Effective Field Theory of Post-Newtonian Gravity with a Spin (x2)

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EFTs and their Setup are Universal

There is a Hierarchy of Scales

[Goldberger et al. 2007]

1. r_s, scale of internal structure, $r_s \sim m$
2. r, orbital separation scale, $r \sim \dfrac{r_s}{v^2}$
3. λ, radiation wavelength scale, $\lambda \sim \dfrac{r}{v}$

We can use EFT!

$v \ll 1$, $nPN \equiv v^{2n}$ correction in GR to Newtonian Gravity

For an EFT of PNG proceed in stages corresponding to each scale

Setup of EFT is Universal

Bottom-Up or Top-Down

Stage 1 Remove the scale r_S of isolated compact object, bottom-up approach

$$S\left[g_{\mu\nu}\right] = -\frac{1}{16\pi G}\int d^4x\sqrt{g}R$$

Integrate out strong field modes $g_{\mu\nu} \equiv g^s_{\mu\nu} + \bar{g}_{\mu\nu}$

$$\Rightarrow S_{\text{eff}}\left[y^\mu, e^\mu_A, \bar{g}_{\mu\nu}\right] = -\frac{1}{16\pi G}\int d^4x\sqrt{\bar{g}}R\left[\bar{g}_{\mu\nu}\right] + \underbrace{\sum_i C_i \int d\sigma O_i(\sigma)}_{\equiv S_{pp} \text{ with Wilson coefficients}}$$

Setup of EFT is Universal

Stage 2 Remove the orbital scale r of binary, top-down

$$\bar{g}_{\mu\nu} \equiv \eta_{\mu\nu} + \underbrace{H_{\mu\nu}}_{\text{orbital}} + \underbrace{\widetilde{h}_{\mu\nu}}_{\text{radiation}}$$

Kenneth Wilson

$$\partial_t H_{\mu\nu} \sim \frac{v}{r} H_{\mu\nu}, \quad \partial_i H_{\mu\nu} \sim \frac{1}{r} H_{\mu\nu}, \quad \partial_\rho \widetilde{h}_{\mu\nu} \sim \frac{v}{r} \widetilde{h}_{\mu\nu}$$

$$S_{\rm eff}\left[y_1^\mu, y_2^\mu, e_{(1)}{}_A^\mu, e_{(2)}{}_A^\mu, \bar{g}_{\mu\nu}\right] = -\frac{1}{16\pi G}\int d^4x\sqrt{\bar{g}}\,R\left[\bar{g}_{\mu\nu}\right] + S_{(1)\rm pp} + S_{(2)\rm pp}$$

Integrate out orbital field modes

$$\Rightarrow e^{iS_{\rm eff(composite)}\left[y^\mu, e_A^\mu, \widetilde{h}_{\mu\nu}\right]} \equiv \int \mathcal{D}H_{\mu\nu}\; e^{iS_{\rm eff}\left[y_1^\mu, y_2^\mu, e_{(1)}{}_A^\mu, e_{(2)}{}_A^\mu, \bar{g}_{\mu\nu}\right]}$$

Stop here for effective action in conservative sector, that is
WITHOUT any remaining orbital scale field DOFs

[**ML** 2010, **ML** & Steinhoff 2014]

To construct EFT – identify Symmetries

1. *General coordinate invariance*, and *parity invariance*
2. *Worldline reparametrization invariance*
3. *Internal Lorentz invariance of local frame field*
4. *SO(3) invariance of body-fixed spatial triad*
5. *Spin gauge invariance*, that is invariance under choice of completion of body-fixed spatial triad through timelike vector
6. Assume isolated object has no intrinsic permanent multipole moments beyond the mass monopole and the spin dipole

To construct EFT – identify Degrees of Freedom

1. *The gravitational field*
   - The metric $g_{\mu\nu}(x)$
   - The tetrad field $\eta^{ab}\tilde{e}_a{}^{\mu}(x)\tilde{e}_b{}^{\nu}(x) = g^{\mu\nu}(x)$

2. *The particle worldline coordinate*

   $y^{\mu}(\sigma)$ a function of an arbitrary affine parameter σ
   Particle worldline position does not in general coincide with object's 'center'

3. *The particle worldline rotating DOFs*

   Worldline tetrad, $\eta^{AB}e_A{}^{\mu}(\sigma)e_B{}^{\nu}(\sigma) = g^{\mu\nu}$

   $\Rightarrow$ Angular velocity $\Omega^{\mu\nu}(\sigma)$ + Spin $S_{\mu\nu}(\sigma)$

   $\Rightarrow$ Worldline Lorentz matrices, $\eta^{AB}\Lambda_A{}^{a}(\sigma)\Lambda_B{}^{b}(\sigma) = \eta^{ab}$
   + conjugate local spin, $S_{ab}(\sigma)$

For EFT with Spin – fix gauge of rotational variables

Effective action of a spinning particle

[Hanson & Regge 1974, Bailey & Israel 1975]

- $u^\mu \equiv dy^\mu/d\sigma$, $\Omega^{\mu\nu} \equiv e^\mu_A \frac{De^{A\nu}}{D\sigma} \Rightarrow L_{\rm pp}\left[u_\mu, \Omega^{\mu\nu}, g_{\mu\nu}\right]$
- $S_{\mu\nu} \equiv -2\frac{\partial L}{\partial \Omega^{\mu\nu}}$ spin as further worldline DOF – classical source

$$\Rightarrow S_{\rm pp} = \int d\sigma \left[-m\sqrt{u^2} - \frac{1}{2} S_{\mu\nu}\Omega^{\mu\nu} + L_{\rm SI}\left[u^\mu, S_{\mu\nu}, g_{\mu\nu}\left(y^\mu\right)\right]\right]$$

For an EFT the gauge of the rotational variables should be fixed at the level of the action

[**ML** 2×PRD 2010, **ML** & Steinhoff JCAP 2014]

Start in the covariant gauge: $e_{[0]\mu} = \frac{p^\mu}{\sqrt{p^2}}$, $\qquad S_{\mu\nu} p^\nu = 0$

- Linear momentum $p_\mu \equiv -\frac{\partial L}{\partial u^\mu} = m\frac{u^\mu}{\sqrt{u^2}} + \mathcal{O}(S^2)$

Gauge freedom of rotational variables can be restored

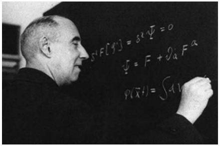
[**ML** & Steinhoff, JHEP 2015]

Introduce gauge invariance in rotational variables

Transform $e^{A\mu}$ from gauge $e_{[0]\mu} = q_\mu \;\rightarrow\; \hat{e}_{[0]\mu} = w_\mu$
with a boost-like transformation in 4D covariant form:

$\hat{e}^{A\mu} = L^{\mu}{}_{\nu}(w,q)e^{A\nu}, \quad q_a,\ w_a$ timelike unit 4-vectors

$\Rightarrow$ Generic gauge: $\hat{e}_{[0]\mu} = w_\mu,\ \hat{S}^{\mu\nu}\left(p_\nu + \sqrt{p^2}\hat{e}_{[0]\nu}\right) = 0$



Ernst Stueckelberg

$\Rightarrow$ Extra term in action appears!

- For minimal coupling $\frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu} = \frac{1}{2}\hat{S}_{\mu\nu}\hat{\Omega}^{\mu\nu} + \frac{\hat{S}^{\mu\rho}p_\rho}{p^2}\frac{Dp_\mu}{D\sigma}$
- Extra term contributes to finite size effects,
  yet carries no Wilson coefficient
- Beyond minimal coupling $S_{\mu\nu} = \hat{S}_{\mu\nu} - \frac{\hat{S}_{\mu\rho}p^\rho p_\nu}{p^2} + \frac{\hat{S}_{\nu\rho}p^\rho p_\mu}{p^2}$

Rotational DOFs should be separated from field DOFs

[**ML**, 2×PRD 2010, **ML** & Steinhoff, JCAP 2014, JHEP 2015]

Worldline tetrad contains both rotational and field DOFs

- $\hat{e}_A{}^\mu = \hat{\Lambda}_A{}^b \tilde{e}_b{}^\mu$: Tetrad field $\eta_{ab}\tilde{e}^a{}_\mu \tilde{e}^b{}_\nu = g_{\mu\nu}$, $\eta^{AB}\hat{\Lambda}_A{}^a\hat{\Lambda}_B{}^b = \eta^{ab}$
- $\frac{1}{2}\hat{S}_{\mu\nu}\hat{\Omega}^{\mu\nu} = \frac{1}{2}\hat{S}_{ab}\hat{\Omega}^{ab}_{\text{flat}} + \frac{1}{2}\hat{S}_{ab}\omega_\mu{}^{ab}u^\mu$, $\omega_\mu{}^{ab} \equiv \tilde{e}^b{}_\nu D_\mu \tilde{e}^{a\nu}$ Ricci rotation coefficients

  $\Rightarrow$ New rotational variables: $\hat{\Omega}^{ab}_{\text{flat}} = \hat{\Lambda}^{Aa}\frac{d\hat{\Lambda}_A{}^b}{d\sigma}$, $\hat{S}_{ab}$

Separation of field from particle worldline DOFs is not complete

- $\hat{\Lambda}_{[0]}{}^a = w^a = \tilde{e}^a{}_\mu w^\mu$ may contain further field dependence
- $\hat{S}_{0i}$ contain further field dependence

  $\Rightarrow$ Field completely disentangled from worldline DOFs
  only once gauge for rotational variables is fixed

Nonminimal coupling action with spin must be constrained

[**ML** & Steinhoff, JHEP 2014, JHEP 2015]

Building blocks ∼Riemann×Spin-Induced Multipoles

Curvature tensors

$E_{\mu\nu} \equiv R_{\mu\alpha\nu\beta} u^\alpha u^\beta, \quad B_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\alpha\beta\gamma\mu} R^{\alpha\beta}{}_{\delta\nu} u^\gamma u^\delta, \quad$ and $D_{[i]} = e^\mu_{[i]} D_\mu$

- Field is vacuum solution at LO
- At most linear in Riemann
- Properties of Riemann, Bianchi identities
- Parity invariance
- Initial rotational gauge fixing and gauge invariance

Spin-induced multipoles

$S^\mu \equiv *S^{\mu\nu} \frac{p_\nu}{\sqrt{p^2}} \simeq *S^{\mu\nu} \frac{u_\nu}{\sqrt{u^2}}, \quad *S_{\alpha\beta} \equiv \frac{1}{2}\epsilon_{\alpha\beta\mu\nu} S^{\mu\nu}$

- $SO(3)$ invariance of body-fixed triad

LO nonminimal couplings to all orders in spin are fixed

[**ML** & Steinhoff, JHEP 2014, JHEP 2015]

LO nonminimal couplings to all orders in spin

New spin-induced Wilson coefficients:

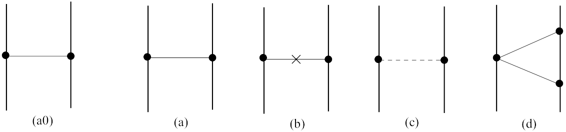
$$L_{\rm SI} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{ES^{2n}}}{m^{2n-1}} D_{\mu_{2n}} \cdots D_{\mu_3} \frac{E_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}}$$
$$+ \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{BS^{2n+1}}}{m^{2n}} D_{\mu_{2n+1}} \cdots D_{\mu_3} \frac{B_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}} S^{\mu_{2n+1}}$$

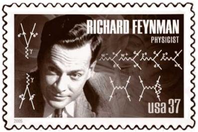
LO couplings up to 4PN

- $L_{ES^2} = -\frac{C_{ES^2}}{2m} \frac{E_{\mu\nu}}{\sqrt{u^2}} S^\mu S^\nu,$ Quadrupole @2PN
- $L_{BS^3} = -\frac{C_{BS^3}}{6m^2} \frac{D_\lambda B_{\mu\nu}}{\sqrt{u^2}} S^\mu S^\nu S^\lambda,$ Octupole @3.5PN
- $L_{ES^4} = \frac{C_{ES^4}}{24m^3} \frac{D_\lambda D_\kappa E_{\mu\nu}}{\sqrt{u^2}} S^\mu S^\nu S^\lambda S^\kappa,$ Hexadecapole @4PN

LO sectors beyond Newtonian

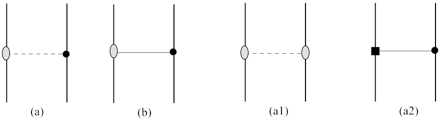
Feynman graphs of non-spinning sector to 1PN order





One-loop diagram – absent from 1PN with NRG fields

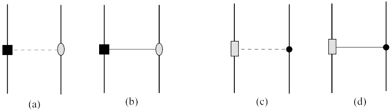
LO Feynman diagrams with spin – to quadratic in spin



LO cubic and quartic in spin sectors

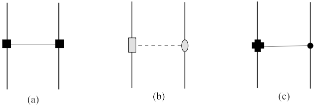
[**ML** & Steinhoff, JHEP 2014]

Feynman diagrams of LO **cubic** in spin sector



- On left pair – quadrupole-dipole, on right – octupole-monopole
- Note analogy of each pair with LO spin-orbit

Feynman diagrams of LO **quartic** in spin sector

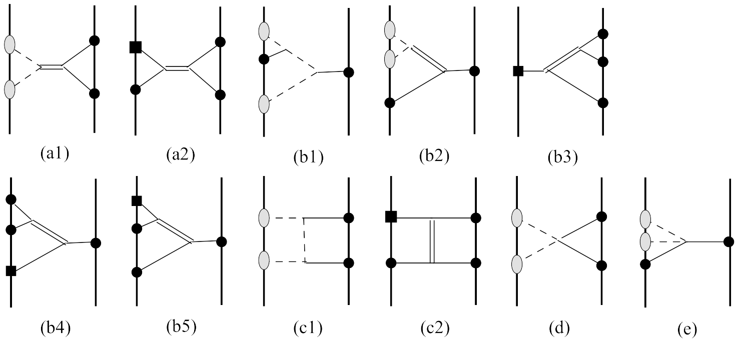


- On left and right – quadrupole-quadrupole and hexadecapole-monopole
- Each is analogous to LO spin-squared
- In middle – octupole-dipole analogous to LO spin1-spin2

NNLO spin-squared sector

[**ML** & Steinhoff, JCAP 2016]

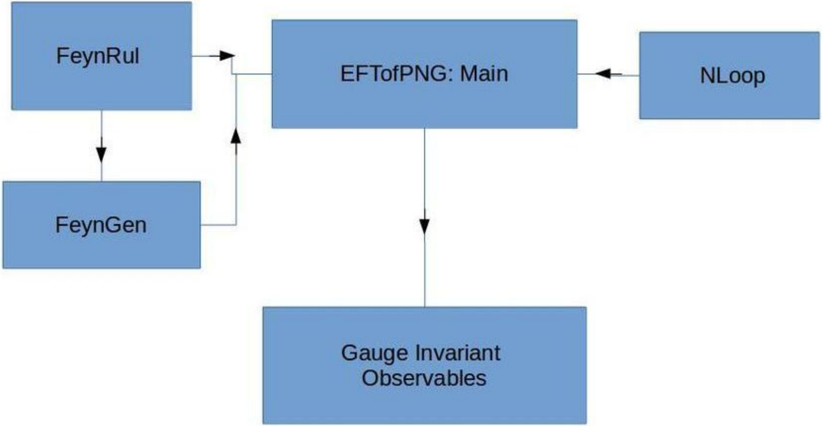
Feynman diagrams of order G^3 with two loops



- Five 2-loop topologies actually fall into 3 kinds
- H topology (graphs c1,c2) – irreducible kind – require more labor

EFTofPNG package version 1.0

[**ML** & Steinhoff 2017]



arXiv:1704.xxxxx, https://github.com/pncbc/eftofpng
→ To be public very soon

Conclusions

EFT of PNG with Spin: Summary of conceptual results

[**ML** & Steinhoff, JHEP 2015]

- EFT formulation for spinning objects
- Spin-induced nonminimal coupling to all orders in spin
- EOMs and Hamiltonians straightforward derivation

EFT of PNG with Spin: Summary of Applications

- NLO S1-S2, SO [**ML**, 2×PRD 2010], S^2 [**ML** & Steinhoff, JHEP 2015]
- NNLO S1-S2 [**ML**, PRD 2011], SO, S^2 [**ML** & Steinhoff, 2×JCAP 2016]
- LO S^3+S^4 [**ML** & Steinhoff, JHEP 2014]
- Public package with spin + observables pipeline [**ML** & Steinhoff 2017]

Spinning and non-spinning sectors progress – now simultaneous! [G. Faye's talk]

Prospective work

- Matching of spin-induced Wilson coefficients
- Radiative sector with spin: Formulation and application of EFT