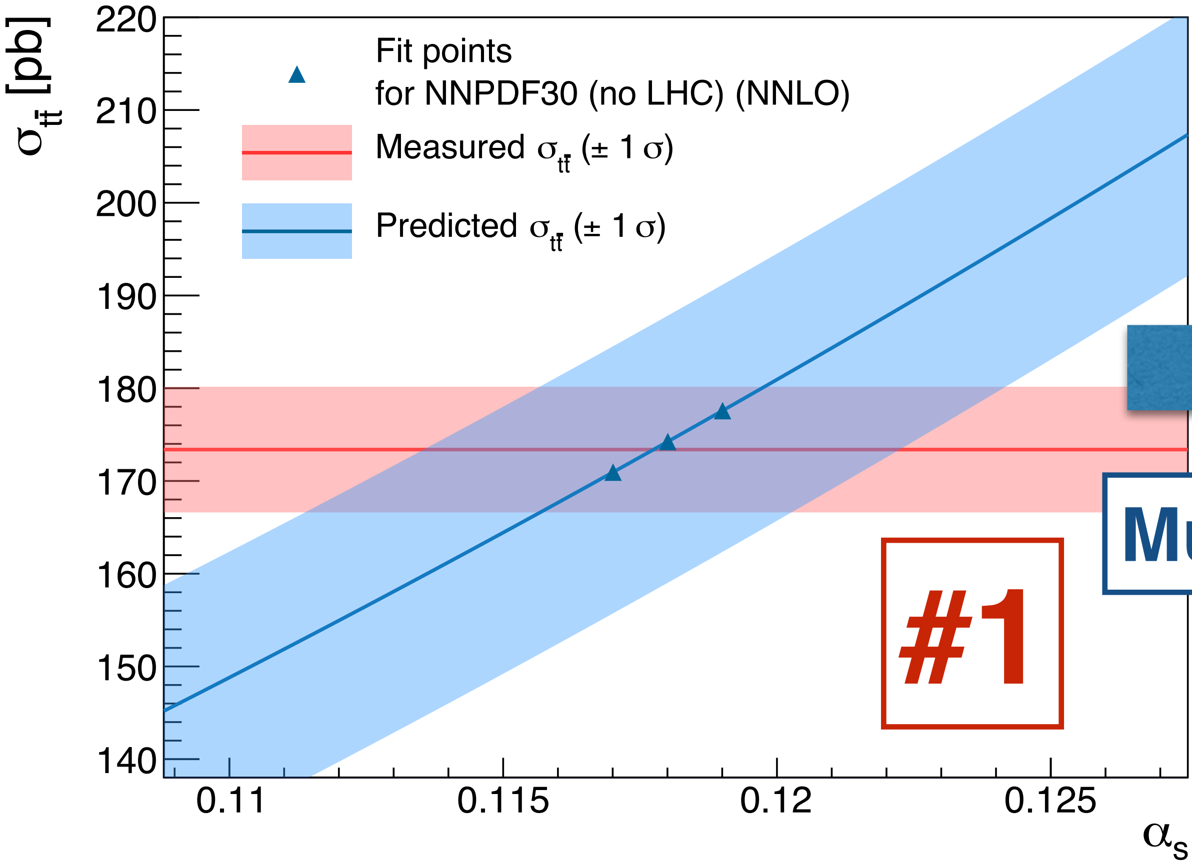
Why σ_{tt}?

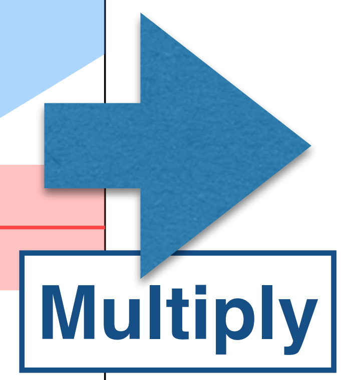


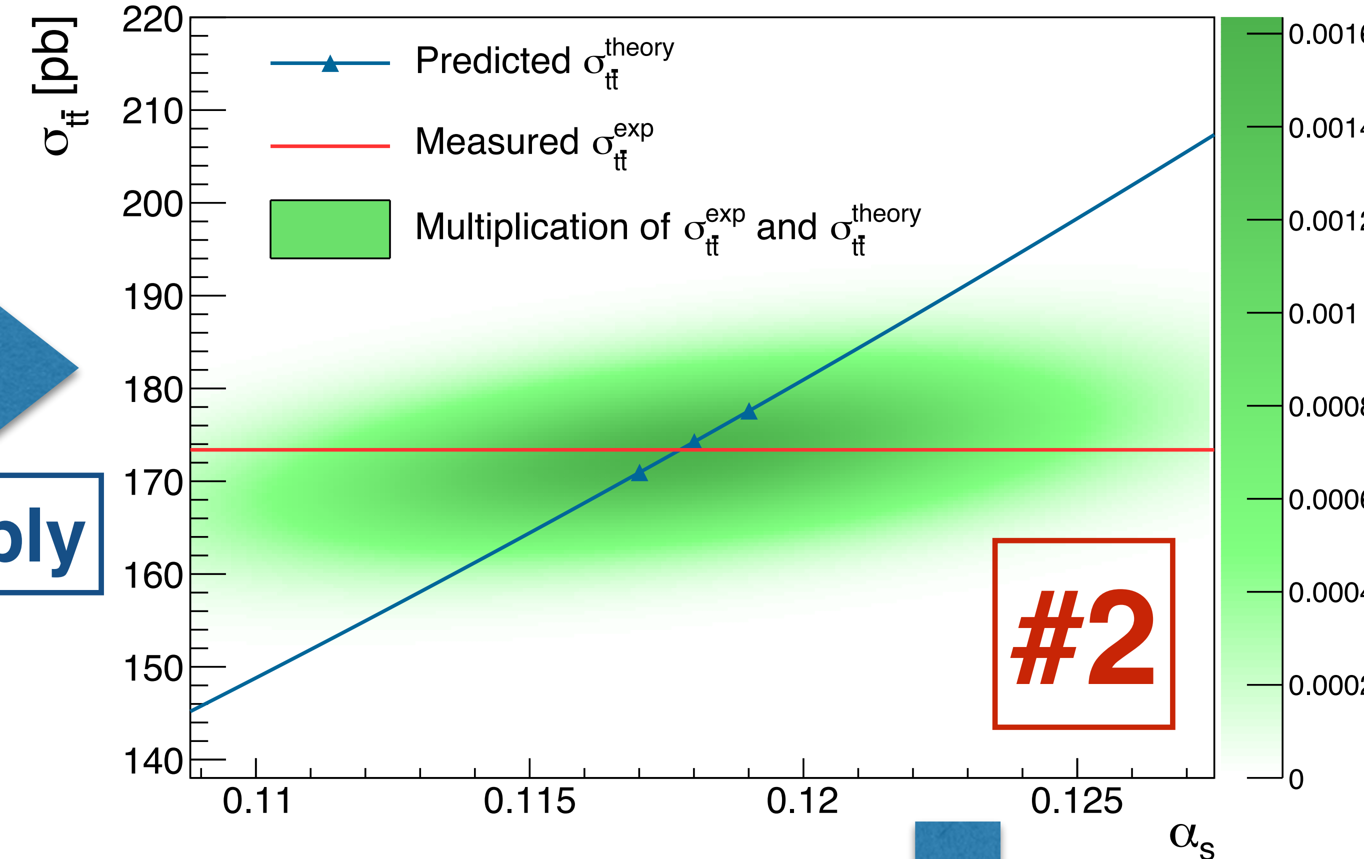
[http://pdg.lbl.gov/2016/reviews/rpp2016-rev-qcd.pdf]

- σ_{tt} is well-measured (known up to NNLO)
- Only few results from hadron colliders in the world average
- Currently one extraction like this available from CMS at 7 TeV [Phys. Let. B 728 (2014)]
- New data available:
  - ATLAS at 7 TeV, 8 TeV and 13 TeV
    [Eur. Phys. J. C 76 (2016) no.11, 642] [Phys. Lett. B 761 (2016) 136]
  - CMS at 7 TeV, 8 TeV and 13 TeV
    [JHEP 1608 (2016) 029] [Phys. Rev. Lett. 116 (2016) no.5, 052002]
  - Tevatron (D0/CDF combination) at 1.96 TeV [Phys. Rev. D 89 (2014), 072001]
- Updates to the PDF sets

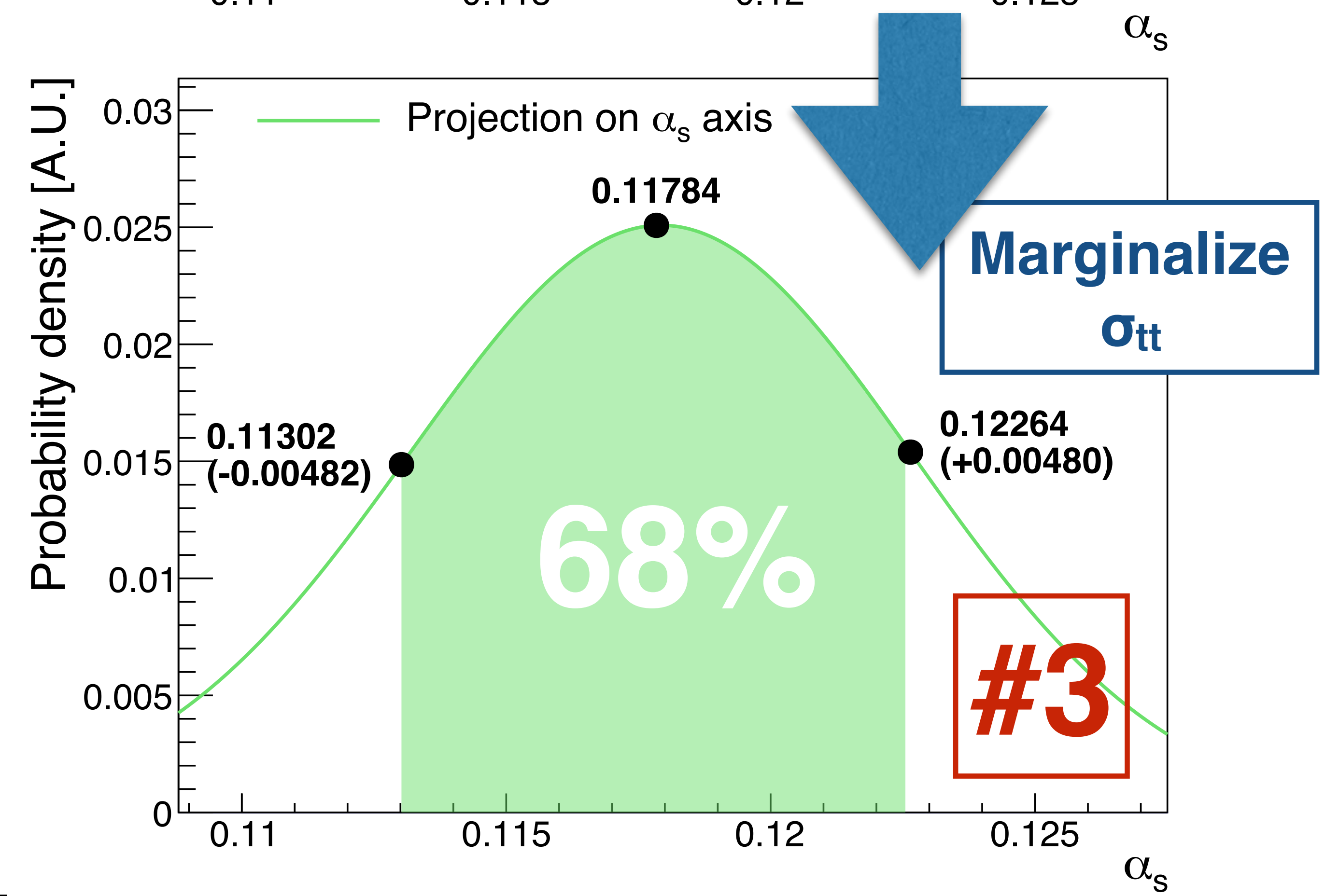
Determining α_s from a σ_{tt} measurement







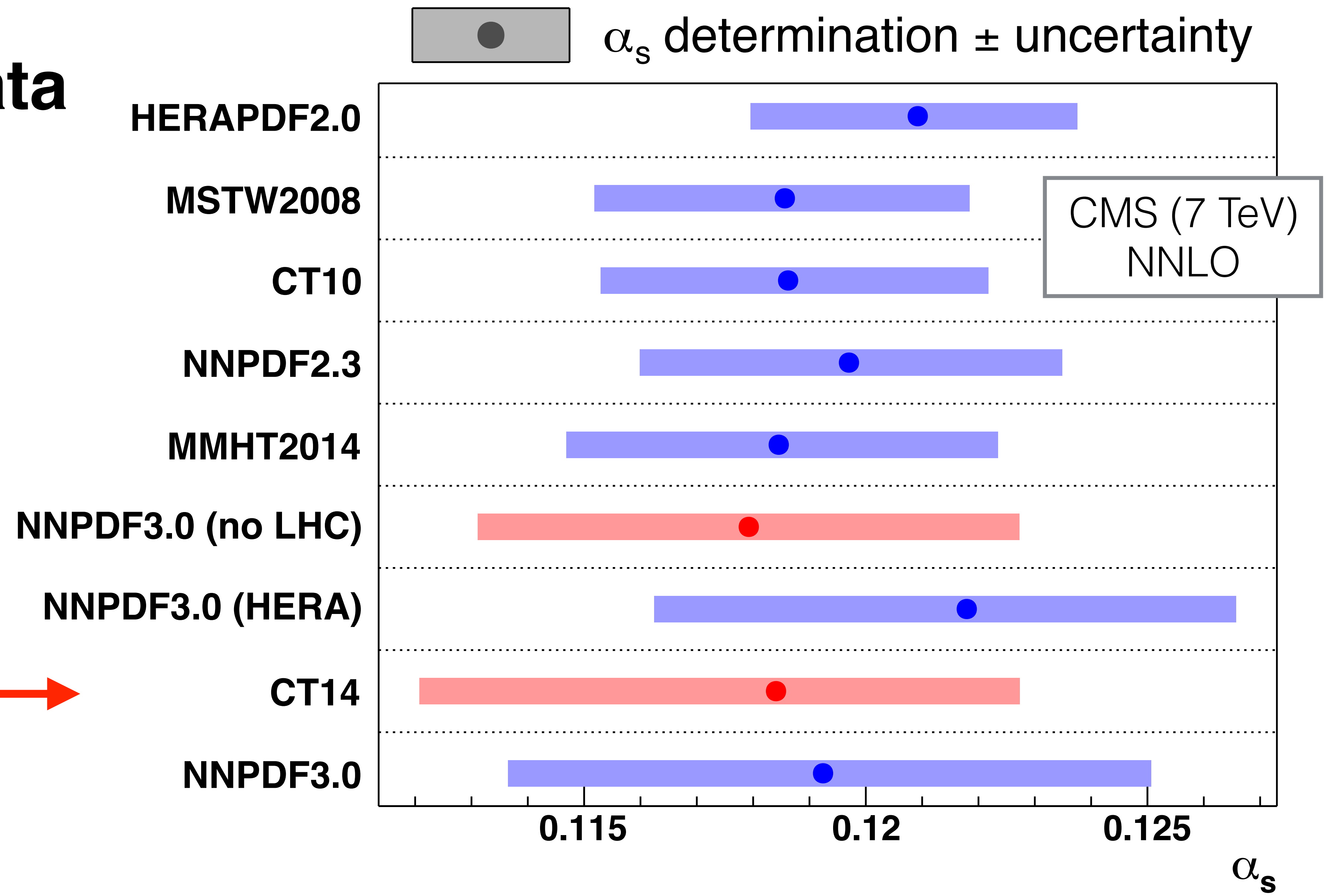
- Shown example:
  CMS 7 TeV, NNPDF3.0 noLHC
- Fit points calculated using `top++2.0`
  [Comput. Phys. Commun. 185 (2014) 2930]
- Procedure from CMS [Phys. Let. B 728 (2014)]
- Dependence of acceptance corrections neglected (effect <1% in region of interest)



Which PDF sets to use?

1. Global fit should **not** include **σ_{tt} data**
2. **Latest** version should be used

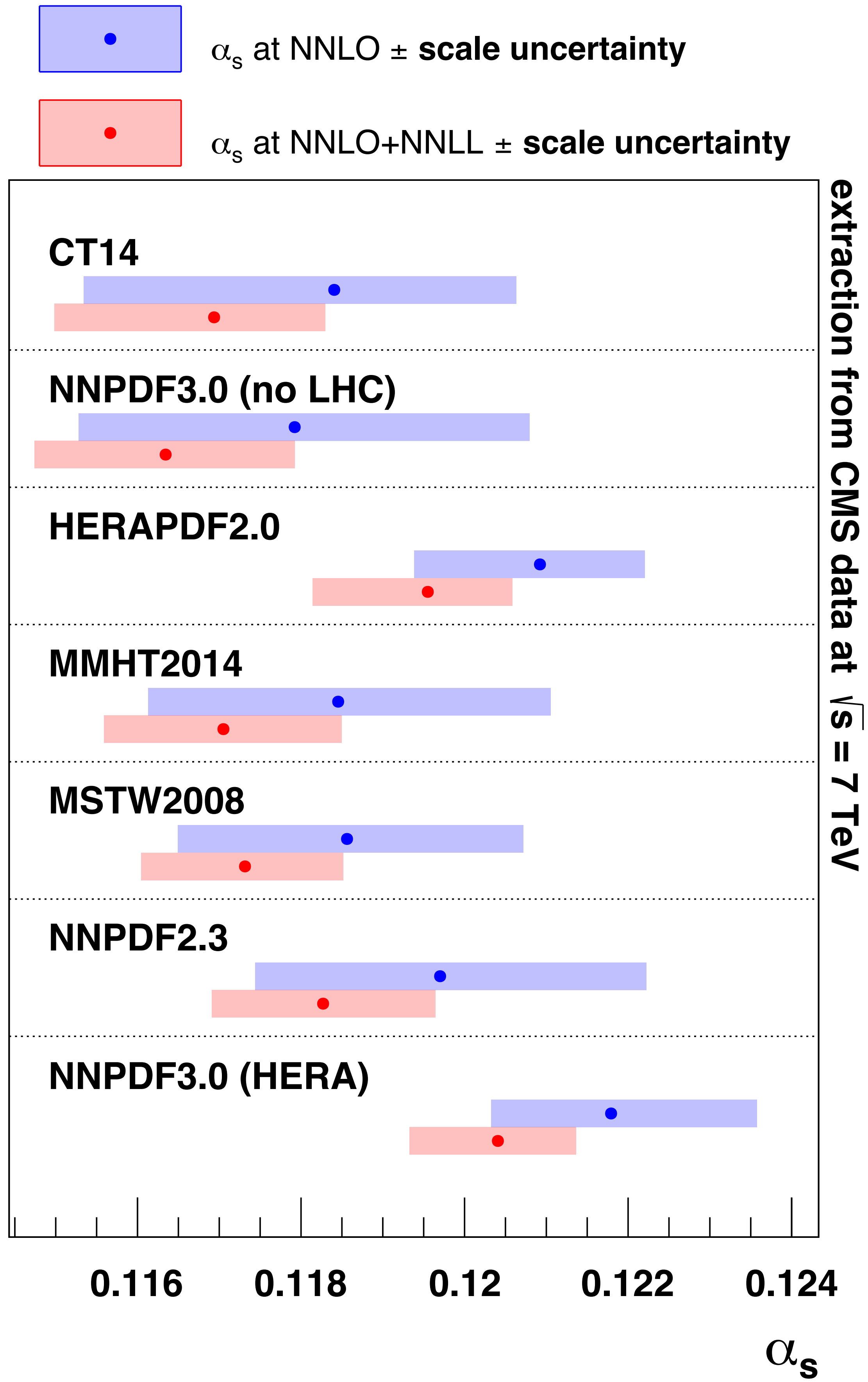
Used PDF sets



- *NNPDF3.0 (no LHC)* still contains σ_{tt} data from Tevatron
  - Tevatron determination only from CT14

Soft gluon resummation (NNLL)

- Using resummation (**NNLO+NNLL**) or without resummation (**NNLO**)
  - **Scale uncertainty** decreases by a **factor of ~2** when using the resummation
  - Determined value of α_s goes **down by ~0.001**
- NNNLO for Higgs gluon fusion shows disagreements with NNLO +NNLL
  - No consensus that NNLL brings reliable reduction of uncertainties



Combining correlated measurements

- One extraction yields **1 central value** and **7 uncertainties** (*statistical, systematic, luminosity, beam energy, pdf, scale, top mass*)

Using NNPDF3.0 noLHC (NNLO)

	Central value	Stat.	Syst.	Lumi.	E_{beam}	PDF	m_{top}	Scale*
ATLAS (7 TeV)	0.12056	0.00092	0.00124	0.00106	0.00097	0.00259	0.00268	0.00198
ATLAS (8 TeV)	0.11651	0.00039	0.00127	0.00173	0.00097	0.00253	0.00290	0.00215
ATLAS (13 TeV)	0.11827	0.00069	0.00233	0.00164	0.00104	0.00204	0.00318	0.00239
CMS (7 TeV)	0.11784	0.00064	0.00129	0.00115	0.00091	0.00266	0.00276	0.00206
CMS (8 TeV)	0.11700	0.00032	0.00135	0.00147	0.00097	0.00252	0.00288	0.00198
CMS (13 TeV)	0.11575	0.00070	0.00324	0.00184	0.00102	0.00207	0.00323	0.00180

*: Assuming a 68% confidence interval (not 100%, flat)

- These uncertainties are (often strongly) **correlated**
- Aim is to **combine** these measurements into a single determination

Maximum Likelihood Estimate Method

- Idea: Fit α_s to the individual probability distribution functions per experiment simultaneously

$$L(\alpha_s) = \prod_i \mathrm{Gauss}(\alpha_s,\, \mu_i + \sum_j \theta_j \delta_{ij},\, \sigma_i) \times \prod_j \mathrm{Gauss}(\theta_j,\, 0,\, 1)$$

  - μ_i : The determination for experiment i (central value of the determination)
  - σ_i : **Statistical uncertainty** for experiment i (from determination)
  - θ_j : The **nuisance parameter** j, normal Gaussian distributed
  - δ_j : Impact of nuisance parameter j (from determination)
- Nuisance parameters take **correlations** into account

Maximum Likelihood Estimate Method

- Idea: Fit α_s to the individual probability distribution functions per experiment simultaneously

$$L(\alpha_s) = \prod_i \mathrm{Gauss}(\alpha_s,\, \mu_i + \sum_j \theta_j \delta_{ij},\, \sigma_i) \times \prod_j \mathrm{Gauss}(\theta_j,\, 0,\, 1)$$

- Theory uncertainties (*PDF, scale, m_t*) excluded from the nuisance parameters
  - Otherwise combination result driven by these parameters (i.e. practically doing a top mass measurement, or a PDF measurement)
  - Combination result strongly dependent on assumed correlation structure
- *Gaussians* replaced by convolutions of *asymmetric Gaussians* to allow **asymmetry** of uncertainties

Maximum Likelihood Estimate Method

$$L(\alpha_s) = \prod_i \mathrm{Gauss}(\alpha_s,\, \mu_i + \sum_j \theta_j \delta_{ij},\, \sigma_i) \times \prod_j \mathrm{Gauss}(\theta_j,\, 0,\, 1)$$

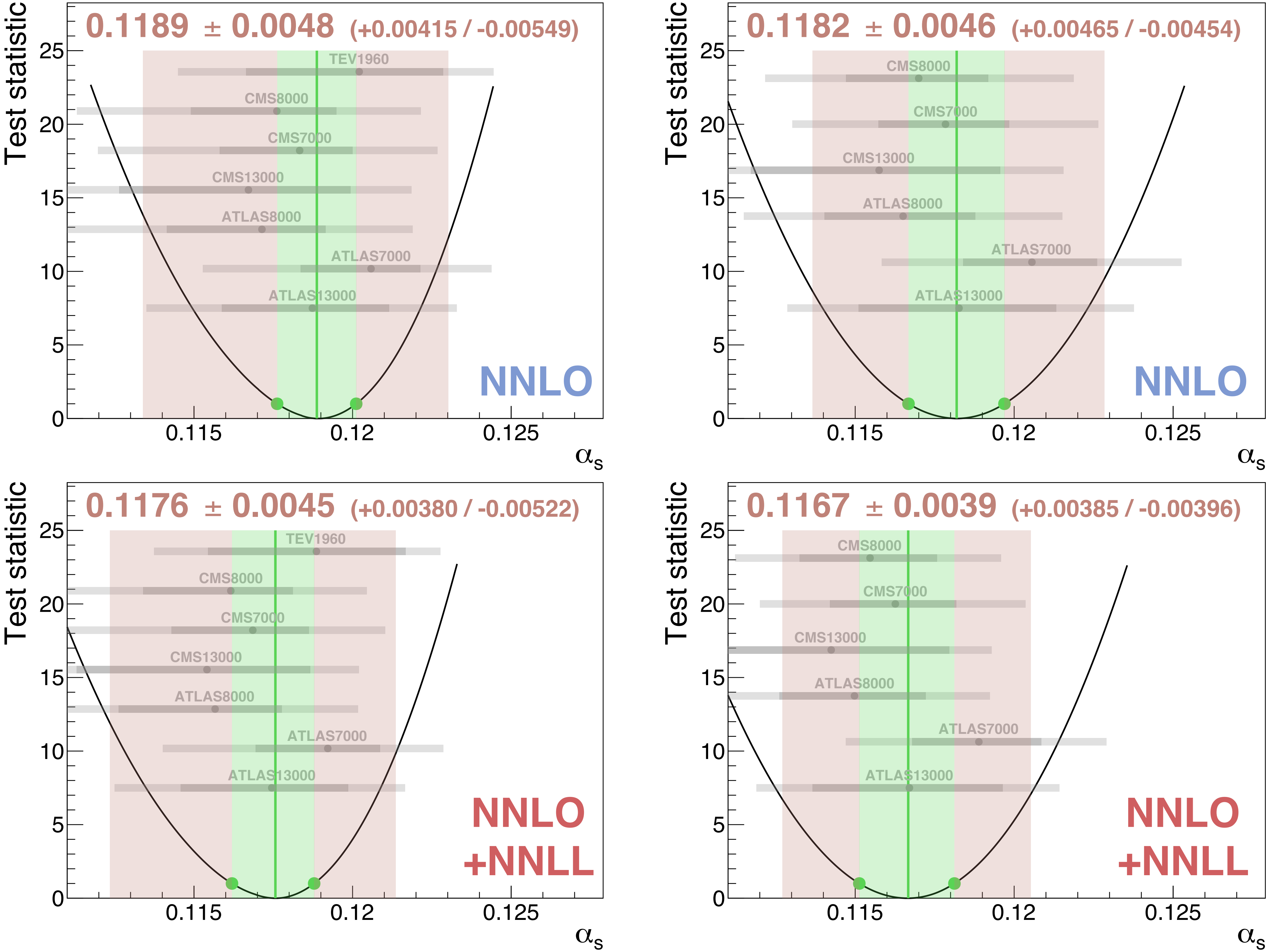
- To extract the uncertainties, a **scan** is performed over α_s, while the nuisance parameters are **profiled**
- For each scan point a **test statistic** q is calculated:

$$q(\alpha_s) = -2\ln\left(\frac{L(\alpha_s,\, \boldsymbol{\theta}_{\alpha_s})}{L(\hat{\alpha}_s,\, \hat{\boldsymbol{\theta}}_{\hat{\alpha}_s})}\right)$$

- $L(\hat{\alpha}_s,\, \hat{\boldsymbol{\theta}}_{\hat{\alpha}_s})$: Likelihood maximised for α_s and **θ**
- $L(\alpha_s,\, \boldsymbol{\theta}_{\alpha_s})$: Likelihood maximised for **θ** (α_s is input)
- -2 and the natural logarithm make q χ^2-distributed (1 dof)

Preliminary results

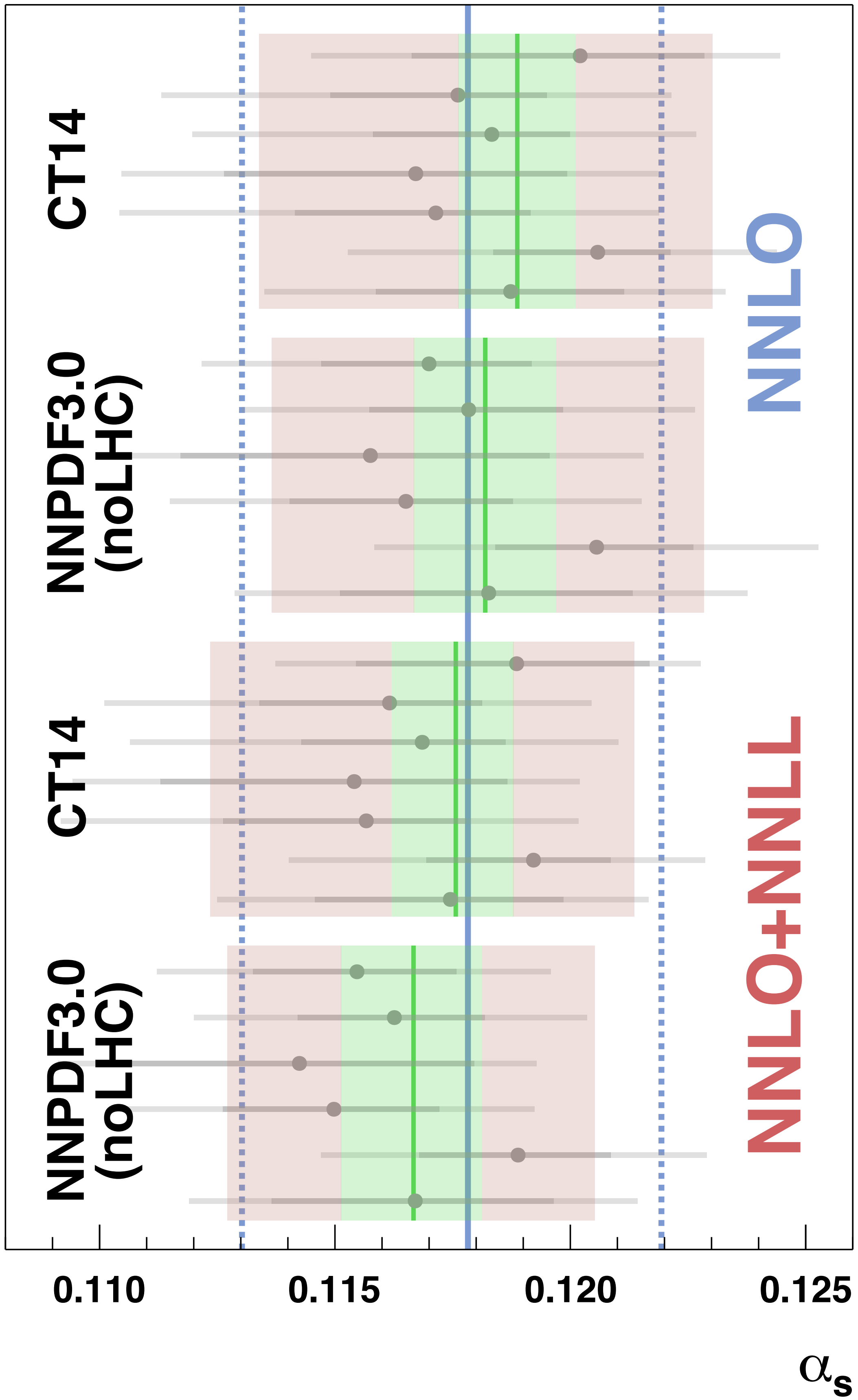
CT14 | NNPDF3.0 noLHC



: Combination uncertainty

: Uncertainty from m_t, PDF and scale

Preliminary results



- Final combination result: straight average between the three PDF sets and with/ without resummation

$$\alpha_s(m_Z) = 0.1178 \, {}^{+0.00411\,(+3.49\%)}_{-0.00480\,(-4.07\%)}$$

Conclusion

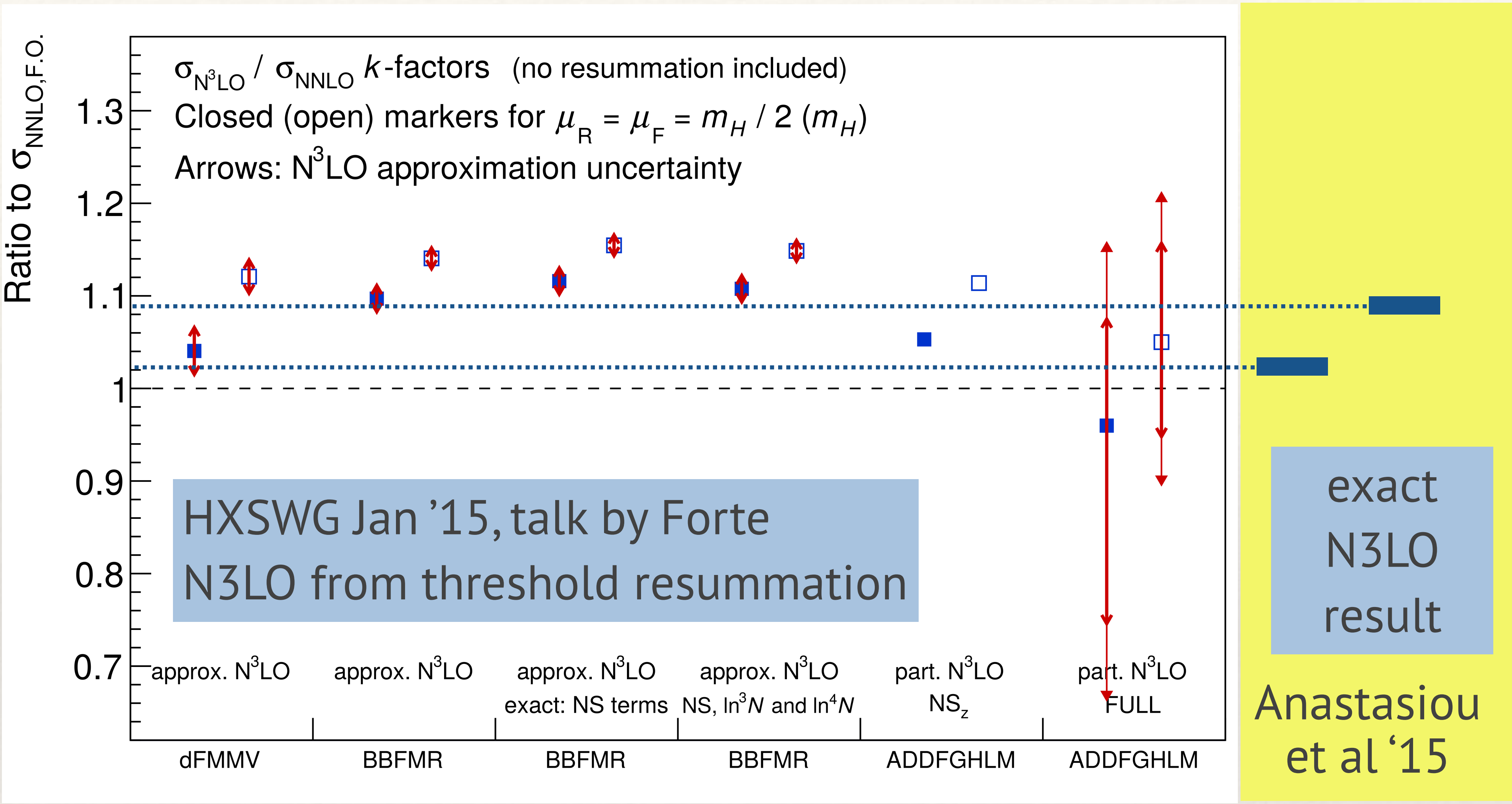
- Developed machinery to determine α_s from σ_{tt} measurements and to combine these
  - Final precision of ~3.8%
- Overall we made conservative choices:
  - Scale uncertainties not treated as a 'flat' probability distribution, but as a Gaussian
  - Chose PDF sets that are obtained without input from top quark production, rather than those with smallest claimed uncertainties
  - Average of NNLO and NNLO+NNLL (instead of just NNLO+NNLL)

Backup

Slide from Gavin Salam (2015)

NNLO v. NNLL+NNLO?

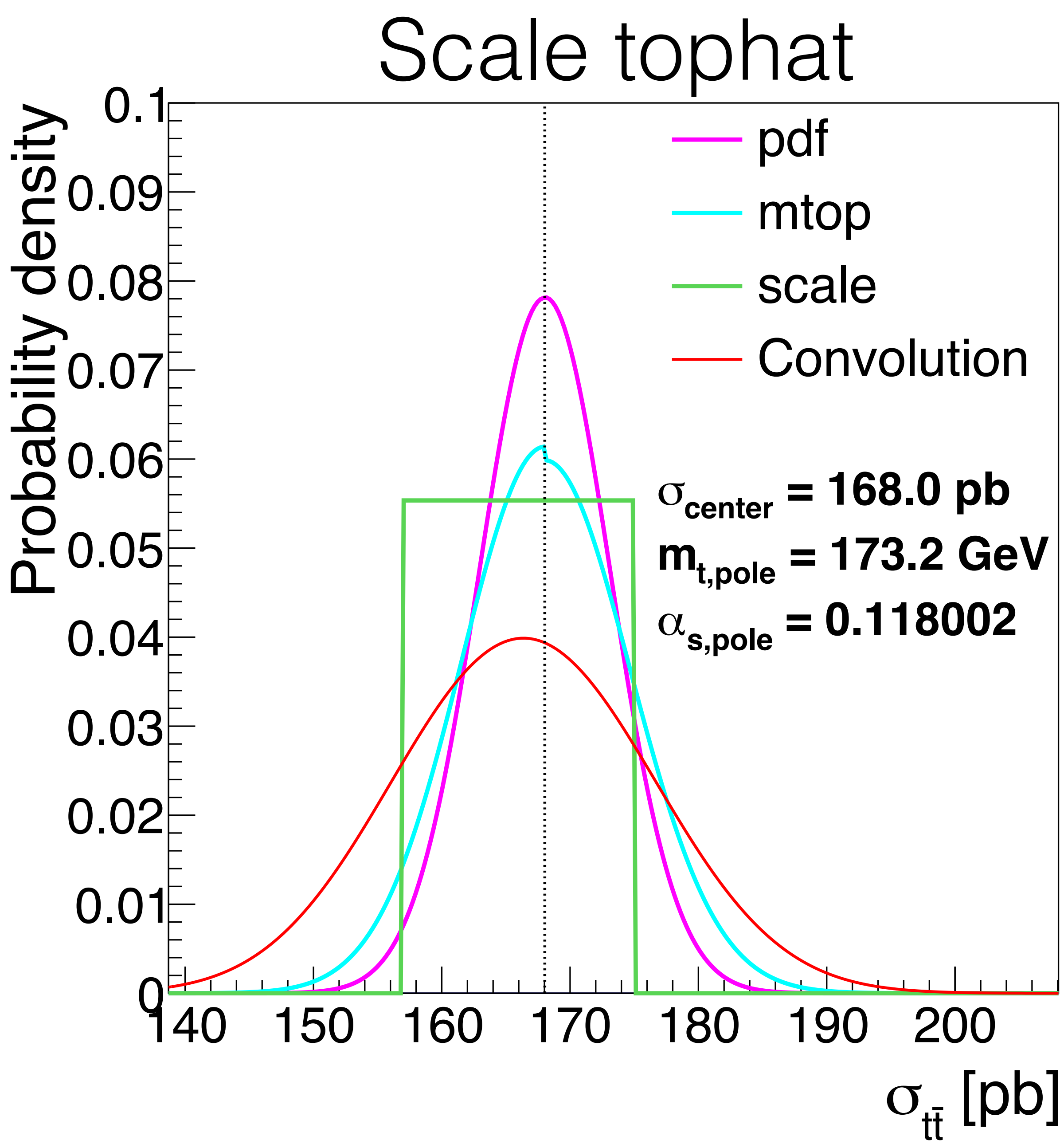
$N^3LO/NNLO$ k-FACTOR in gluon fusion → Higgs



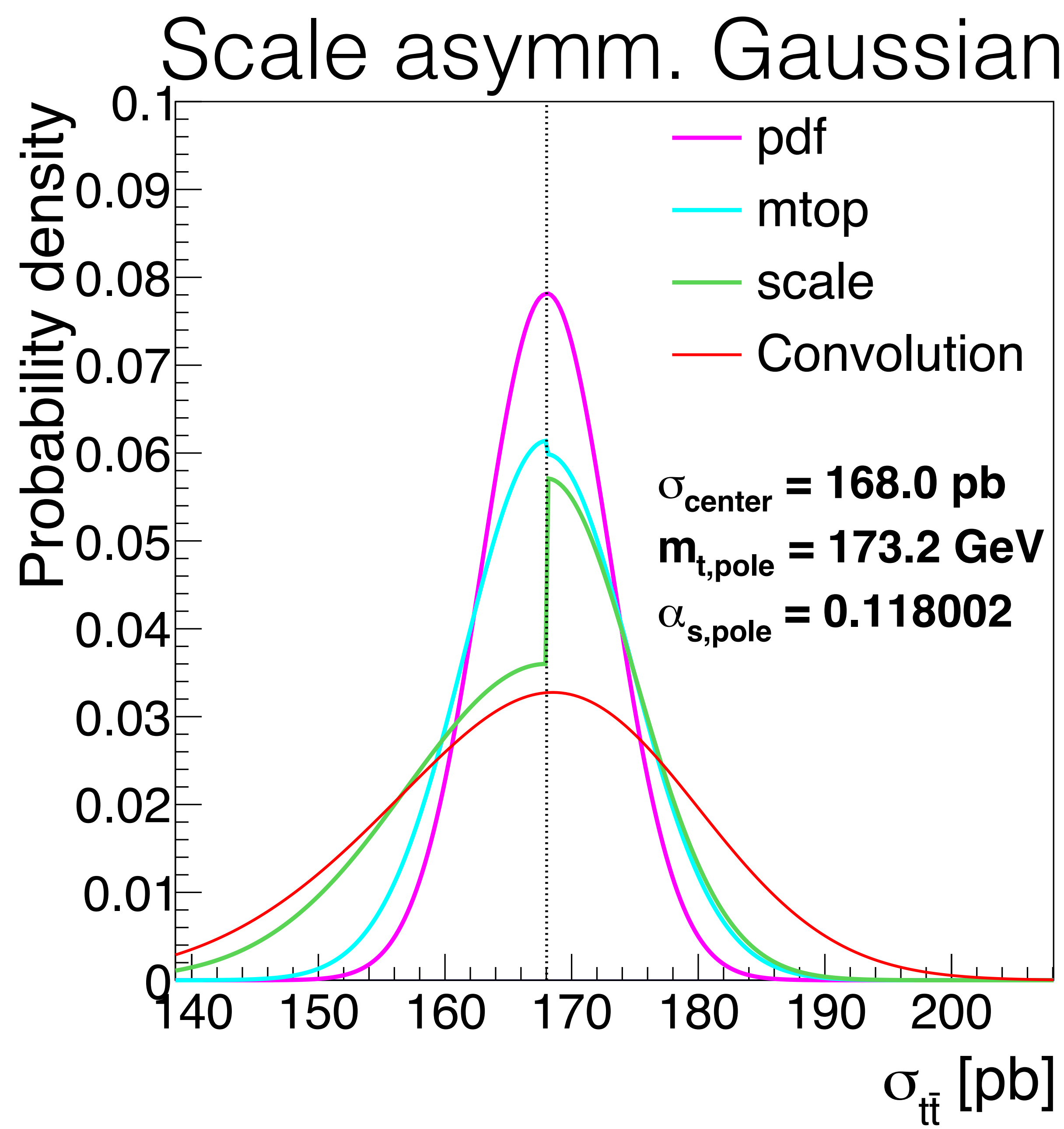
In case of Higgs production (only process known at N3LO), threshold approx.for N3LO was off by 2–10%.

We will consider results with and without NNLL

Extracting α_s (5) — Scale: Tophat vs. Gaussian



$\alpha_s = 0.1201 \pm 0.0032$



$\alpha_s = 0.1196 \pm 0.0038$

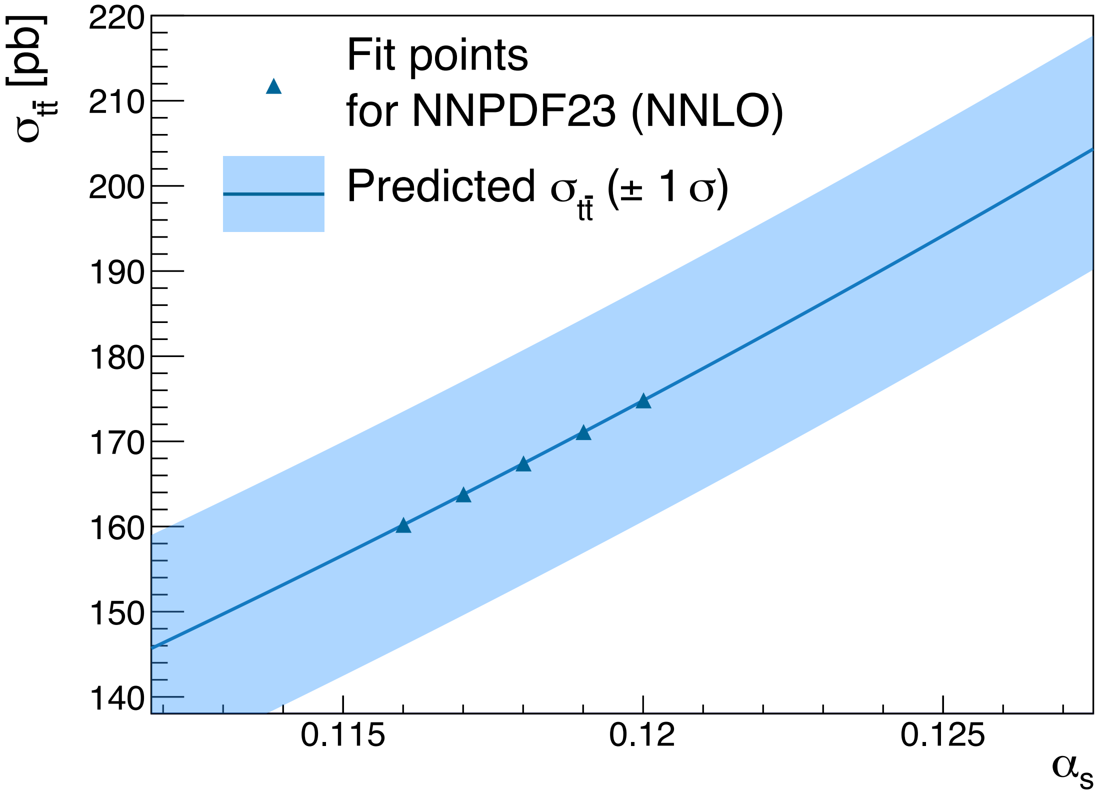
From CMS data at 7 TeV Using NNPDF2.3 NNLO

- Asymm. Gaussian is slightly more conservative

Extracting α_s from σ_{tt} measurements

Compare **theory** with **experiment**

- Theory dependence by fitting various evaluations of $\sigma_{tt}(\alpha_s)$ by `top++2.0` [Phys. Rev. Lett. 110, 252004]



Extracting α_s from σ_{tt} measurements

Compare **theory** with **experiment**

- Theory dependence by fitting various evaluations of $\sigma_{tt}(\alpha_s)$ by `top++2.0` [Phys. Rev. Lett. 110, 252004]

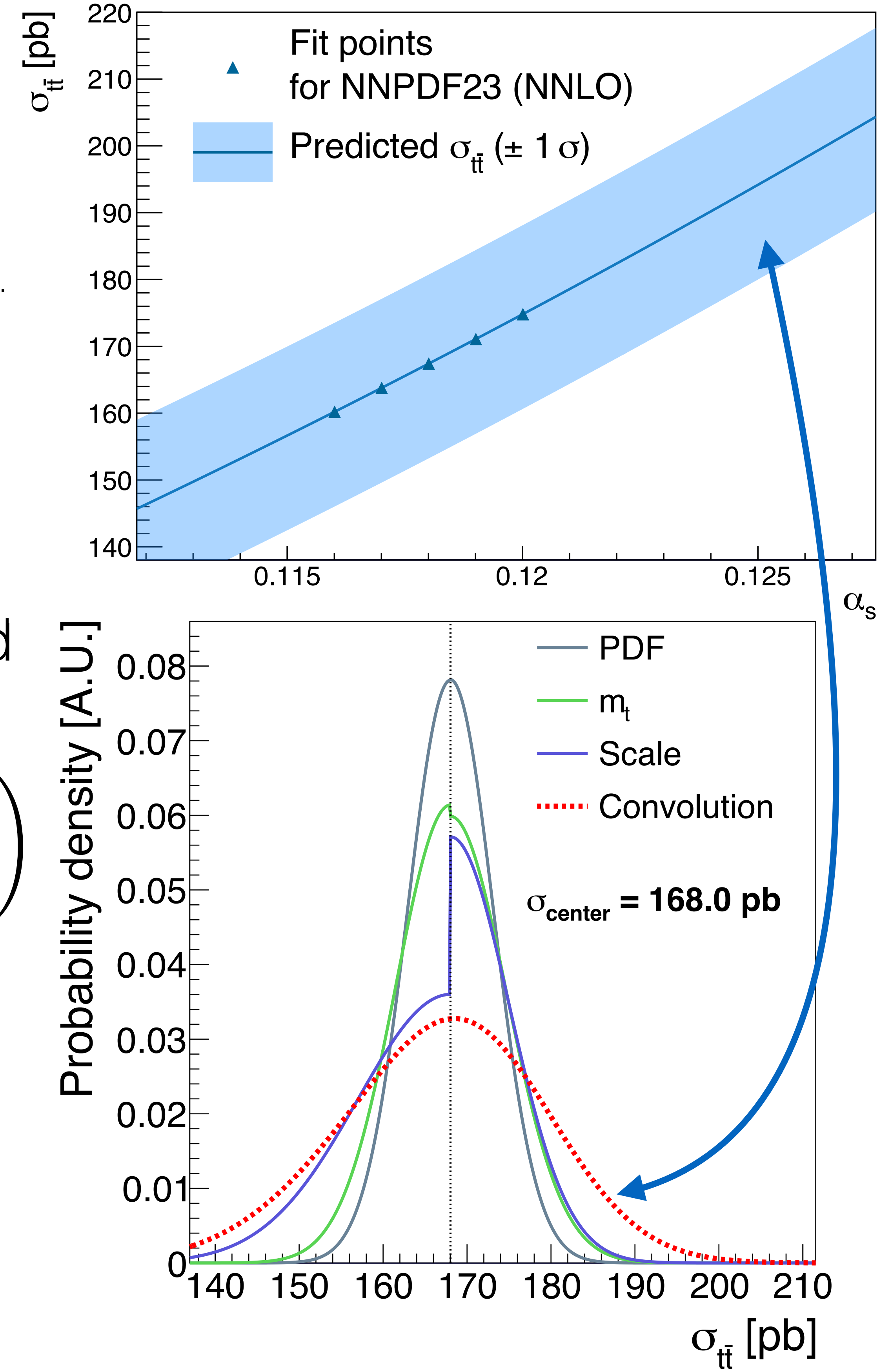
- Uncertainties from the **pdf**, **scale** and the **top mass** taken into account

- Theory uncertainty composed of convoluted asymmetric Gaussians:

$$\text{Asym. Gauss.}(x) = \frac{1}{\sqrt{2\pi}\sigma_\pm} \exp\left(\frac{(x-\mu)^2}{\sigma_\pm^2}\right)$$

$$\sigma_\pm = \begin{cases} \sigma_+ & \text{if} \quad x > \mu \\ \sigma_- & \text{if} \quad x < \mu \end{cases}$$

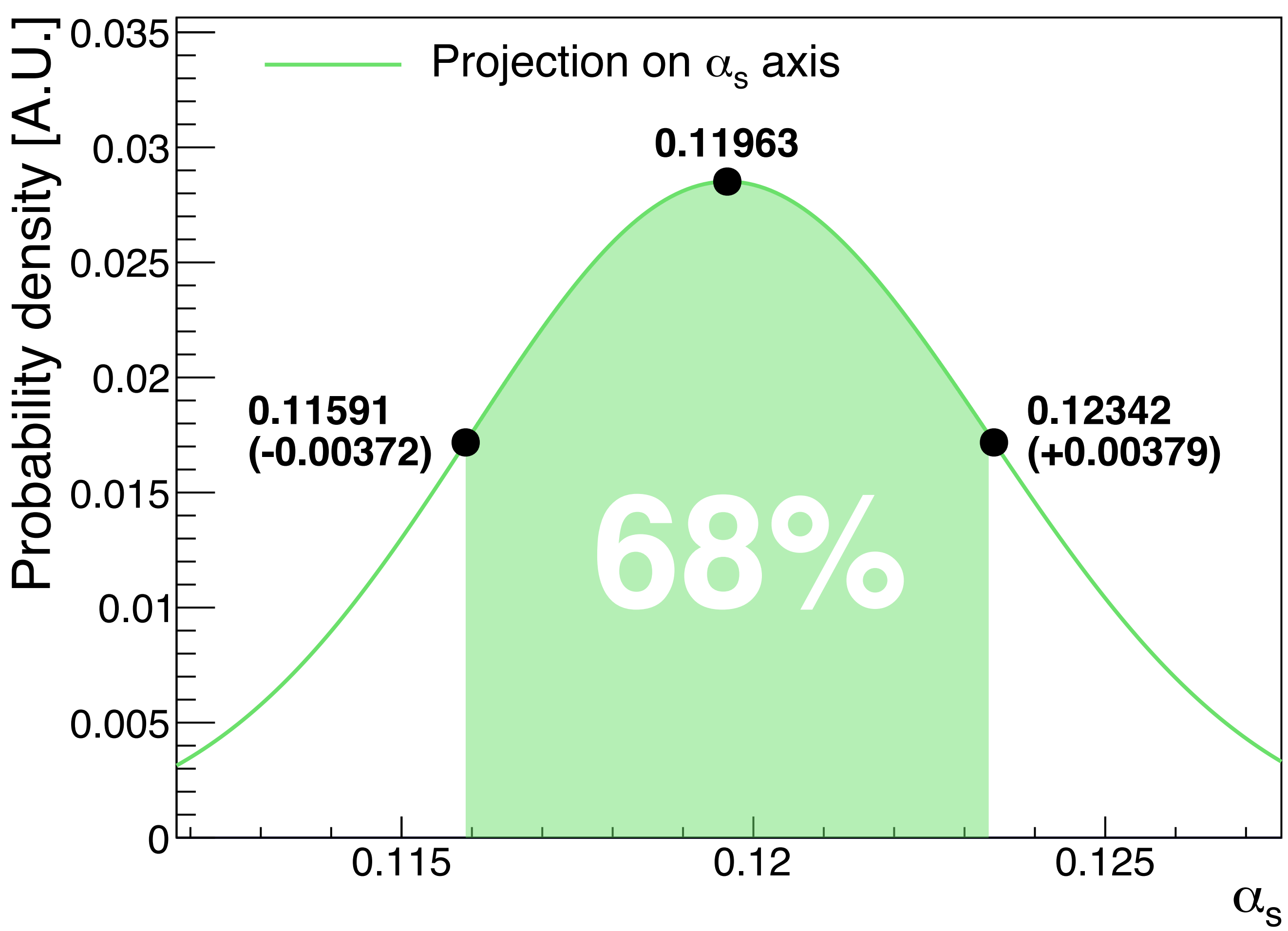
- Magnitude of uncertainty assumed not to depend on the cross section



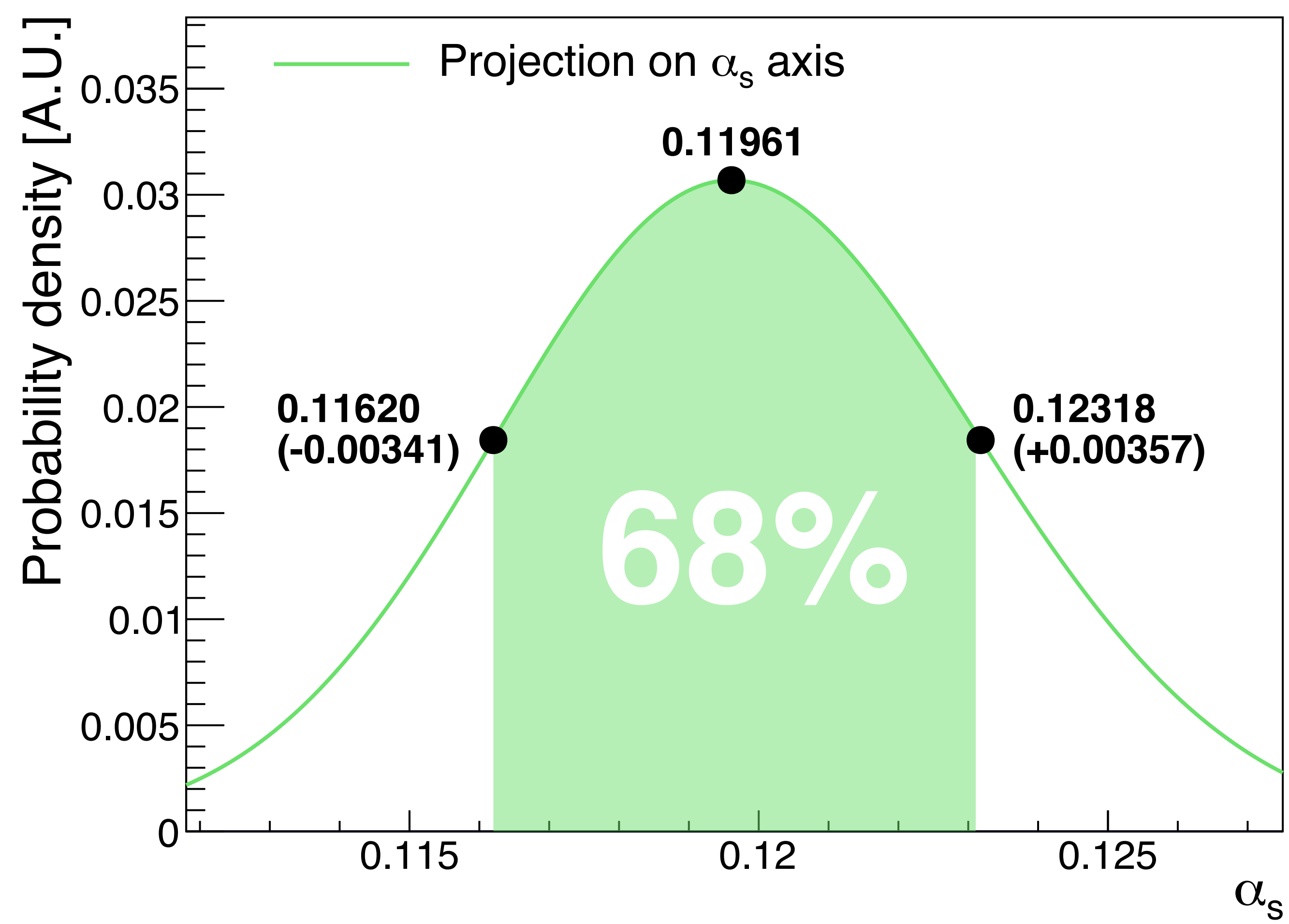
Estimating impact per error source

- Impact per source of uncertainty is evaluated by repeating the extraction with an error source omitted

With PDF error



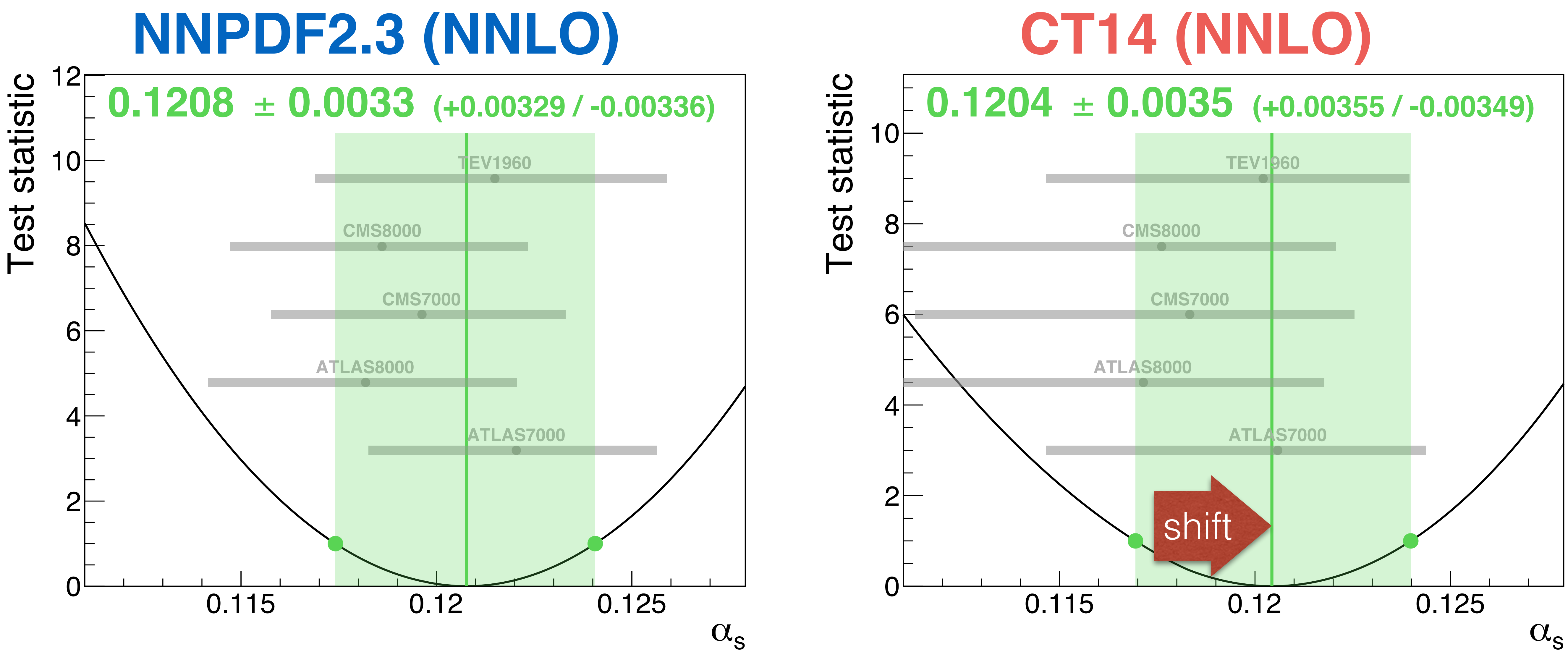
Without PDF error



$$\text{PDF error}^{+} = \sqrt{0.00379^2 - 0.00357^2} = 0.00128$$

$$\text{PDF error}^{-} = -\sqrt{-0.00372^2 - -0.00341^2} = -0.00148$$

Preliminary combination results

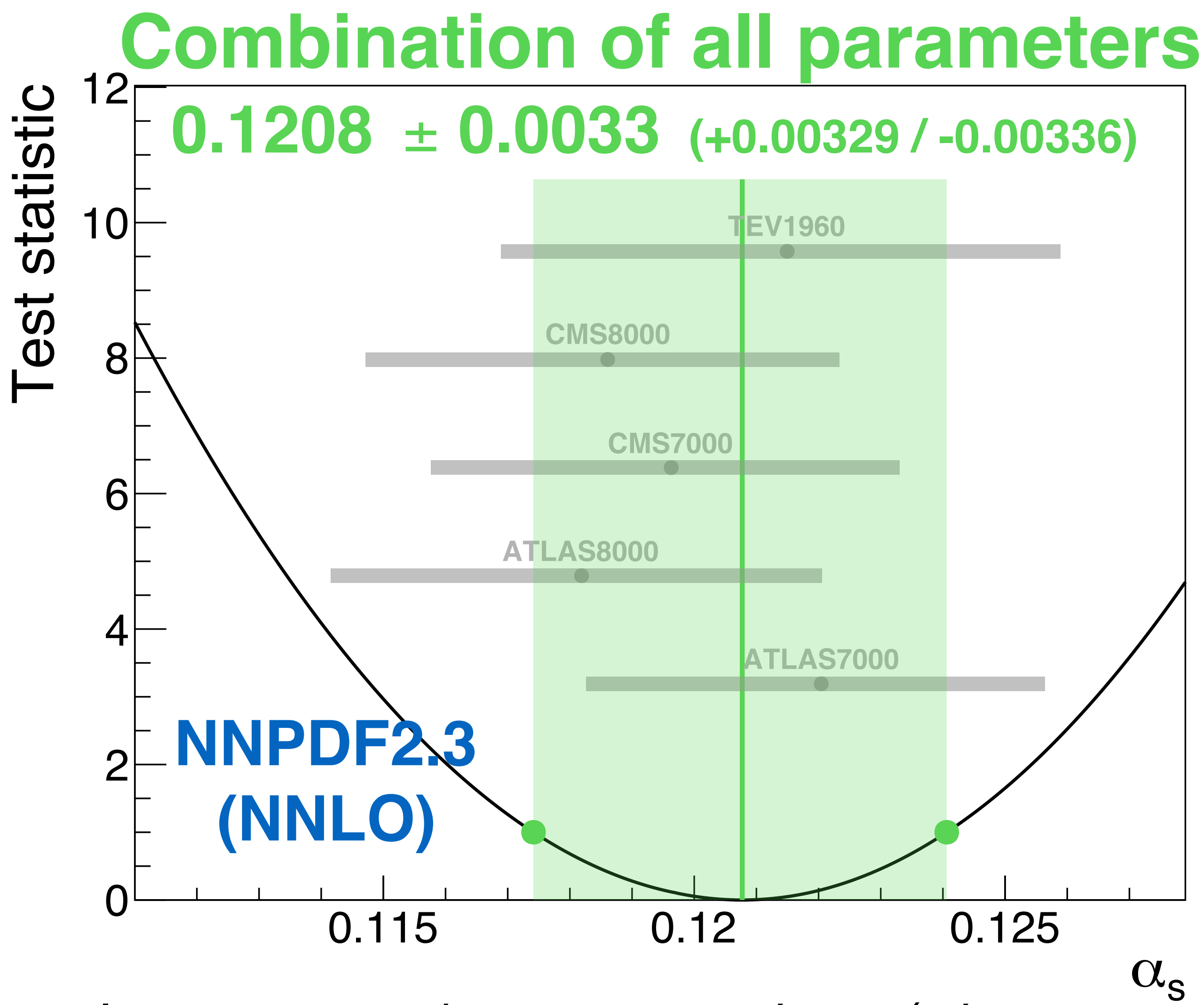


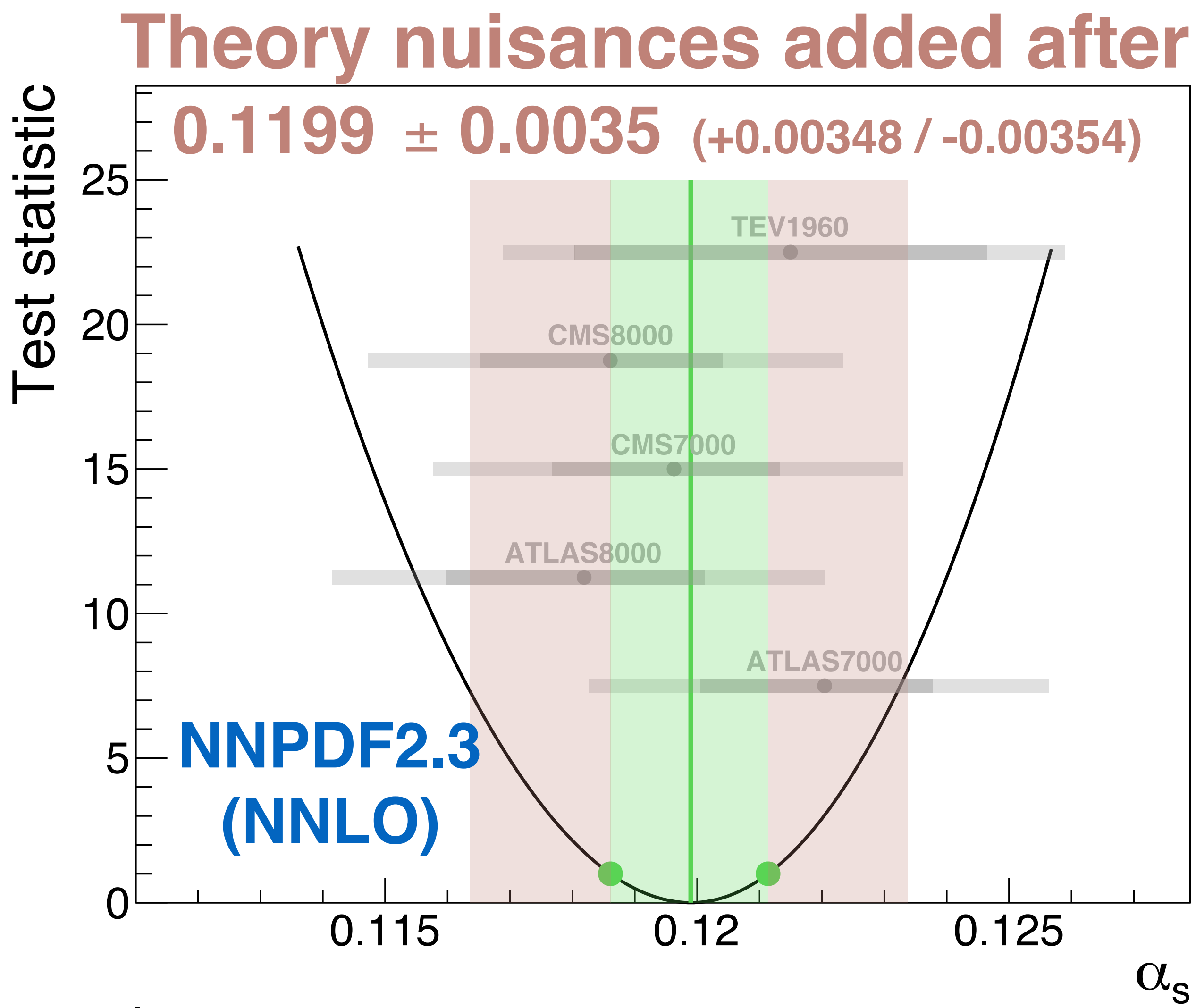
- Slightly asymmetric probability distribution functions return a reasonable combination
- Asymmetric functions are strongly influenced by the nuisance parameters
  - Different combination techniques (BLUE[1]) show the same pattern

1: **B**est **L**inear **U**nbiased **E**stimate [Nucl. Instrum. Methods A 270 (1988)]

Preliminary combination results

- **Theory nuisance parameters** are very large, and tend to drive the final determination
  - Solution is to add theory uncertainties after combining



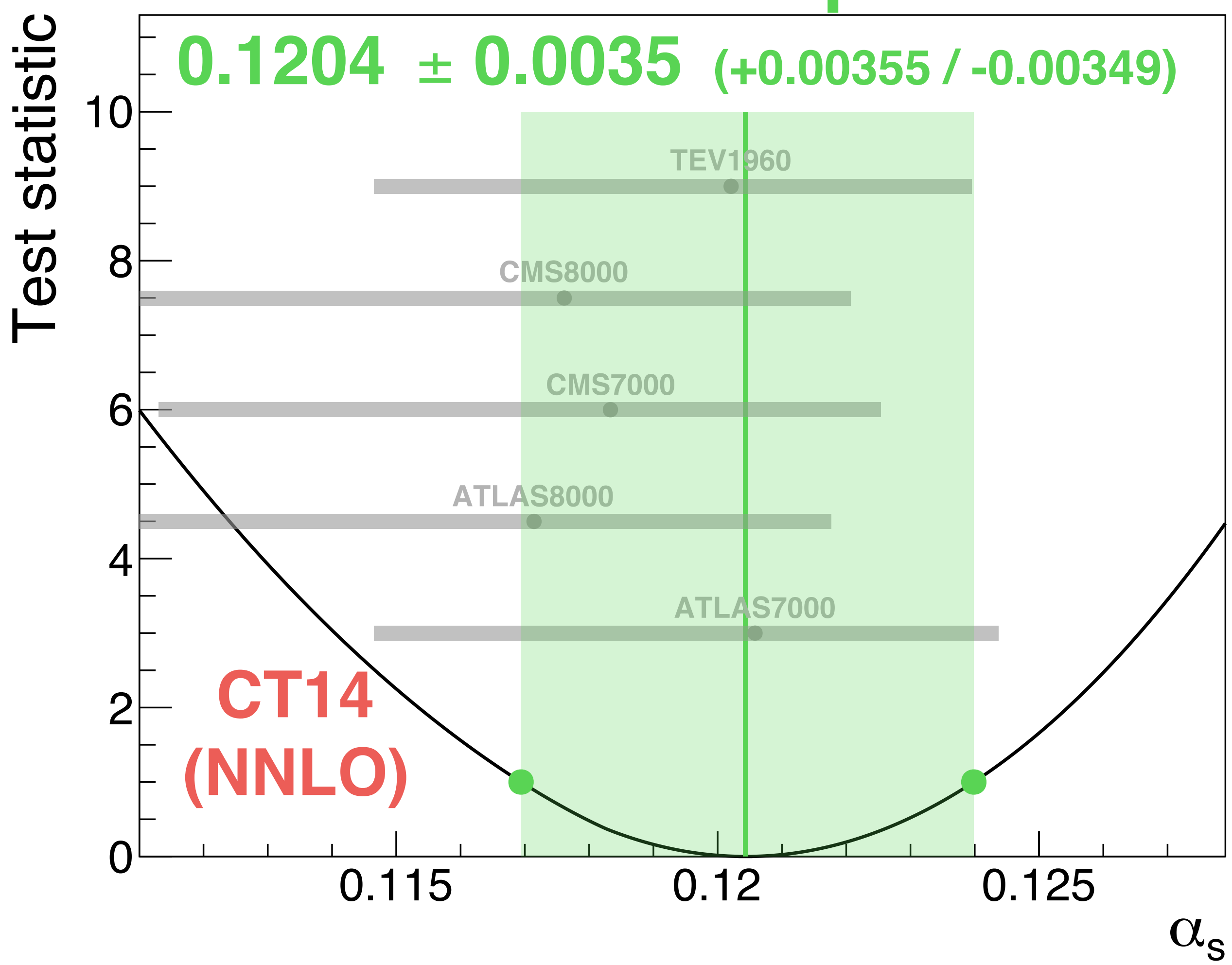


- Increases the uncertainty (since some nuisance parameters are not optimally fitted)
- For **NNPDF2.3 (NNLO)**, final determinations are not too different

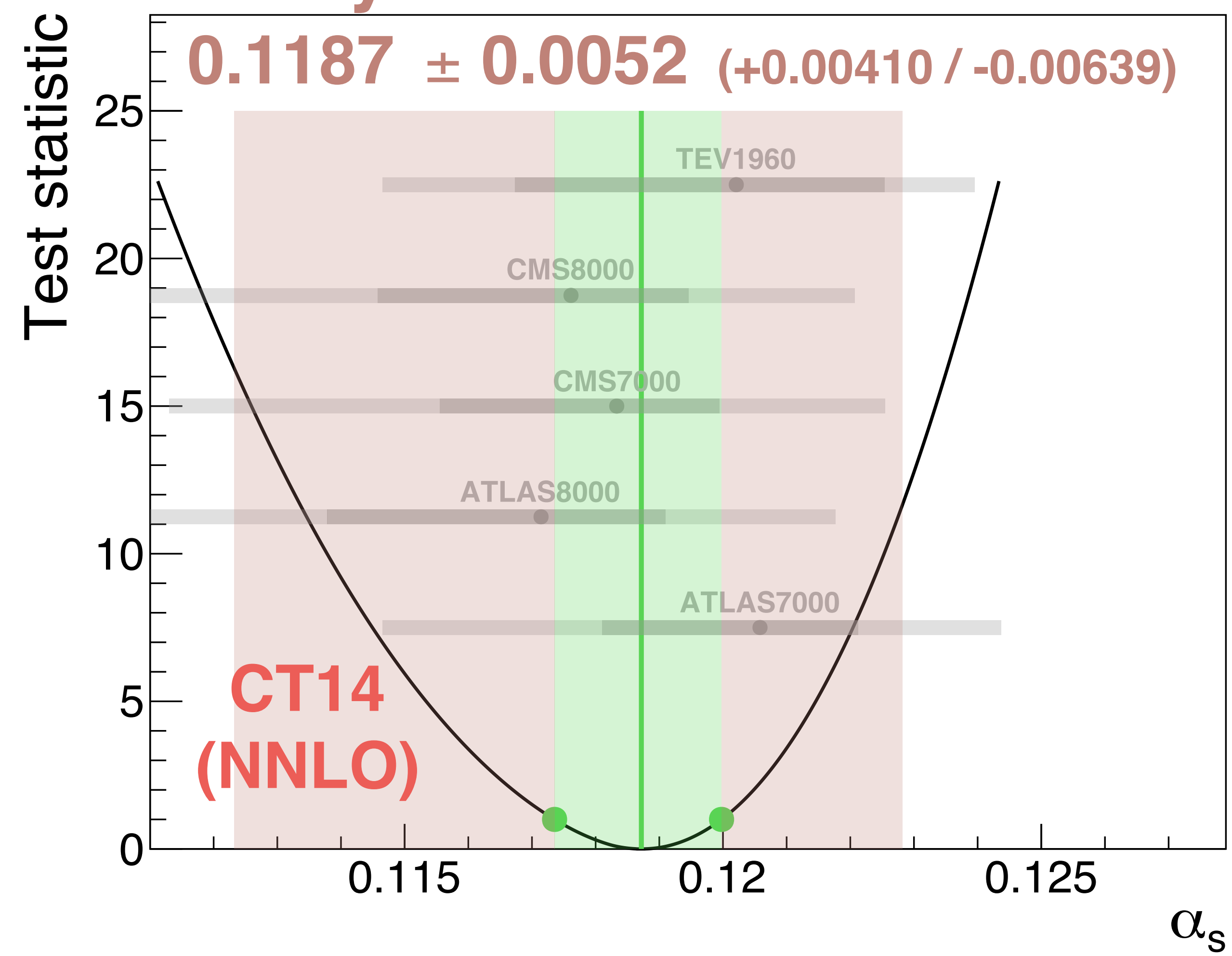
Preliminary combination results

- For **CT14 (NNLO)**: Larger theory uncertainties and more asymmetry, differences are more pronounced
    - Uncertainty goes up, central value shifts (but well within the 1σ band)

Combination of all parameters



Theory nuisances added after



- Same method under different combination schemes yields similar results

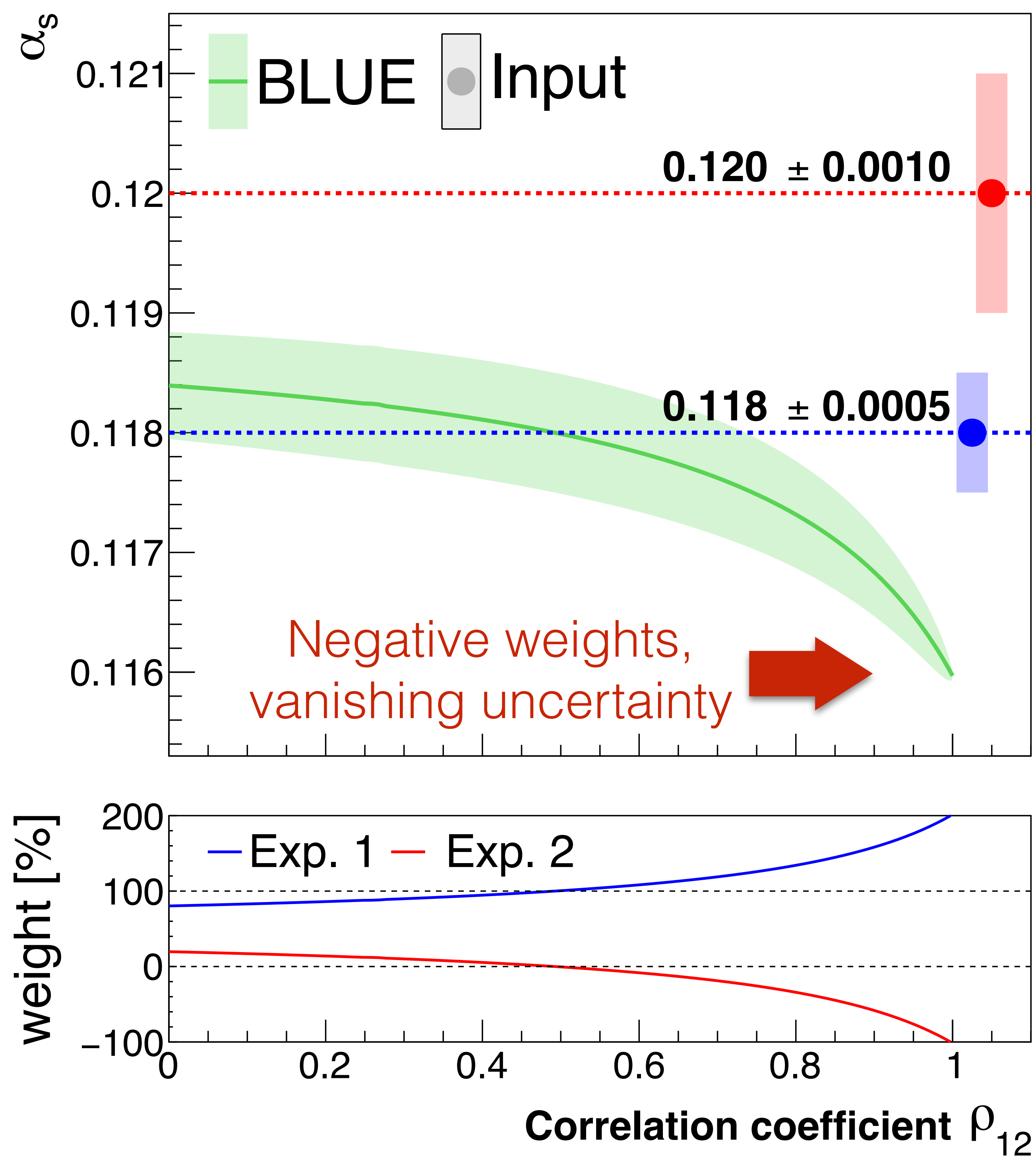
Combining correlated measurements

- One extraction yields **1 central value** and **7 uncertainties** (*statistical, systematic, luminosity, beam energy, pdf, scale, top mass*)
  - Many uncertainties are correlated between experiments
- Combinations have been performed using the **BLUE**[1] method:

$$y_{BLUE} = \sum_i w_i \, y_i \qquad \sigma^2_{BLUE} = w^T \, \mathbf{E} \, w$$

  - Weights found by minimising σ_{BLUE}
- Correlation coefficients ρ have to be set carefully
  - ρ = 1.0 is not conservative

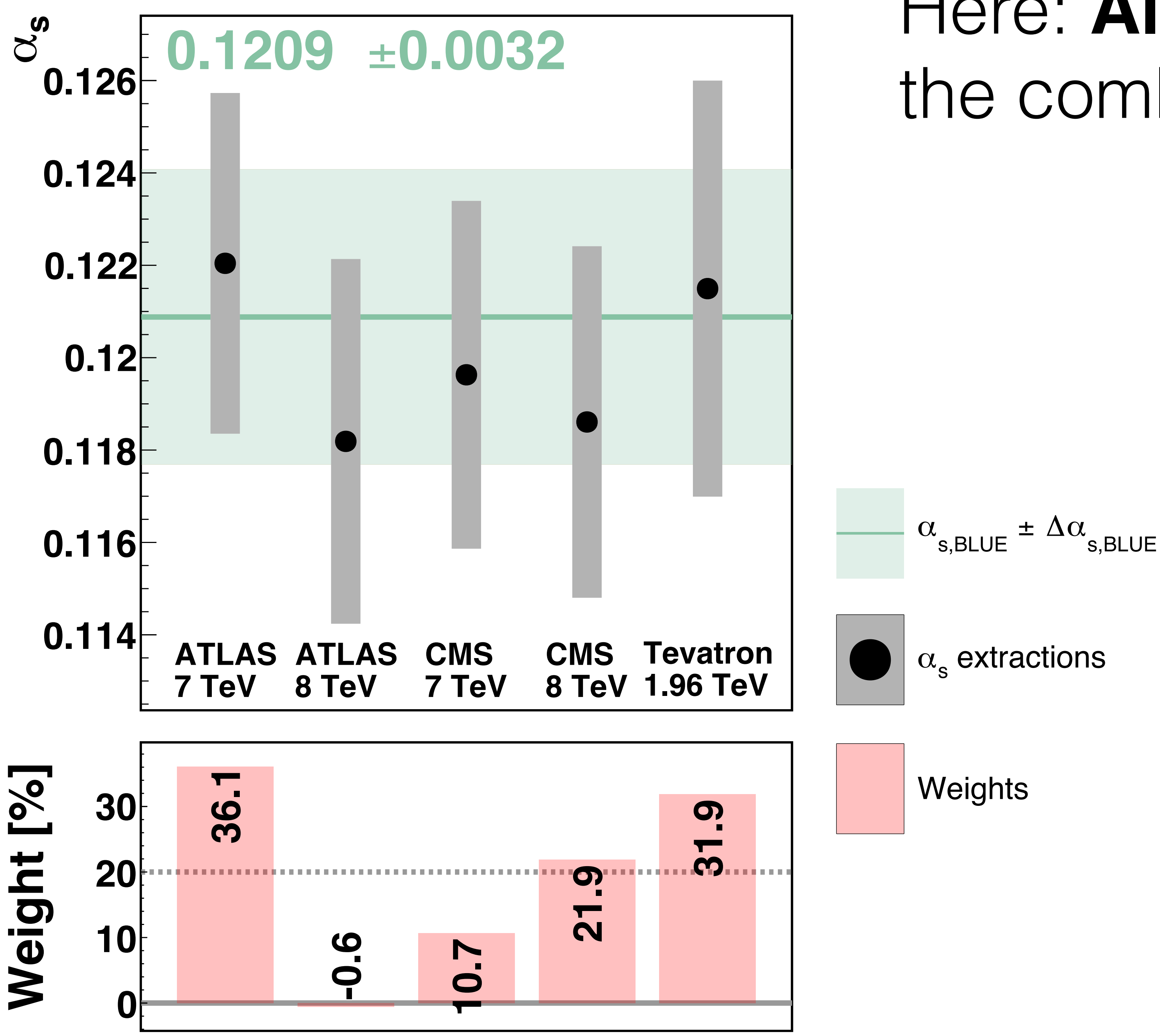
Example combination



1: **B**est **L**inear **U**nbiased **E**stimate
[Nucl. Instrum. Methods A 270 (1988)]

Preliminary combination results

Full combination for NNPDF2.3 at NNLO



Here: **All** error sources used in the combination

Alternative:

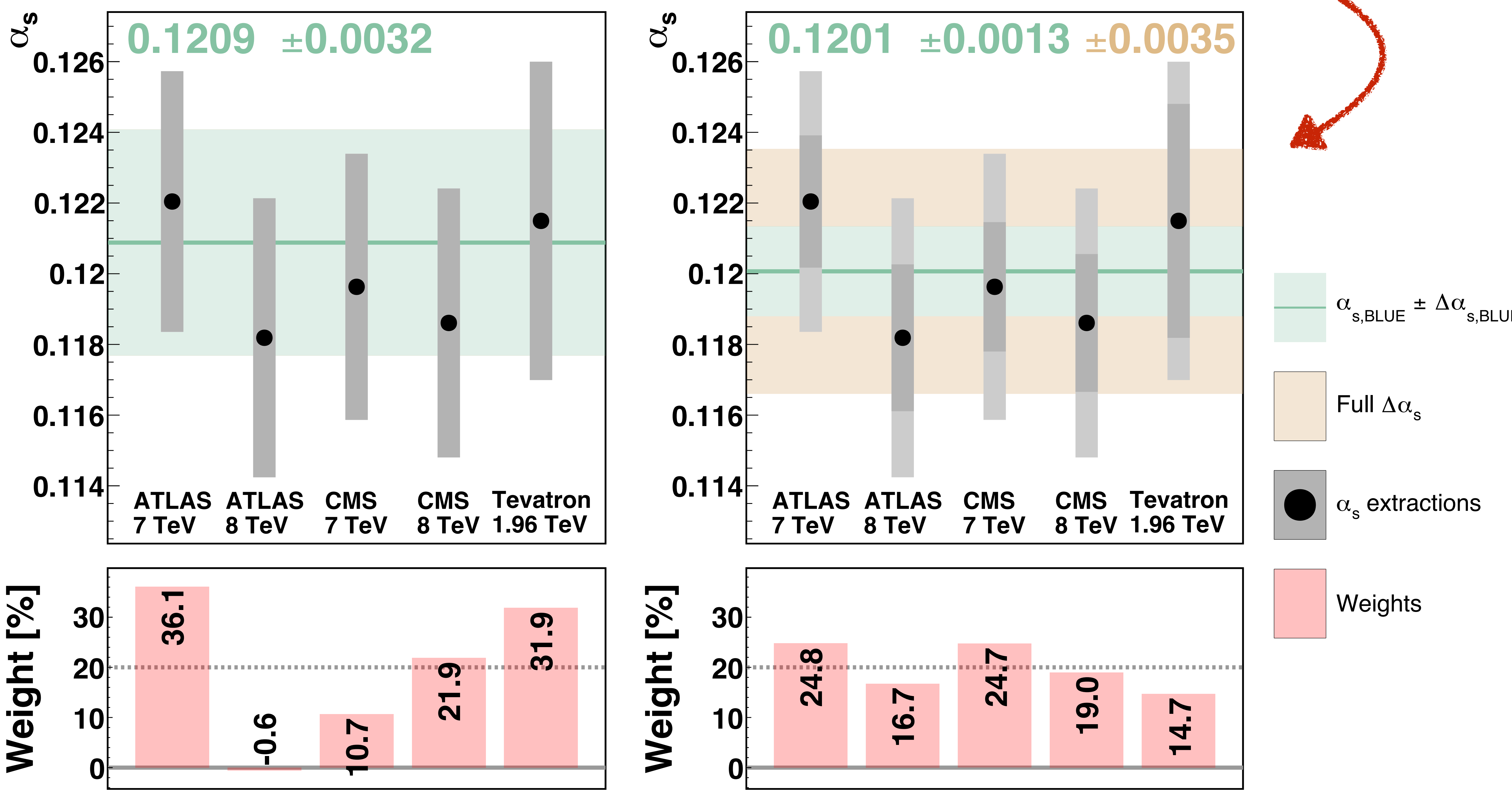
- Run the combination without some error sources
- Add these error sources after the combination

Can be useful to study the effects of the (strongly correlated) theory uncertainties

Preliminary combination results

Full combination for NNPDF2.3 at NNLO

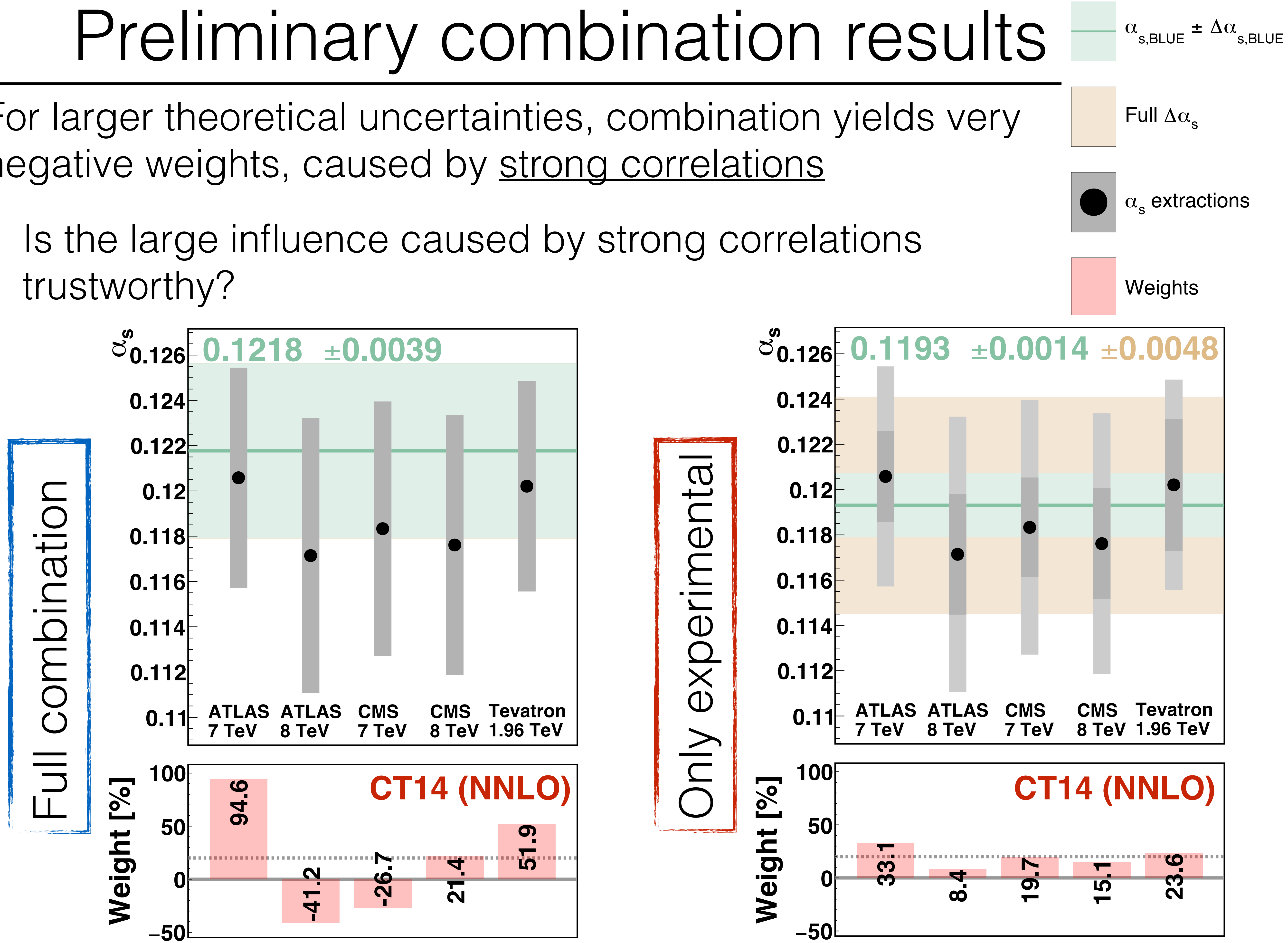
Combination of only experimental uncertainties
(theoretical uncertainties added after combination)



Preliminary combination results

For larger theoretical uncertainties, combination yields very negative weights, caused by strong correlations

- Is the large influence caused by strong correlations trustworthy?



Overview of theory uncertainties

- For $\sigma_{tt,\ theory}(\ \alpha_s\ )$, uncertainties include:

1. Uncertainty due to pdf (**pdf**)
   Calculated by `top++2.0` by computing σ_{tt} for all members of the pdf set

   - For the *replicas* type pdfs, uncertainty due to pdf is simply the standard deviation of σ_{tt} for different members. Calculation can be a bit more involved depending on the pdf.

2. Uncertainty due to scale (**scale**)

   - By recomputing σ_{tt} in `top++2.0` at different renormalisation scale variations ($1/2 \leq \mu_R/\mu_F \leq 2$), and taking minimum and maximum variations

3. Uncertainty due to uncertainty on the top mass (**mtop**)

   - Recompute σ_{tt} in `top++2.0` at $(m_{top,\ pole} + \Delta m_{top,\ pole})$ and $(m_{top,\ pole} - \Delta m_{top,\ pole})$

   - Experimental σ_{tt} also depends on $m_{top,\ pole}$, so bounds should be scaled:

$$\sigma_{t\bar{t}}^{+} = \sigma_{t\bar{t}}(m_{\text{top}} + \Delta m_{\text{top}}^{\text{pole}})_{\text{theory}} \cdot \frac{\sigma_{t\bar{t}}(m_{\text{top}})_{\text{experimental}}}{\sigma_{t\bar{t}}(m_{\text{top}} + \Delta m_{\text{top}}^{\text{pole}})_{\text{experimental}}}$$

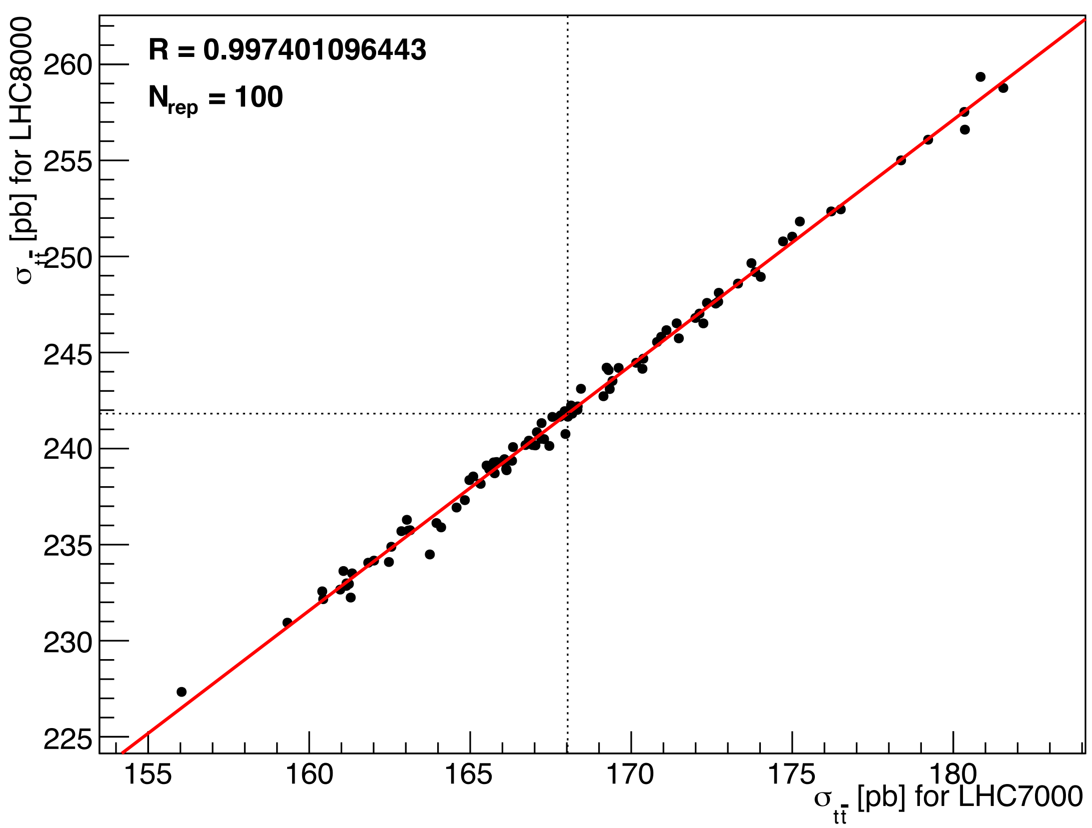
$$\sigma_{t\bar{t}}^{-} = \sigma_{t\bar{t}}(m_{\text{top}} - \Delta m_{\text{top}}^{\text{pole}})_{\text{theory}} \cdot \frac{\sigma_{t\bar{t}}(m_{\text{top}})_{\text{experimental}}}{\sigma_{t\bar{t}}(m_{\text{top}} - \Delta m_{\text{top}}^{\text{pole}})_{\text{experimental}}}$$

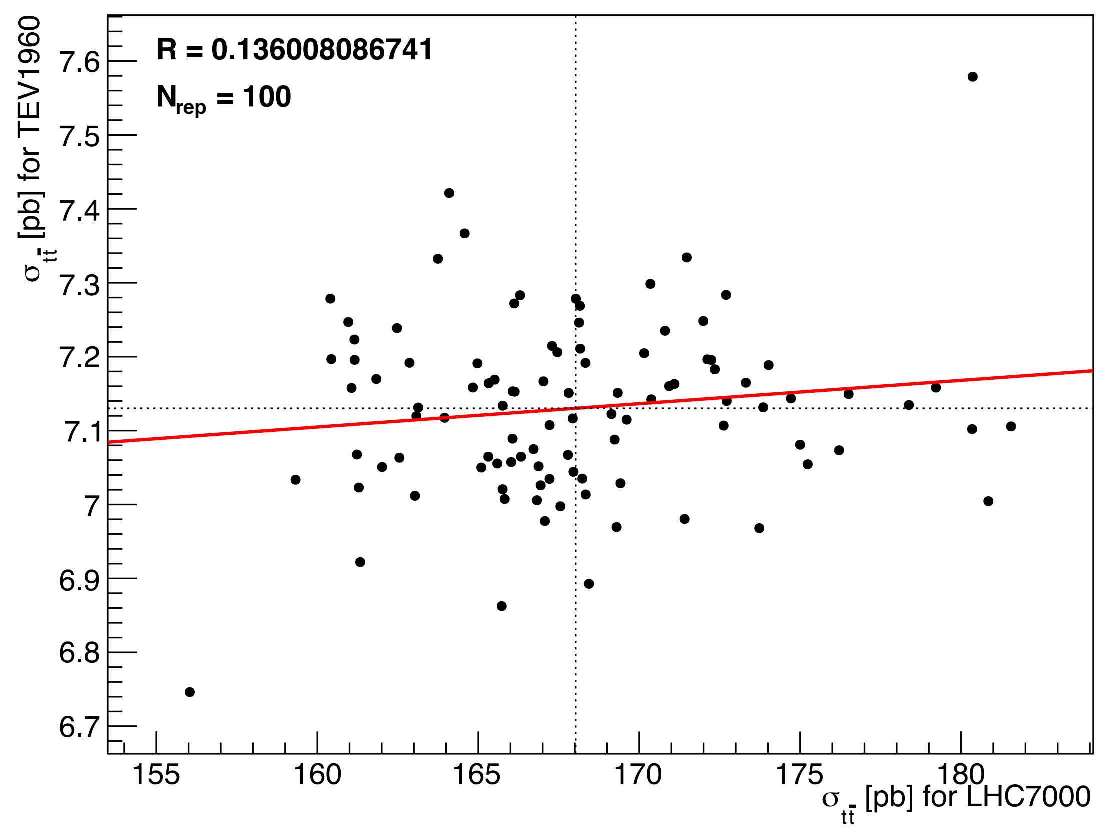
Correlations: Experimental uncertainties

- Breakdown of chosen correlation values:
  - **stat**: ρ = 0.0 between all measurements
  - **syst**: ρ = 1.0 for measurements at the same experiment, 0.0 elsewhere
  - **lumi**: Partly correlated for measurements at the same center of mass energy, 0.0 elsewhere
    - Bunch current uncertainty is the same for CMS and ATLAS (100% correlated)
    - Individual luminosity determinations are considered uncorrelated
  - **Ebeam**: ρ = 1.0 between all LHC experiments, ρ = 0.0 between LHC and Tevatron

Correlations: Theoretical uncertainties

- Breakdown of chosen correlation values:
  - **scale**: ρ = 1.0 between all LHC measurements, ρ = 0.5 between LHC and Tevatron measurements
  - **mtop**: ρ = 1.0 for all measurements
  - **pdf**: ρ can be determined by calculating the correlation coefficient of the PDF members





pdf plotted here: NNPDF2.3 (NNLO)

Maximum Likelihood Estimate Method

- Idea: Fit α_s to the individual probability distribution functions per experiment simultaneously
- Correlations are split up:

"Fully correlated uncertainties" (100%) —> **Nuisance parameters**
"Uncorrelated uncertainties" (0%) —> **Statistical uncertainties**

  - The nuisance parameters affect individual experiments simultaneously, and are fitted together with α_s

- Correlation coefficients between 0 and 1 are split up in a nuisance parameter and a statistical uncertainty
  - E.g. Luminosity has a correlated part at LHC (the uncertainty from the Van der Meer scans) and an uncorrelated part (from long-term luminosity monitoring per experiment)

Maximum Likelihood Estimate Method

$$L(\alpha_s, \boldsymbol{\theta}) = \prod_i \text{Gauss.}(\alpha_s,\ \mu_i + \sum_j \theta_j \delta_j,\ \sigma_i) \times \prod_j \text{Gauss.}(\theta_j,\ 0,\ 1)$$

- μ_i : The determination for experiment i
- σ_i : Statistical uncertainty for experiment i
- θ_j : The nuisance parameter j
- δ_j : Impact of nuisance parameter j

- Same likelihood estimate as in the *combine* tool
- Gaussians replaced by convolutions of asymmetric Gaussians when working with asymmetry
- Second part can strongly influence the final determination if a nuisance parameter has a large δ