Detecting Additional Polarization Modes with LISA

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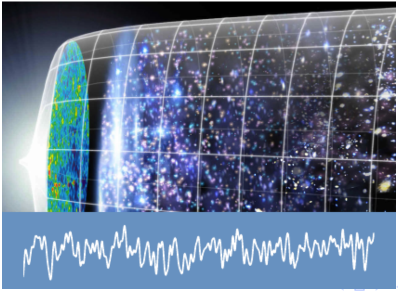
Outline

1. Polarization of GW
2. LISA sensitivity to additional modes
3. Mode extraction
4. Comments on LISA

Motivation

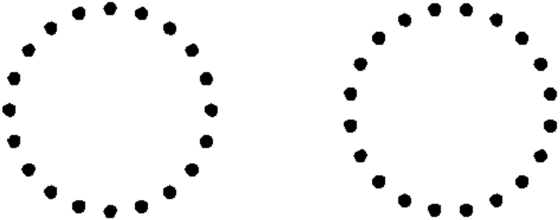
- Existence of scalar fields (e.g. Higgs)
  $\Rightarrow$ Extensions of GR

- SGWB possible from unresolved sources
  $\Rightarrow$ Analyze the polarization content of such a signal

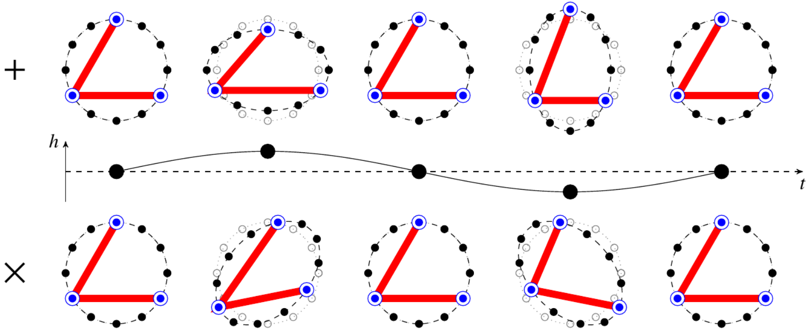


Polarization

Polarization (tensor modes from GR)



Polarization (tensor modes from GR)

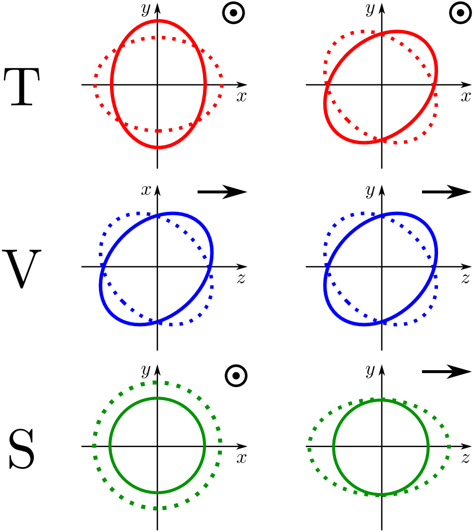


eLISA/NGO Yellow Book, 2012

Polarization (additional modes)

- GR: h_+ and $h_\times$
- Metric theories of gravity: up to six polarizations
- Not all of them disappear within the frame of a given theory
- 2T, 2V, 2S

Polarization (additional modes)



Polarization (additional modes)

$$h_{ij}(\omega t - \boldsymbol{k}\cdot\boldsymbol{x}) = \sum_A h_A(\omega t - \boldsymbol{k}\cdot\boldsymbol{x}) e_{ij}^A$$

$$e_{ij}^{+} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad e_{ij}^{\times} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$e_{ij}^{x} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad e_{ij}^{y} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$e_{ij}^{b} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad e_{ij}^{l} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Some examples

Theories	**Polarization modes**
GR	$+, \times$
Metric $f(R)$ gravity	$+, \times, b, l$
Palatini $f(R)$ gravity	$+, \times$
Scalar-tensor theory (massive)	$+, \times, b, l$
Brans-Dicke theory (massive)	$+, \times, b, l$
Brans-Dicke theory (massless)	$+, \times, b$
Massive bimetric	$+, \times, b, l, x, y$
$\vdots$	$\vdots$

$\Rightarrow$ There is a way to discriminate those theories!

Several questions

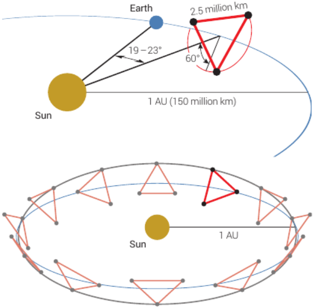
Without any assumption, a SGWB would contain a mixture of all modes:

- Is LISA sensitive to additional modes?
- SNR for each mode?
- How to extract the information regarding the additional modes?

Sensitivity

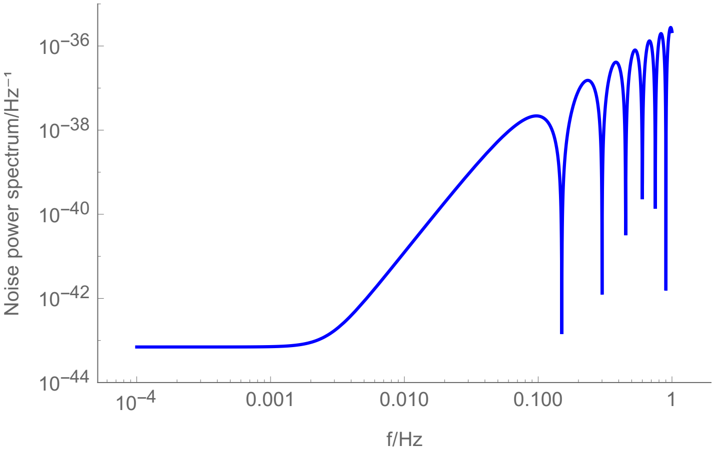
Sensitivity to additional modes

- 3 arms, 2.5 millions km
- TDI (X, α, ζ, E, P, U)
- LISA with 3 arms $\equiv$ 2 detectors
- Simultaneous detection of 2 polarizations
- How sensitive?



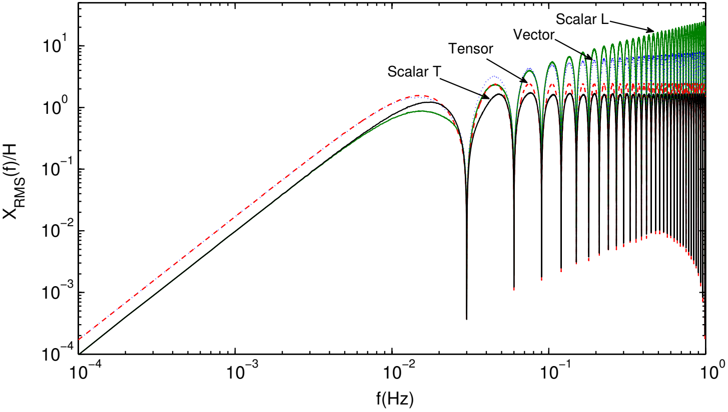
LISA: sensitivity to other modes

Noise power spectrum $S_X(f)$



LISA: sensitivity to other modes

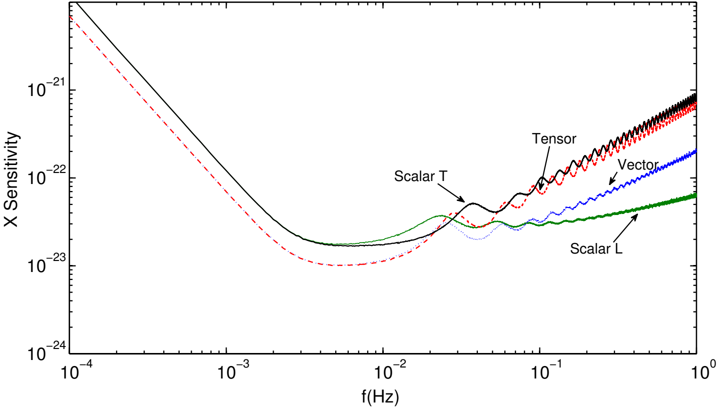
RMS GW response $X_{\mathrm{RMS}}(f)$



Tinto, Alves, PRD 82, 122003 (2010)

LISA: sensitivity to other modes

$$X \text{ Sensitivity } SNR \cdot \sqrt{S_X(f)B}/X_{\mathrm{RMS}}$$



Tinto, Alves, PRD 82, 122003 (2010)

[SNR=5, T=1 year]

Extraction of the modes from a SGWB

SGWB with all polarization modes

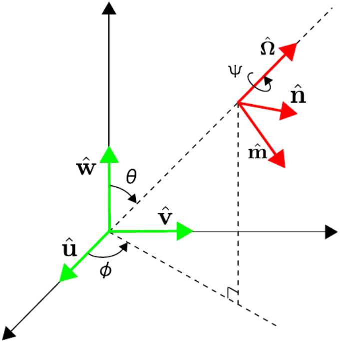
SGWB from many unresolved sources (e.g. produced during inflation):

- isotropic
- independently polarized
- stationary
- Gaussian

$\Rightarrow$ Model-independant

$$h(t,\mathbf{x}) = \sum_A \int_{S^2} d\hat{\mathbf{\Omega}} \int_{-\infty}^{\infty} df\ \tilde{h}_A(f,\hat{\mathbf{\Omega}}) e^{2\pi i f(t-\hat{\mathbf{\Omega}}\mathbf{x}/c)} F_A(\hat{\mathbf{\Omega}}),$$

F_A: antenna pattern function

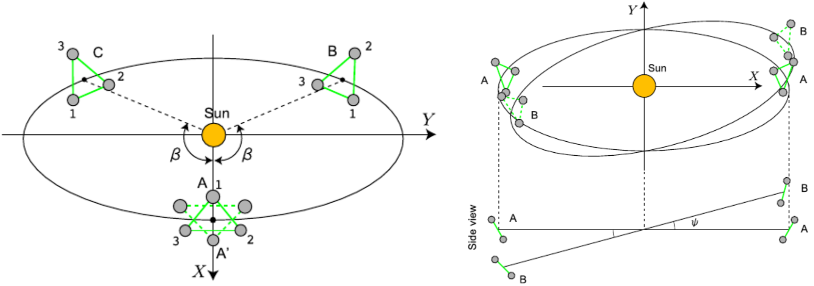


Mode separation: recipe

1. Network of detectors: cross-correlation
   (to discriminate from stochastic detector noise)
2. Sensitivity to each mode
3. Correlation matrix

Nishizawa et al. PRD 79, 082002 (2009)
Nishizawa, Taruya, Kawamura, PRD 81, 104043 (2010)

Network of detectors



Nishizawa, Ayama, 2013

Energy densities

$$\Omega^A_{\mathrm{GW}}(f) = \frac{1}{\rho_{\mathrm{crit}}} \frac{d\rho^A_{\mathrm{GW}}}{d\ln(f)} \propto f^3 S^A_h(f)$$

$$\begin{aligned} \Omega^T_{\mathrm{GW}} &= \Omega^+_{\mathrm{GW}} + \Omega^\times_{\mathrm{GW}} \\ \Omega^V_{\mathrm{GW}} &= \Omega^x_{\mathrm{GW}} + \Omega^y_{\mathrm{GW}} \\ \Omega^S_{\mathrm{GW}} &= \Omega^b_{\mathrm{GW}} + \Omega^l_{\mathrm{GW}} \end{aligned}$$

Power-spectral density, noise spectrum

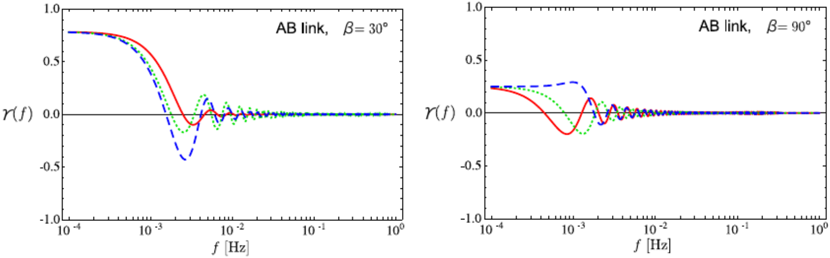
$$
\begin{aligned}
s(t) &= h(t) + n(t) \\
\langle \tilde{h}^*_A(f, \hat{\boldsymbol{\Omega}}) \tilde{h}_{A'}(f', \hat{\boldsymbol{\Omega}}') \rangle &= \delta(f - f') \frac{1}{4\pi} \delta^2(\hat{\boldsymbol{\Omega}}, \hat{\boldsymbol{\Omega}}') \delta_{AA'} \cdot \frac{1}{2} S_h^A(|f|) \\
\langle \tilde{n}_I(f) \tilde{n}_J(f') \rangle &= \frac{1}{2} \delta(f - f') \delta_{IJ} \cdot P_I(|f|)
\end{aligned}
$$

Optimal SNR for each mode

$$SNR^M \propto \int_0^\infty df \left[\frac{(\Omega_{\mathrm{gw}}^M(f))^2 \det \mathbf{F}(f)}{f^6 \mathcal{F}_M(f)} \right]^{(1/2)} ,$$

$$F_{MM'} = \sum_{\text{detector pairs } (I,J)} \int_0^{T_{\mathrm{obs}}} \mathrm{d}t \frac{\gamma_{IJ}^M(t,f)\gamma_{IJ}^{M'}(t,f)}{P_I(f)P_J(f)},$$

Overlap reduction functions $\gamma_{IJ}^{A}(f)$



Nishizawa, Ayama, 2013

Mode extraction

Define the statistics

$$\begin{aligned} Z_{IJ} &\propto \left| f^3 \right| \tilde{s}_I^*(f) \tilde{s}_J(f) \\ &= \sum_M \Omega_{\mathrm{GW}}^M \gamma_{IJ}^M(f) + \text{ noise} \end{aligned}$$

$$\langle Z_{IJ} \rangle = \sum_M \Omega_{\mathrm{GW}}^M \gamma_{IJ}^M(f)$$

Mode extraction

$$\begin{pmatrix} \langle Z_{12} \rangle \\ \langle Z_{23} \rangle \\ \langle Z_{31} \rangle \end{pmatrix} = \begin{pmatrix} \gamma_{12}^T & \gamma_{12}^V & \gamma_{12}^S \\ \gamma_{23}^T & \gamma_{23}^V & \gamma_{23}^S \\ \gamma_{31}^T & \gamma_{31}^V & \gamma_{31}^S \end{pmatrix} \begin{pmatrix} \Omega_{\mathrm{GW}}^T \\ \Omega_{\mathrm{GW}}^V \\ \Omega_{\mathrm{GW}}^S \end{pmatrix}$$

$\boldsymbol{Z}$ $\boldsymbol{\Pi}$ $\boldsymbol{\Omega}$

$$\boxed{\boldsymbol{\Omega} = \boldsymbol{\Pi}^{-1} \boldsymbol{Z} \qquad (\det \boldsymbol{\Pi} \neq 0)}$$

Potential problems/solutions for LISA

- Full polarization extraction: requires a **network** (future projects?)
- Autocorrelation technique:
  (Tinto, Armstrong, 2013 (1205.4620))
  1 single detector with correlated noise
  $\sim$ 2 colocated/coaligned detectors with uncorrelated noise
- But not yet for the full polarization content
- Correlation at different times, noise problem to solve

Conclusion

- LISA sensitive to additional modes
- Higher sensitivity to longitudinal scalar/vector modes
- Possible to extract the polarization content from a SGWB signal
- Single cluster: simultaneous detection of two polarizations (3 arms)
- Network: Separate detection of all modes
- All modes with a single cluster?

Thank you for your attention!