Casimir-Polder shifts on quantum levitation states[1]

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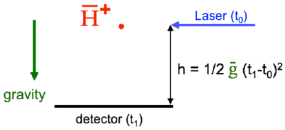
With discussions with V. V. Nesvizhevsky and A. Yu. Voronin

[1] P-P. Crépin, G. Dufour, R. Guérout, A. Lambrecht and S.Reynaud, *Physical Review A 95 (2017) 032501*

Gravitational Behavior of Antihydrogen at Rest[2]

Test the equivalence principle for antimatter by timing the free fall of antihydrogen $\overline{\mathrm{H}}$ released from trap
Experiment under construction at CERN
Current experimental bound[3] : $-65g \leq \overline{g} \leq 110g$



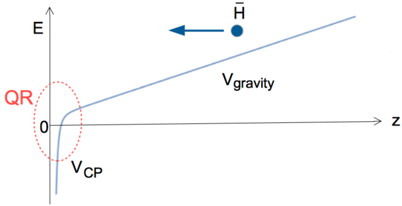
- START : the extra e^+ is photodetached
- STOP : annihilation of $\overline{\mathrm{H}}$ on the detector after its free fall

Expected accuracy for $\overline{g}$: 10^{-2}

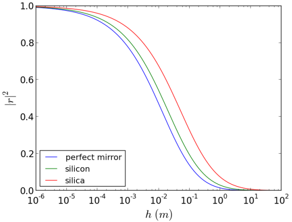
[2] P. Indelicato et al. *Hyperfine Interact* (2014) 228:141-150
[3] ALPHA collab. *Nature Communications* 4 (2013) 1785

Quantum levitation states

At small distances (<1 μm), $\overline{\text{H}}$ is sensitive to the Casimir-Polder potential : quantum reflection (QR) occurs[4]



Potential landscape



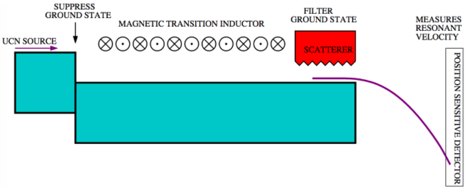
Reflectivity

$\overline{\text{H}}$ is trapped between *Gravity* (↓) and *Quantum Reflection* (↑)
→ quantum levitation state

[4] G. Dufour et al. *J. Mod. Phys.* Conf. Ser. 30 (2014) 1460265

Outlooks

- Using QR for spectroscopic measurements[5] : in future generations of GBAR, spectroscopic measurements on antihydrogen atoms in quantum levitation states
- Analogy with the GRANIT experiment[6] to measure resonance transitions between the gravitationally quantum states of neutrons



- GOAL : determine precisely quantum levitation states

[5] A. Yu. Voronin, V. V. Nesvizhevsky et al. *J. Mod. Phys. Conf. Ser.* , 30 (2014) 1460266

[6] M. Kreuz, V. V. Nesvizhevsky et al. *Nucl. Instr. Meth. A* 611 (2009) 326

Scattering length approximation[7] :

$$\mathcal{E}_n^1 = \lambda_n \epsilon_{\overline{g}} + m\overline{g}a \tag{1}$$

- $\lambda_n \epsilon_{\overline{g}}$: energy of quantum bouncers,
  $\epsilon_{\overline{g}} = \left(\frac{\hbar^2 m \overline{g}^2}{2}\right)^{1/3}$ $\quad$ (0.6 peV for $\overline{g} = g$),
  λ_n are zeros of the Airy function Ai
- $m\overline{g}a$: CP shift due to QR, a is the *scattering length*

Transition frequencies :

$$\omega_{mn} = \frac{\mathcal{E}_n^1 - \mathcal{E}_m^1}{\hbar} = (\lambda_n - \lambda_m)\epsilon_{\overline{g}} \tag{2}$$

Measure of ω_{mn} would give a direct access to the value of $\overline{g}$!
Perform a full quantum treatment of free fall and QR → improve (1)

[7] A.Y. Voronin, P. Froelich and V. V. Nesvizhevsky *P. R. A* 83 (2011) 032903

Schrödinger equation

Schrödinger equation :

$$\psi''(z) + F(z)\psi(z) = 0$$

$F(z) = \frac{2m}{\hbar}(E - V(z))$

$$\begin{cases} V(z) = m\bar{g}z + V_{CP}(z) & \text{if } z > 0 \\ V(z) = \infty & \text{if } z \leq 0 \end{cases}$$

Liouville transformation :

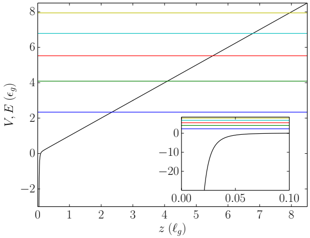
$\boldsymbol{z}(z), \ \boldsymbol{\psi}(\boldsymbol{z}) = \sqrt{\boldsymbol{z}'(z)}\psi(z)$

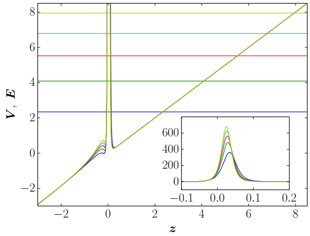
$$\boldsymbol{\psi}''(\boldsymbol{z}) + \boldsymbol{F}(\boldsymbol{z})\boldsymbol{\psi}(\boldsymbol{z}) = 0$$

$\boldsymbol{F}(\boldsymbol{z}) = \boldsymbol{E} - \boldsymbol{V}(\boldsymbol{z})$

$\boldsymbol{V}(\boldsymbol{z}) = \boldsymbol{z} - \boldsymbol{V}_{CP}(\boldsymbol{z})$

$\boldsymbol{E} = \frac{E}{\epsilon_{\overline{g}}}$: preserves energy shifts

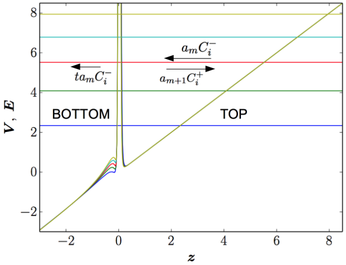




Cavity resonances

New picture : Fabry-Perot cavity

- TOP mirror : perfectly reflecting due to gravity
- BOTTOM mirror : partially reflecting due to QR



Above and below the bottom mirror, quasi-stationary states :

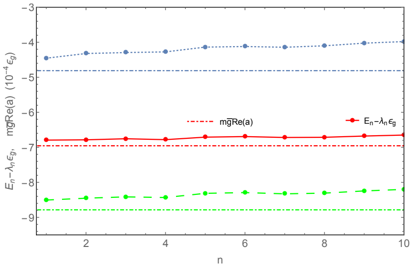
$$\psi_m(z) = \frac{a_m}{2}(\mathrm{Ci}^+(z - z_t) + \mathrm{Ci}^-(z - z_t)), \quad \mathrm{Ci}^\pm(z) = \mathrm{Ai}(z) \pm i\mathrm{Bi}(z)$$

m : number of bounces, ρ : round-trip factor, $a_{m+1} = \rho\, a_m$

Resonances E_n (n labels energy levels) correspond to :

$$\rho \in \mathbb{R}, \quad \rho \simeq 1$$

We solve numerically Schrödinger equation and find $\rho(E)$ and also E_n.
Comparison with scattering approximation $E_n - \lambda_n \epsilon_{\overline{g}}$ ($= m\overline{g}a$ in s.l.a.) :



Approximation works until a fraction of $10^{-4}\epsilon_{\overline{g}}$.
We need a more precise description.

Round-trip factor = QR amplitude r + propagation phase factor :

$$\rho \simeq -re^{2i\theta(-E/\epsilon_{\overline{g}})}, \quad \tan\theta(x) = \frac{\mathrm{Ai}(x)}{\mathrm{Bi}(x)}$$

Resonance condition :

$$2\theta(-E_n/\epsilon_{\overline{g}}) + \arg(-r) = 2n\pi$$

Effective range approximation

$$\arg(-r) = ?$$

New complex length $\mathcal{A}(k)$, such as $\mathcal{A}(0) = a$ and

$$r = -\frac{1-ik\mathcal{A}(k)}{1+ik\mathcal{A}(k)}, \quad \hbar k \equiv \sqrt{2mE}$$

Effective range theory suggests[8] for $V_4 = -C_4/z^4$ potential :

$$k\mathcal{A}(k) = -ikl\,\alpha(kl), \quad l = \frac{\sqrt{2mC_4}}{\hbar}$$
$$\alpha(K) = 1 + i\frac{\pi}{3}K + \left(\frac{8}{3}(\gamma + \ln 2) - \frac{28}{9} - \frac{2\pi}{3}i + \frac{4}{3}\ln K\right) K^2$$

For Casimir-Polder potential $(V(z) \to -C_4/z^4)$:

$$\alpha(K) = \alpha_0 + i\frac{\pi}{3}K + \left(\alpha_2 + \frac{4}{3}\alpha_0 \ln K\right) K^2$$

where α_0 and α_2 are determined by a fit.

Resonance condition becomes :

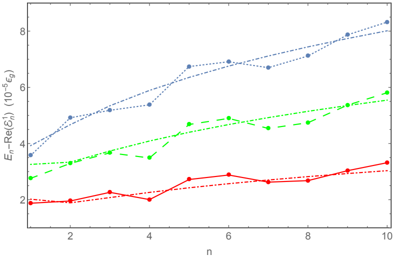
$$\theta(-E_n/\epsilon_{\overline{g}}) - \mathrm{Re}(\arctan(k_n l\,\alpha(k_n l))) = n\pi$$

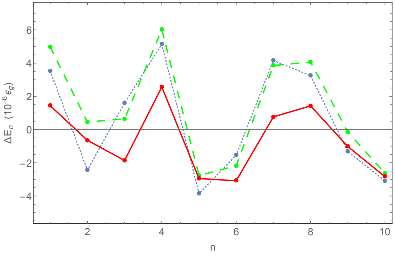
[8] I. Spruch, T. O'Malley and I, Rosenberg, *Phys. Rev. Lett.* 5 (1960) 375

Results

Correction to the scattering length approximation : $E_n - \lambda_n \epsilon_{\overline{g}} - m\overline{g}a$

$$\Delta E_n = E_n^{\text{ana}} - E_n^{\text{num}}$$





Analytical method would be sufficient to calculate quantum levitation states energies and deduce from the spectroscopy measurements the value of $\overline{g}$ with an accuracy better than $10^{-5}\overline{g}$!

THANK YOU FOR YOU ATTENTION !

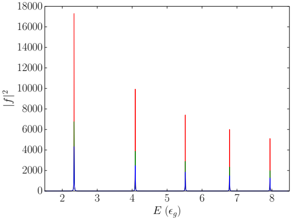
Width of resonances

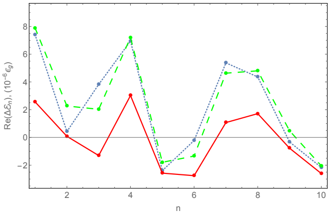
Extend analytically ρ to $\mathbb{C}$.

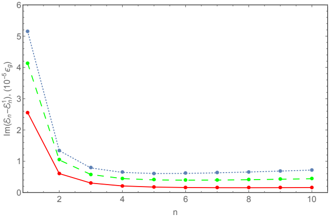
Cavity response function : $f(E) = \frac{\rho(E)}{1-\rho(E)}$

Complex resonances $\mathcal{E}_n$: $\rho(\mathcal{E}_n) = 1$.

Fit $|f|^2 \simeq \frac{A_n}{(E - \text{Re}\,\mathcal{E}_n)^2 + (\text{Im}\,\mathcal{E}_n)^2}$







- Energies are still known with an accuracy of a few $10^{-6}\epsilon_{\overline{g}}$
- Good approximation of the lifetime in cavity :

$$\tau = \frac{\hbar}{2m\overline{g}b},\ b = -\text{Im}(a)$$